Superfluid–insulator transition in dirty ultracold Fermi gas

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Superfluid–insulator transition in an ultracold Fermi gas in the external disorder potential of the amplitude V_0 is studied as a function of its concentration n and the magnetic field B in the presence of the Feshbach resonance. We find the zero temperature phase diagrams in the plane (B, n) at a given V_0 and in the plane (V_0, n) at a given B. We study the transition between cases of classical and quantum random potentials which can be implemented by tuning intensity of speckles. Our results for Bose–Einstein condensation side of the diagram are also valid for a Bose gas.

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Using the Feshbach resonance in the magnetic field B one can study lots of interesting physics in ultracold Fermi gases (see the recent review article [1] and references therein). In the vicinity of the Feshbach resonance $B = B_0$ the scattering length of two fermions typically changes as

$$a = a_0 \frac{\Delta B}{B_0 - B}, \qquad (1)$$

where we omitted the non-resonant term. As a result by the decreasing magnetic field the Fermi gas can be transformed from the phase of weakly attracting fermions (at $B > B_0$) to the phase of repelling each other compact composite bosons, dimers made of two fermions with opposite spins (at $B < B_0$). In a clean Fermi gas all mentioned above phases are superfluid. Far enough from the resonance at $B > B_0$ superfluidity is described by the Bardeen–Cooper–Schrieffer (BCS) theory, while on the other side, at $B < B_0$, the theory of Bose–Einstein condensation (BEC) of composite bosons works. Thus, reduction of magnetic field B leads the gas through the BCS–BEC crossover.

The aim of this paper is to consider both sides of BCS–BEC crossover in a „dirty" Fermi gas, i.e. in the gas situated in a three-dimensional (3D) random potential. Such a random potential can be created, for example, by superposing a 3D speckle pattern on the ultracold gas. Obviously, a strong enough disorder can localize the Fermi gas on BCS side and the BEC condensate on BEC side, destroying superfluidity in both cases. For brevity, we call the localized phase „insulator" and the localization transition „superfluid–insulator" (SI) transition. In this paper we are talking about SI transition in uniform infinite gas but our results can be applicable to experiments with traps.

Expansion of BEC condensate of ultracold Bose gases in the disorder potential of one-dimensional speckles has been recently studied experimentally [2–4]. It was found that the disorder stops expansion at some distance. In this case, however, a big role may be played by rare very high hills of the random potential. Apparently several laboratories are planning similar studies of SI transition in a potential created by 3D speckles. One can expect that in this case the rare high hills are less important and theory of SI transition in infinite system is relevant.

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In a Fermi gas in a fixed external random potential the SI transition can be driven by the decreasing concentration of fermions n at a given magnetic field B, or by the decreasing B at a given n. Therefore, one can think about the SI phase diagram of a Fermi gas in the plane (B, n). In this paper, we find the zero temperature SI border line $n(B)$ on such a phase diagram (see Fig. 1). Because B and n can be independently controlled this diagram can be verified experimentally. We characterize disorder by the amplitude of the random potential energy V_0 (mean square deviation of random potential $V(r)$ from average value) and the characteristic size of potential wells and hills R. In the first part of this paper we assume that both V_0 and R are so large that if m is the mass of the fermion

$$V_0 \gg \frac{\hbar^2}{mR^2}. \qquad (2)$$

This means that we are dealing with a classical random potential. We also assume that $n(B)$ is so large that the average number of atoms in well $nR^3 \gg 1$. Below we verify that near the SI border this inequality actually follows from (2).

Let us first consider BCS phase corresponding to $B > B_0$. Here the criterion of superfluidity coincides with the condition that the Fermi level of weakly interacting Fermi

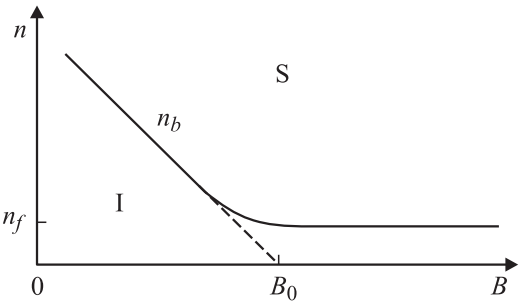

Figure 1. The phase diagram of the SI transition. Magnetic field B is plotted on the horizontal axis, while the fermion concentration n is plotted on the vertical one. S stands for superfluid and I for insulator. B_0 is the Feshbach resonance point. Critical concentrations n_f and $n_b(B)$ are straight lines given by Eqs. (3) and (10) respectively for the classical random potential, and by Eqs. (18) and (19) for the quantum one.

gas reaches the mobility edge in a given external potential. Roughly speaking, this happens when the Fermi energy of the gas $E_F = (\hbar^2/2m)(3\pi^2 n)^{2/3}$ becomes larger than the amplitude V_0 of the random potential. This condition leads to the critical concentration of SI transition $n = n_f$ on the side of free fermions ($B > B_0$)

$$n_f = C_f R^{-3} \left(\frac{V_0}{\hbar^2/mR^2} \right)^{3/2}. \quad (3)$$

Thus, the segment of the SI border at $B > B_0$ is horizontal as shown in Fig. 1. It is known that in a classical long range potential the coefficient C_f can be found using the idea that in the classical potential (2) the mobility edge coincides with the classical percolation level V_p [5,6]. This is the level at which Fermi gas lakes formed in the random potential wells merge to create the infinite cluster or the Fermi sea. In a generic three-dimensional gaussian potential with the distribution function

$$F(V) = \frac{1}{V_0\sqrt{2\pi}} \exp(-V^2/2V_0^2), \quad (4)$$

this level corresponds to occupation of $\theta_c = 17\%$ of the space by lakes [6,7]. This gives

$$V_p = -0.96V_0. \quad (5)$$

Now we can find n_f as the total concentration of fermions in wells deeper than V_p. Inequality (2) lets us use Thomas–Fermi (TF) approximation

$$n_f = \frac{1}{3\pi^2} \int_{-\infty}^{V_p} \left[\frac{2m(V_p - V)}{\hbar^2} \right]^{3/2} F(V) dV. \quad (6)$$

For a gaussian potential this leads to the coefficient in Eq. (3)

$$C_f = \frac{2}{3\pi^{5/2}} \int_{-\infty}^{V_p/V_0} [V_p/V_0 - x]^{3/2} \exp(-x^2/2) dx \approx 0.008. \quad (7)$$

Let us switch to the less trivial BEC side of the diagram which corresponds to $B < B_0$. In this case, interaction of dimers plays the crucial role. Following Ref. [1] we refer to the scattering length of the two dimers as a_{dd}. Then the uniform gas of interacting dimers has the positive chemical potential

$$\mu(n) = \frac{4\pi\hbar^2 (n/2) a_{dd}}{2m} = \frac{\pi\hbar^2 n a_{dd}}{m}. \quad (8)$$

Here we took into account that the concentration of dimers is $n/2$, while the dimer mass is $2m$. If $\mu(n)$ is larger than the amplitude of the random potential V_0, the gas of dimers can screen the random potential redistributing a small fraction of its density from the hills of the random potential to the wells. On the other hand, if $\mu(n) \ll V_0$ the gas is fragmented in many disconnected lakes. Thus, the condition of delocalization of dimers and, therefore, the condition of superfluidity in this case is roughly speaking $\mu(n) = V_0$. Substituting the nontrivial result of Ref. [8]

$$a_{dd} = 0.6a \quad (9)$$

into Eq. (8) and using Eq. (1) we get for the SI border concentration of fermions n_b on the dimer side ($B < B_0$)

$$n_b(B) = C_b R^{-3} \frac{R}{a} \frac{V_0}{\hbar^2/mR^2} = C_b R^{-3} \frac{R}{a_0} \frac{V_0}{(\hbar^2/mR^2)} \frac{B_0 - B}{\Delta B}. \quad (10)$$

In order to estimate the numerical coefficient C_b we consider the percolation level $2V_p$ in the potential energy of a dimer $2V(r)$. The local concentration $n(r)/2$ of dimers adjusts to external potential according to the Gross–Pitaevskii equation (GPE)

$$\mu\psi(r) = \left[-\frac{\hbar^2\nabla^2}{4m} + 2V(r) + \frac{4\pi\hbar^2 a_{dd}}{2m} |\psi(r)|^2 \right] \psi(r), \quad (11)$$

where μ is the condensate chemical potential, $2V(r)$ is the potential acting on a dimer, and the condensate wave function $\psi(r)$ is normalized to total number of dimers, $\int dr |\psi(r)|^2 = N/2$, where N is the total number of fermions. Thus, $|\psi(r)|^2$ has the meaning of the local concentration of dimers $n(r)/2$. Let us show that near the SI border one can use the TF approzimation and drop the kinetic energy term of GPE. This can be done if the healing length $l_h = [(n/2)a_{dd}]^{-1/2} \ll R$. Using the above estimate for the critical concentration n_b we get that for the classical disorder potential (Eq. (2)) at the BEC side SI border

$$\frac{l_h(n_b)}{R} = \left(\frac{\hbar^2/mR^2}{V_0} \right)^{1/2} \ll 1. \quad (12)$$

Thus, one can proceed in the TF approximation, where at every point local concentration of the condensate $n(r)$ satisfies equation

$$\frac{\pi\hbar^2 n a_{dd}}{m} + 2V(r) = \mu. \quad (13)$$

The chemical potential μ is determined by normalization of concentration of dimers $n(r)/2$ to the total number of dimers $N/2$ and grows with increasing N. If $\mu < 2V_p$ we get only disconnected Bose gas lakes. If $\mu > 2V_p$ the merging lakes form the Bose sea or the infinite cluster. Thus, similarly to the BCS side on the BES side the SI transition also happens when $\mu = 2V_p$. For a gaussian potential with the help of Eq. (9) this gives for C_b in Eq. (10)

$$C_b = \frac{\sqrt{2}}{0.6\pi^{3/2}} \int_{-\infty}^{V_p/V_0} [V_p/V_0 - x] \exp(-x^2/2) dx \approx 0.01. \quad (14)$$

For more realistic [9] distribution of the speckle potential we do not know the percolation threshold, but on the basis of approximate universality [6] of the θ_c we guess that C_b is the same as for guassian potential within 20%.

Thus, the SI border $n(B)$ consists of the two straight lines shown in Fig. 1. At $B < B_0$ it follows the line with the negative slope, Eq. (10), and at $B > B_0$ the border line is horizontal, Eq. (3). Eq. (10) is valid until $n_b(B) \gg n_f$. At $B = B_0 - \delta B$, where $\delta B = \Delta B[V_0/\hbar^2/ma_0^2]^{1/2} \ll B_0$ Eq. (10) crosses over to Eq. (3). In the interval of the width $2\delta B$ around $B = B_0$ interaction is strong, i. e. $n(B)|a|^3 \sim k_F|a| \sim l_h/|a| \sim 1$.

We can not calculate $n(B)$ in this crossover interval. In Fig. 1 instead of leaving the crossover interval blank we connect the two asymptotic straight lines by an arbitrary smooth monotonic crossover. In principle it is possible that the real phase diagram is non-monotonic and has minimum in the crossover range, similarly to the curves of Ref. [10]. It is, however, difficult to see physical grounds for such a behavior in the presence of a strong disorder.

The fact that the critical concentration n_b exceeds n_f is easy to understand. Indeed, at a given n dimers have much smaller chemical potential than weakly interacting fermions. Thus, dimers need a larger concentration n in order to get delocalized. In the similar way one can understand the growth of $n(B)$ with the decreasing B at $B < B_0$. The farther from the resonance, the more ideal the Bose gas of dimers is, the smaller is its chemical potential. Again, to compensate for this trend $n(B)$ should grow with the decreasing B.

Above we assumed that the number of particles in a well of the random potential is large, $nR^3 \gg 1$, and used the mean field approximation on the BEC side, ignoring discreetness of particles. As we see from Fig. 1 the minimum value of the border concentration $n(B)$ is n_f. Therefore, inequality $n_f R^3 \gg 1$ guarantees that everywhere on the border $n(B)R^3 \gg 1$. Substituting Eq. (3) into $n_f R^3 \gg 1$ we arrive at inequality (2). Thus, it is the single condition of validity of the above theory of the SI border.

It is clear from the above discussion that the insulating phase on the BEC side consists of disconnected lakes, populated by dimers. One can use the term Bose glass [11] for this phase, because it has no excitation gap.

Until now we assumed that the disorder is classical in the sense of inequality (2). Let us now discuss what happens for a quantum random potential, obeying the opposite strong inequality

$$V_0 \ll \frac{\hbar^2}{mR^2}. \tag{15}$$

In an experiment one can move from inequality (2) to inequality (15) by scaling down the intensity of the light beams creating speckles, while keeping the rest of the speckle set up (including R) fixed. How will then the phase diagram in (B, n) plane change?

Let us start this discussion from the BCS side of the diagram ($B > B_0$) and concentrate on the disorder induced density of states (DOS) at small energies. For simplicity, we assume that we are dealing with a gaussian potential $V(r)$ which two point correlation function decays as $1/r^3$ or faster at $r \gg R$. According to inequality (15) the wells of the size R do not have levels. In this case, the characteristic energy of the low energy tail of DOS is determined by wells of the size $L \gg R$, which are large enough to get a level [6,12,13]. The typical depth of such wells $V(L)$ is much smaller than V_0, namely $V(L) = V_0(R/L)^{3/2} \ll V_0$. This happens due to the cancellation of the majority of $(L/R)^3$ contributions of wells and hills of the size R. In the volume L the number of wells of the size R can typically be larger or smaller by $(L/R)^{3/2}$ than number of hills of this size. This is why we arrive at typical fluctuation of average potential $V(L)$.

Using condition of the level existence $V(L) = \hbar^2/mL^2$, we find the characteristic size of the well, which has a single level

$$L_c = R\left(\frac{\hbar^2}{mR^2V_0}\right)^2. \tag{16}$$

Substituting L_c into $V(L)$ we arrive at the characteristic energy scale of the low energy tail [6,12,13]

$$V_t = C_t V_0\left(\frac{V_0}{\hbar^2/mR^2}\right)^3. \tag{17}$$

The energy which separates localized and delocalized states, the mobility edge, is also of the order of V_t. For a quantum random potential (Eq. (15)) $V_t \ll V_0$ and the concentration of fermions which can be localized in the tails or, in other words, the critical concentration of SI transition, n_f, is very small, too

$$n_f \sim R^3\left(\frac{V_0}{\hbar^2/mR^2}\right)^6, \quad V_0 \ll \hbar^2/mR^2. \tag{18}$$

Let us switch now to the BEC side of the phase diagram ($B < B_0$). In this case, the tails of DOS can accommodate more dimers in the band of energies V_t because we can condense many bosons at one level. Only if the chemical potential of bosons given by Eq. (8) becomes larger than V_t the states become delocalized. Thus, n_b can be estimated equating μ and V_t. Using Eqs. (8), (9) and Eq. (17) we get for $V_0 \ll \hbar^2/mR^2$

$$n_b \sim R^{-3}\frac{R}{a}\left(\frac{V_0}{\hbar^2/mR^2}\right)^4 \sim R^{-3}\frac{R}{a_0}\left(\frac{V_0}{\hbar^2/mR^2}\right)^4\frac{B_0 - B}{\Delta B}. \tag{19}$$

This result can be also obtained from the condition [14] that the SI transition happens at $\mu\tau/\hbar \sim 1$, where τ is the relaxation time of a delocalized boson with the energy μ. In terms of radius of the cloud at a given number of atoms it was also obtained in Ref. [15].

In the language of GPE the estimates we arrived above correspond to the solution, where all three terms in the right side of Eq. (11) play comparable roles. In other words, expectations of the kinetic energy term, of the random potential term and of the repulsion energy are of the same order of magnitude at SI border. Note, that at the same time the amplitude V_0 of the bare potential is much larger than the other terms. Only the quantum mechanical averaging makes the disorder potential energy $V(L_c)$ equal to the other terms. For a Fermi gas, the idea of such averaging is known, for a long time [6,12,13]. For a weakly non-ideal Bose gas idea of quantum mechanical „smoothing“ of disorder potential was explored only recently [9,16]. However, SI phase diagram of an infinite, uniform in average gas could

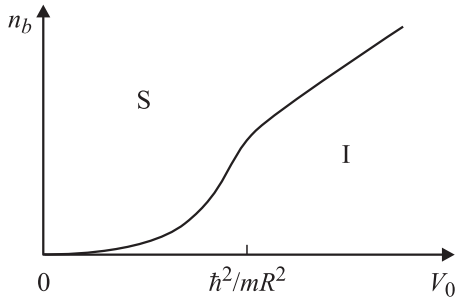

Figure 2. Schematic phase diagram of SI transition for a Fermi gas at $B < B_0$ or for a weakly non-ideal Bose gas. The critical concentration of SI transition n_b is plotted against the disorder amplitude V_0. S stands for superfluid and I for the insulator. The crossover between regimes of quantum and classical random potentials takes place at $V_0 = \hbar^2/mR^2$.

not be studied in Refs. [9,16] because they dealt with an one-dimensional disorder potential.

The crossover between Eqs. (18) and (19) happens in the interval of the width $2\delta B = \Delta B(a_0/R) \times [V_0/(\hbar^2/mR^2)]^2 \ll B_0$ around B_0. In this interval interaction is strong, $n|a|^3 \sim k_F|a| \sim l_h/|a| \sim L_c/|a| \sim 1$ and we again have no theory.

Let us discuss applicability of the mean field theory (GPE) for calculation of n_b. GPE is applicable if at $n = n_b$ the characteristic length L_c or if $n_b L_c^3 \gg 1$. It is clear that $n_b \gg n_f$. Multiplying this inequality by L_c^3 and using Eqs. (16) and (19) we arrive at necessary inequality $n_b L_c^3 \gg n_f L_c^3 \sim 1$. Thus, the mean field theory is applicable for calculation of n_b. (Mean field approach fails and one arrives at single-particle regime [9] only at $n \ll n_b$.)

We see from Eqs. (18) and (19) that in the case of a quantum random potential $V_0 \ll \hbar^2/mR^2$ both critical concentrations n_f and n_b decrease very rapidly with the decreasing V_0. As a result, while the whole phase diagram in this case still looks like Fig. 1, the concentrations n_f and n_b are dramatically smaller than for a classical random potential.

In Fig. 2 we summarize our results for the BEC phase again plotting the critical concentration of SI transition n_b as a function of the amplitude of the random potential V_0 (or the intensity of the speckle-building light), while scattering length a and the characteristic scale of disorder R are fixed. The fourth order parabola in the beginning of $n_b(V_0)$ curve is given by Eq. (19). At $V_0 = \hbar^2/mR^2$ this parabola crosses over to Eq. (10). Our results obtained for the BEC phase of dimers shown in Fig. 2 are clearly applicable to a generic weakly non-ideal Bose gas with the scattering amplitude $a > 0$.

Let us make some numerical estimates. We assume that characteristic size of the trap is $4 \cdot 10^4$ nm and the characteristic of the white spot in the speckle is $R = 210^3$ nm. Assuming that the average distance between atoms is 200 nm we arrive at 1000 atoms in one white spot. Thus, macroscopic approach of this paper is valid. In this conditions the trap size is of the order of $20R$. It is know from the percolation theory [6] that this guarantees that fluctuations of percolation threshold are less than 5%, so that our percolation approach is reasonable as well.

In conclusion I have studied the zero temperature phase diagram of the superfluid–insulator phase transition for a Fermi gas going through Feshbach resonance with the changing magnetic field and for a Bose gas. I dealt with uniform infinite gases, did not consider the role of the inverse parabolic profile $n(r)$ in the trap, and did not study dynamics of the BEC phase expansion when the trap is eliminated. A likely scenario of this expansion (similar to that of Refs. [14,17]) is as follows. If the maximum concentration of the gas in the center of the trap $n_m < n_b$ there is no expansion. On the other hand, if $n_m > n_b$ only bosons from the central domain $r < r_0$ where $n(r) > n_b$ leave reducing original $n(r)$ to the flat $n(r) = n_b$ at $r < r_0$.

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