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A software package for calculating the reachable plasma configuration in Globus-M2 tokamak

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The paper presents a software package for experiment planning on the Globus-M2 spherical tokamak, enabling the calculation of reachable plasma configurations while taking into account the electromagnetic system's limitations. The reachable region is calculated using a linear plasma model with variable parameters under specified constraints on the voltages and currents in the poloidal magnetic field coils. The algorithm uses experimental discharge data and provides an interactive interface for the sequential evaluation of the reachable plasma shape.

Keywords: tokamak, plasma shape, reachability area, linear model.

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The Globus-M2 tokamak [1] at the Ioffe Institute is a spherical tokamak with major radius $R = 0.36$ m, minor radius $a = 0.24$ m, and aspect ratio $A = 1.5$.

To create the required plasma shape, the experiment specifies the programs of currents in the poloidal field coils. Besides, it is necessary to consider that the power supply of electromagnetic system coils is provided by high-power sources. Therefore, it is necessary to establish a knowingly feasible scenario of plasma discharge with account of the restrictions on the voltages and currents in the tokamak coils. In this paper the plasma shape means a set of gaps between the plasma boundary and the limiter (Fig. 1).

Since the linear model reachability set under restricted controls in the finite interval of time is limited [2], to search for feasible configurations of the plasma shape, a software complex was developed [3]. The algorithm is based on the method [4] to find the feasible outputs of the linear plasma model in the tokamak with variable parameters [5] taking into account of the specified limitations.

The method [4] is based on using a matrix relation between inputs and outputs of the linear dynamic system at kth step. The matrix M_k relates the control signals in the current and previous time steps with the output signal in the current time step, and the matrix H_k relates the control signals in the current and previous time step with the output signal in the current and previous time steps. The matrix relation is written for the plasma models considered further as

$$y_k = M_k U_k,$$
$$Y_k = H_k U_k, \tag{1}$$

where y_k — column vector of outputs at kth step, u_k — column vector of inputs at kth step, $Y_k = [y_2^{\mathrm{T}}, \dots, y_k^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{33(k-1)}$ — combined vector of outputs, $U_k = [u_1^{\mathrm{T}}, u_2^{\mathrm{T}}, \dots, u_{k-1}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{6(k-1)}$ — combined vector of inputs. The absence of elements u_k and y_1 in the combined vectors is related to zero initial conditions and absence of feedthrough matrix D in the plasma model [5]. Restrictions on the inputs and outputs in every time steps are specified as intervals

$$u_{k,\min}^{(i)} \leqslant u_k^{(i)} \leqslant u_{k,\max}^{(i)},$$
$$y_{k,\min}^{(j)} \leqslant y_k^{(j)} \leqslant y_{k,\max}^{(j)}. \tag{2}$$

Hereinafter the upper indices indicate the vector element. Therefore, the problem of finding the reachability set (RS) for each component of the output vector reduces to finding the maximum and minimum under conditions (1) and (2):

$$\min\{y_k^{(j)} = M_k^{(j)} U_k\},$$
$$\max\{y_k^{(j)} = M_k^{(j)} U_k\}. \tag{3}$$

The experimental data of plasma discharges of Globus-M2 tokamak includes measurements of voltages and currents in the poloidal field coils, and also coordinates of the center of the plasma and gaps between the plasma boundary and the limiter with the discretization interval 100 μs reconstructed by magnetic measurements (Fig. 1, c). Reconstruction of the plasma was carried out with the use of the FCDI-IT algorithm [6]. The restrictions on these signals may be specified both for the deviations from the scenario and for the full signals (Fig. 2). Therefore, the limitations for the deviations at each step are defined as follows:

$$\begin{cases} y_{\min} \leqslant y_k + \delta y_k \leqslant y_{\max} \\ \delta y_{\min} \leqslant \delta y_k \leqslant \delta y_{\max} \end{cases}$$
$$\rightarrow \max(\delta y_{\min}, y_{\min} - y_k) \leqslant \delta y_k \leqslant \min(\delta y_{\max}, y_{\max} - y_k), \tag{4}$$

similarly for the input signals.

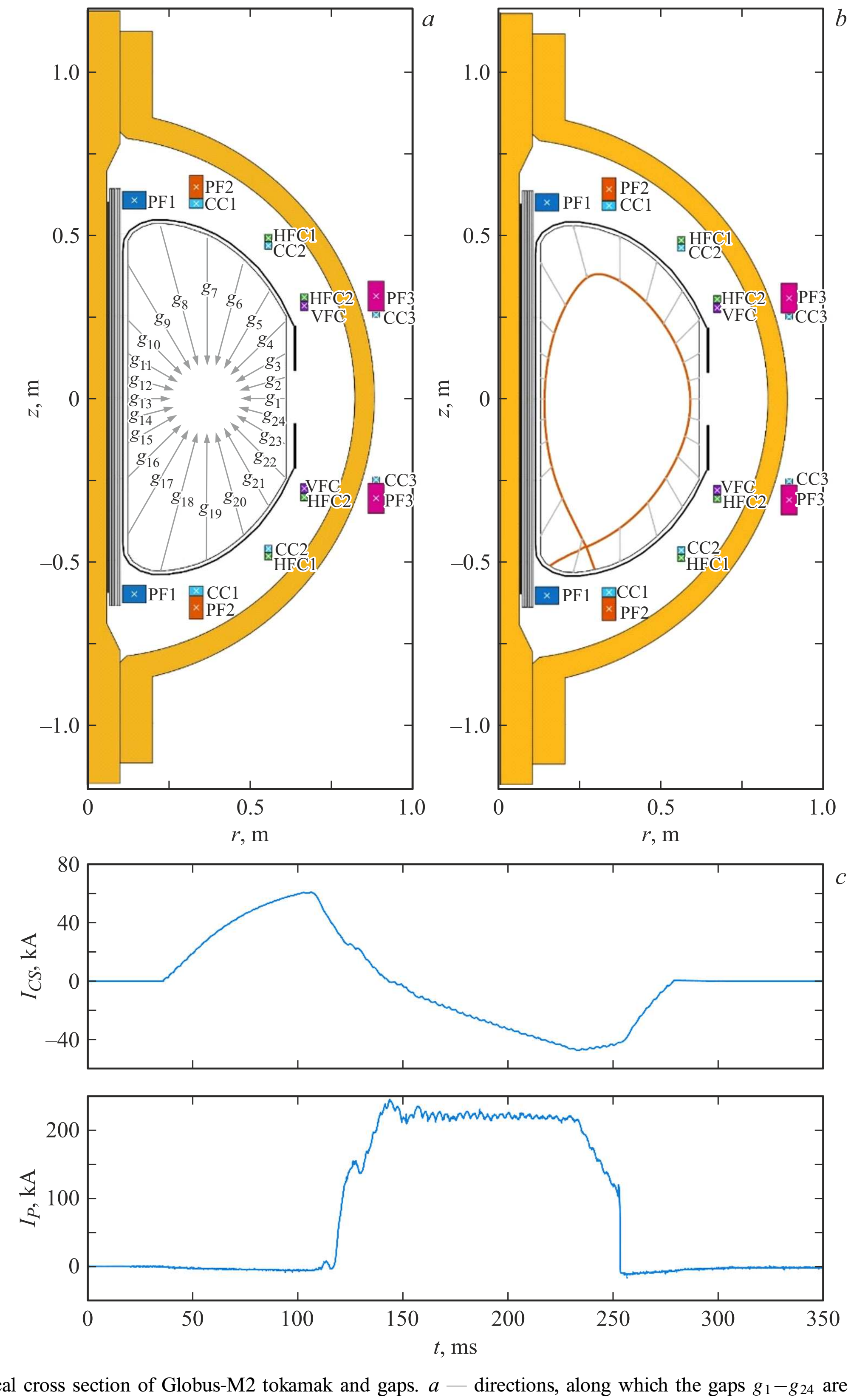

Figure 1. Vertical cross section of Globus-M2 tokamak and gaps. a — directions, along which the gaps g_1–g_{24} are measured between the plasma boundary and tokamak limiter; b — gaps between the limiter and plasma boundary for a certain configuration; c — current in the central solenoid and plasma current in discharge No 41805.

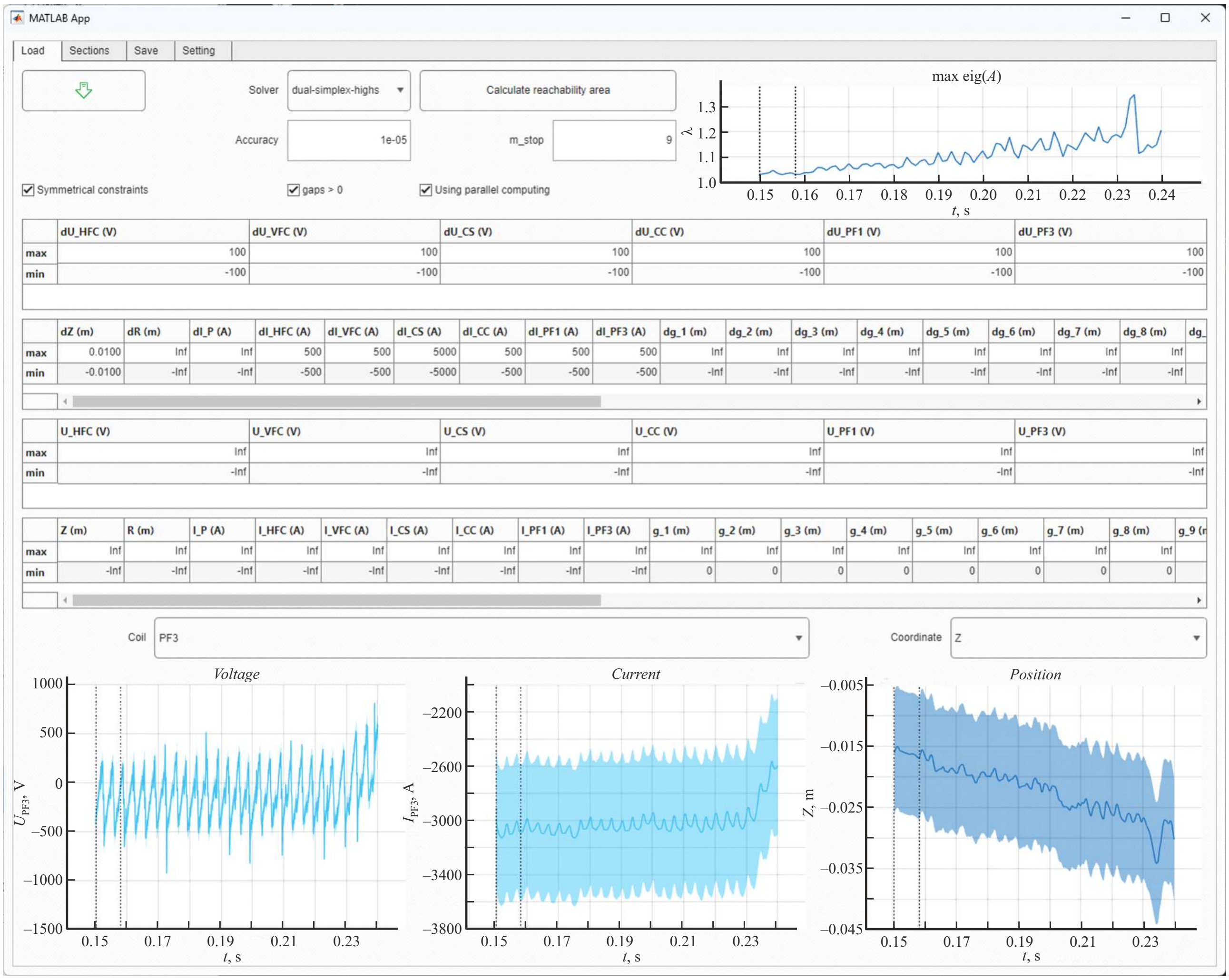

Figure 2. Application interface, tab „Load", discharge No 41805. A selection of restrictions is available for control and output signals, currents and voltages are shown in the coils, and the area of potential deviations in these signals.

Linear plasma models used in this paper were calculated for the equilibria reconstructed by the FCDI-IT algorithm [5,6] using the signals of magnetic diagnostics of Globus-M2 tokamak. The models were obtained according to the method described in [7], based on the Kirchhoff laws for the current circuits in the coils, vacuum vessel and plasma, and also the equation of the balance of forces acting at the plasma.

Let us describe in more detail the generation of models. The FCDI-IT algorithm reconstructed the distribution of plasma toroidal current density $J(r, z)$ with minimization of the residual between the modeled and measured values of poloidal flux in the magnetic loops and plasma current, and also the residual between the modeled distribution of current in the vessel and its estimate calculated using Faraday law [6]. The plasma is modeled by one chain with current density $J(r, z)$, the center of which is located in the point with coordinates (r_P, z_P). The Kirchhoff law is written as follows:

$$M_{PC}\frac{dI_C}{dt} + M_{PV}\frac{dI_V}{dt} + L_P\frac{dI_P}{dt} + R_P I_P + \frac{\partial\Psi_P}{\partial r_P}\frac{dr_P}{dt} + \frac{\partial\Psi_P}{\partial z_P}\frac{dz_P}{dt} = 0, \qquad (5)$$

where I_C — vector of tokamak coil currents, I_V — vector of tokamak chamber currents, $I_P = \int\int J(r, z)dz\,dr$ — plasma composite current, R_P — plasma resistance, L_P — plasma intrinsic inductance, Ψ_P — magnetic flux via plasma circuit, M_{PC} — inductance matrix between plasma and coils, M_{PV} — matrix of inductance between plasma and chamber.

Plasma resistance was estimated as $R_P = -\Delta\psi_a/\bar{I}_P\Delta t$, where $\Delta\psi_a$ — deviation of the magnetic poloidal flux in magnetic axis of plasma obtained from equilibrium reconstruction within the flat-top current Δt, $\bar{I}_P$ — averaged flat-top plasma current.

Linearized equations of the balance of forces are as follows

$$\frac{\partial F_r}{\partial r_P}\delta r_P + \frac{\partial F_r}{\partial z_P}\delta z_P + \frac{\partial F_r}{\partial I_C}\delta I_C + \frac{\partial F_r}{\partial I_V}\delta I_V + \frac{\partial F_r}{\partial I_P}\delta I_P = 0,$$

$$\frac{\partial F_z}{\partial r_P}\delta r_P + \frac{\partial F_z}{\partial z_P}\delta z_P + \frac{\partial F_z}{\partial I_C}\delta I_C + \frac{\partial F_z}{\partial I_V}\delta I_V + \frac{\partial F_z}{\partial I_P}\delta I_P = 0. \tag{6}$$

The vertical component of force F_z includes only Ampère forces acting on the plasma, and the horizontal one, except for the Ampère forces $F_{r,vac}$ also includes a hoop force $F_{r,hoop}$, which also depends on the plasma pressure. Since in the equilibrium the total force acting on the plasma is zero ($F_r = F_{r,vac} + F_{r,hoop} = 0$), then $F_{r,hoop} = -F_{r,vac}$.

The hoop force may also be approximated as [8]:

$$F_{r,hoop} = \frac{\mu_0 I_P^2}{2}\left(\ln\frac{8r_P}{a} + \beta_P + \frac{l_i}{2} - \frac{3}{2}\right). \tag{7}$$

According to this approximation, the hoop force does not depend on z_P, it depends logarithmically on r_P and quadratically on I_P, which makes it possible to estimate the derivatives included into the linearized equation

$$\frac{\partial}{\partial r_P}F_{r,hoop} = \frac{\mu_0 I_P^2}{2r_P}, \qquad \frac{\partial}{\partial z_P}F_{r,hoop} = 0,$$

$$\frac{\partial}{\partial I_P}F_{r,hoop} = \mu_0 I_P\left(\ln\frac{8r_P}{a} + \beta_P + \frac{l_i}{2} - \frac{3}{2}\right) = \frac{2F_{r,hoop}}{I_P} = -\frac{2F_{r,vac}}{I_P}. \tag{8}$$

Therefore, to account for the hoop force the equation of the balance of forces did not use the plasma pressure, but used only the Ampère forces calculated on the basis of the current coils measured in the experiment I_C and restored distributions of plasma current $J(r, z)$ and chamber currents I_V.

As a result, the equations were obtained, which are linearized for small deviations of currents from the reconstructed equilibrium. Besides, for the small deviations of currents and displacements of plasma the problem of finding the magnetic flux using available currents was solved, and the largest closed line of the flux level was found, and from its deviation the coefficients were found, which linked the plasma shape with the current deviations in the linear model. These coefficients together with the coefficients of linearized Kirchhoff equations and balance of forces define matrices A_k, B_k and C_k in the linear model. The state vector x has a physical sense of the vector of current deviations and includes six currents in the coils, ten current modes of the vacuum vessel and plasma current [9]. It should be noted that the presented program may operate with other linear models, for example obtained by linearization of the code that solves the equilibrium problem. In the form of state-space representation the plasma dynamics is written as follows [5]:

$$\begin{cases} x_{k+1} = A_k x_k + B_k \delta u_k, \\ \delta y_k = C_k x_k, \end{cases} \tag{9}$$

where $x \in \mathbb{R}^{17}$ — state vector, $\delta u = [\delta U_{\text{HFC}}, \delta U_{\text{VFC}}, \delta U_{\text{CS}}, \delta U_{\text{CC}}, \delta U_{\text{PF1}}, \delta U_{\text{PF3}}] \in \mathbb{R}^6$ — deviations of voltages from scenario values in the control coils, $\delta y = [\delta Z, \delta R, \delta I_P, \delta I_{\text{HFC}}, \delta I_{\text{VFC}}, \delta I_{\text{CS}}, \delta I_{\text{CC}}, \delta I_{\text{PF1}}, \delta I_{\text{PF3}}, \delta g_1, \ldots, \delta g_{24}] \in \mathbb{R}^{33}$ — deviations by position, plasma current, current in the coils and gaps. In this case, provided that the state vector is zero at the initial time, the matrices M_k, H_k for $k \geq 2$ are calculated recurrently using the auxiliary matrix Q [4]:

$$Q_2 = B_1, \quad Q_{k+1} = A_k Q_k,$$

$$M_k = C_k Q_k,$$

$$H_k = \begin{bmatrix} H_{k-1} & 0_{33(k-1)\times 6} \\ & M_k \end{bmatrix}, \tag{10}$$

where $M_k \in \mathbb{R}^{33\times 6(k-1)}$, $H_k \in \mathbb{R}^{33(k-1)\times 6(k-1)}$, $Q_k \in \mathbb{R}^{17\times 6(k-1)}$, $0_{33(k-1)\times 6}$ — zero matrix of the corresponding size. The restrictions on deviations in voltages in the control coils were set at $|\delta u| \leqslant 100\,\text{V}$, on poloidal currents — $|\delta I_{\text{PF}}| \leqslant 500\,\text{A}$,on the center solenoid current — $|\delta I_{\text{CS}}| \leqslant 5\,\text{kA}$, by vertical position — $|\delta Z| \leqslant 1\,\text{cm}$, the area of shape variations is restricted by the size of the vacuum vessel. The restrictions on the voltage determine the rate of current changes in the coils, which affects the characteristic time of plasma evolution. The result of solving the problem for the model of this shot is shown in Fig. 3. The additional result of the algorithm's output includes the deviated scenario signals that implement this configuration. The results of simulation using the found controls confirmed the feasibility of the calculated configurations (Fig. 4).

The user interface of application [3] for the definition of the reachable configurations of plasma shape consists of two main tabs: „Load" and „Sections"). The „Load" tab (Fig. 2) implements the option of loading the model of type (9), setting the restrictions on the signals according to (4) and viewing the plots of experimental signals. Besides, the selection of the algorithm and precision of solving the optimization problem (3) is available: simplex-method [10] or the internal point method [11]. It is possible to solve the problem (3) using the parallel computing for the simultaneous calculation of RS in different components of outputs $y^{(j)}$.

The tab „Sections" (Fig. 3 and 4) displays the vertical cross section of Globus-M2 tokamak, a plasma boundary is plotted (in the form of a 24-angle, dashed line) using scenario signals, and potential deviations are shown for each component. Therefore, the plasma configurations beyond the area limited with a solid line are non-feasible. The plot provides a view of experimental signals and

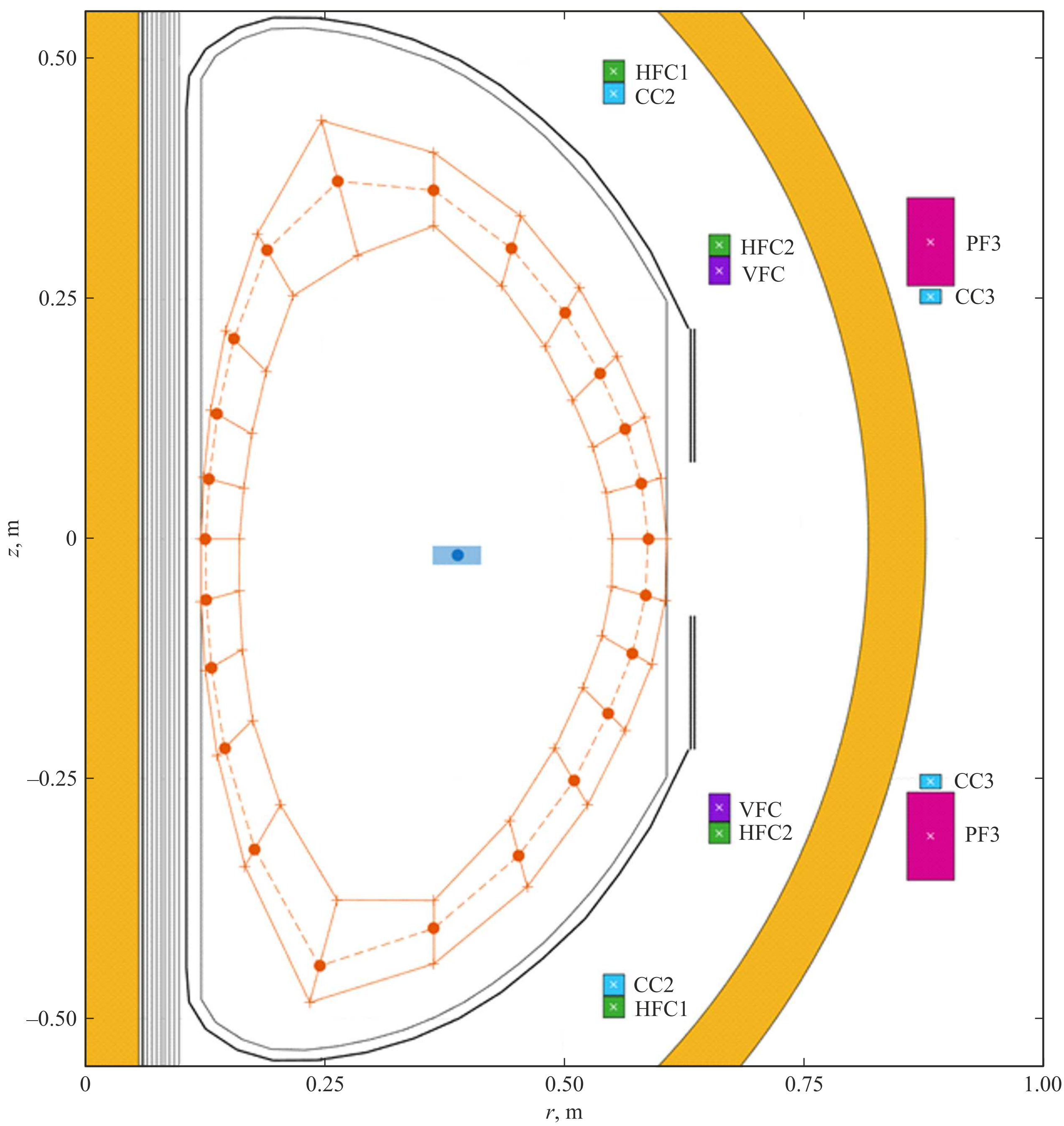

Figure 3. Application interface, element of tab „Sections“, discharge No 41805, $t = 158$ ms. The range of potential deviations of each component of the vertical (poloidal) section of plasma.

deviations from them, implementing one of the potential plasma configurations. In this tab the user selects one of the reachable values of the output signal, after which the algorithm of RS estimation is restarted with the updated restrictions (2), where one of the inequalities is replaced with equality. As a result, in several sequential steps the feasible plasma shape configuration is formed (Fig. 4). The presented software complex is planned for introduction in the Globus-M2 spherical tokamak.

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Conflict of interest

The authors declare that they have no conflict of interest.

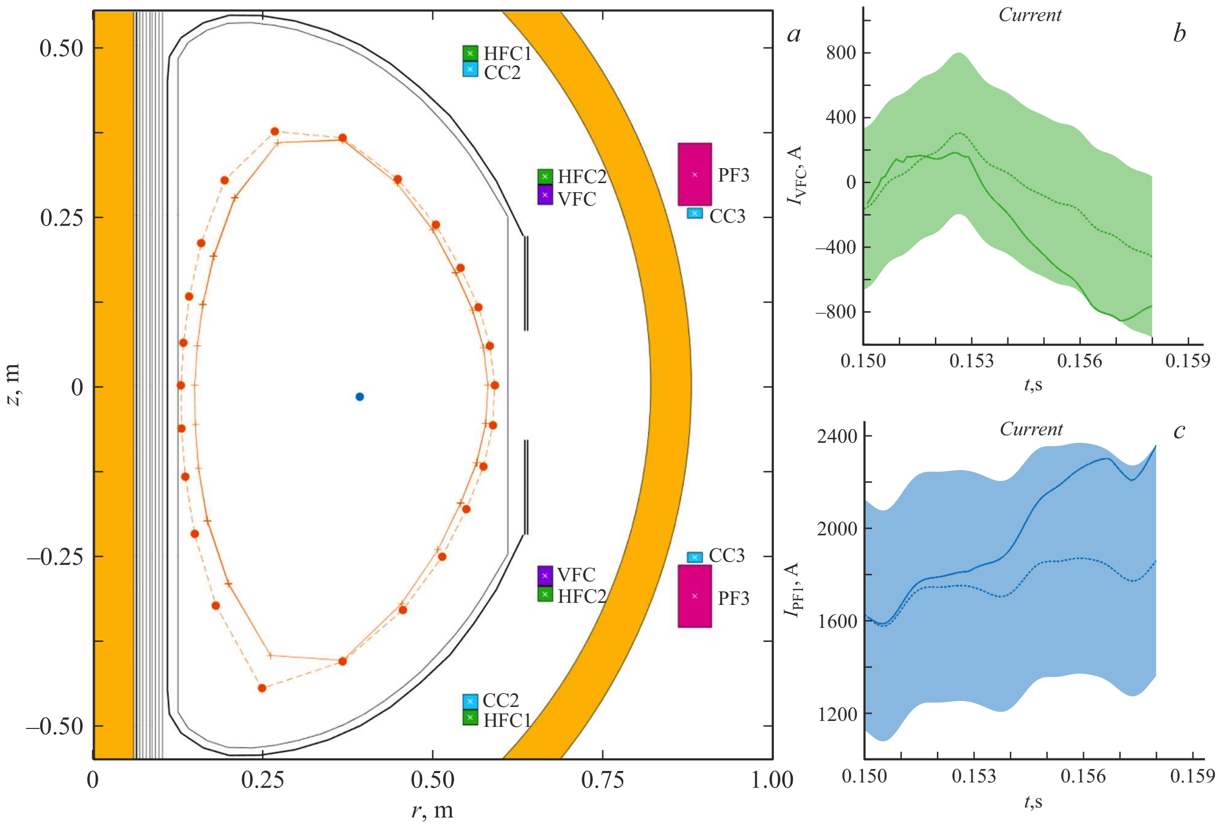

Figure 4. Application interface, elements of tab „Sections", discharge No 41805, $t = 158$ ms. a — potential configuration of vertical (poloidal) section of plasma found by sequential refinement of gaps. Current dynamics: b —in the vertical field coil (VFC), c — in the poloidal field (PF1) coil. Dotted line — experimental signal, solid line — deviated signal implementing this configuration, fill — area of permissible deviations.

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