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Study of phase singularities in optical vortices with high-order fractional topological charges in discrete photonic systems

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We demonstrate the stabilization of optical vortices with higher-order fractional topological charges in a discrete photonic system — a ring-shaped array of optical fibers. It is shown that the system supports stable supermodes with charges $p = 3/2$ and 5/2, which is confirmed by interference patterns and phase front analysis. The study of the phase profile evolution from the near-field zone ($R = 0.3r_0$) to the far-field zone ($R = 10r_0$) for the mode with $p = 3/2$ proves the preservation of its topological structure. Here, R is the radius of a circle centered on the fiber array center, and $r_0 = 100\,\mu$m is the characteristic distance scale. The obtained results open up prospects for light control in photonic devices leveraging the topological stabilization of complex light fields.

Keywords: optical vortex, higher-order fractional topological charges, ring arrays of optical fibers, interference, phase distribution.

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Modern optics demonstrates the stable interest in the study of singular optical fields, with optical vortices taking a special place among them — beams of light with a spiral wave front and phase singularity on the axis. The key characteristic of the optical vortices is a topological charge l, which defines the quantity of phase incursions 2π around the distribution axis and the value of the orbital angular moment [1–4]. In the recent years the significant attention of the researches has been drawn to the optical vortices with fractional topological charges (FTC), occupying an intermediate position between the integer vortices and plane waves [5–8].

Optical vortices with high-order fractional topological charges (HOFTC) are of special theoretical and practical interest, where $|l| > 1$. Such fields have some unique properties, including a complex multi-field structure, increased density of the orbital angular moment, and extended capabilities for coding information in the multiplexing systems with the orbital angular moment [9–11]. However, the fundamental limitation for their practical use is the exclusive instability when propagating in the continuous media, where they demonstrate quick disintegration into a set of single-charge vortices [12].

A promising area for solving the problem of HOFTC stabilization is using discrete photon systems, in particular, circular arrays of optical fibers [13–15]. Such systems have their dispersion properties, which may effectively suppress the dynamic instabilities specific for continuous media. Circular arrays of bound fibers with controlled anisotropy are of special interest, since they make it possible to form topologically stable supermodes with the specified characteristics.

Therefore, the key scientific problem solved in this paper is searching for the conditions and demonstration of the potential existence of topologically stable (not disintegrating when propagating) stationary modes with high-order fractional topological charges in a discrete system with the specified geometry of anisotropy. For this purpose, a detailed model is presented further, and the stability criteria are specified.

Let us consider a circular array made of N anisotropic optical fibers located in the vertices of the regular N-angle with radius of R_0. Optical properties of each fiber are described with a tensor of dielectric permittivity with the main values ε_e and ε_o, corresponding to the extraordinary and ordinary axes. The model is built on some key assumptions. First, the connection between the fibers is deemed to be weak ($k \ll \beta$, where k — connection coefficient that specifies the amplitude of interaction, and β — is the constant for propagation of the main mode in the isolated fiber), which makes it possible to take into account the interaction only between the nearest fibers. Second, low internal anisotropy of fiber materials is assumed. This condition is written as $\Delta\eta = \eta_e - \eta_o \ll \eta_{avg}$, where η_e and η_o — effective refractive indices of fundamental modes polarized in parallel to the local extraordinary and ordinary axes of anisotropy of each fiber, accordingly, and $\eta_{avg} = (\eta_e + \eta_o)/2$ is their mean average. Besides, the fibers are deemed to be homogeneous along their length, which simplifies the analysis of mode propagation in the system.

The dynamics of the optical field in the considered array of optical fibers is described with a discrete non-linear Schrödinger's equation such as

$$i\left(\frac{d\mathbf{E}_n}{dz}\right) + \Sigma_m \mathbf{C}_{nm}\mathbf{E}_m + \gamma|\mathbf{E}_n|^2\mathbf{E}_n + \Delta\beta_n\mathbf{E}_n = 0, \qquad (1)$$

where $\mathbf{E}_n$ is a complex amplitude of the field in n-th fiber, $\mathbf{C}_{nm}$ is a coupling matrix that determines interaction between the fibers, γ is a non-linear Kerr coefficient taking into account the non-linear effects, and $\Delta\beta_n$ describes anisotropic disturbance introduced by the features of dielectric permittivity of each fiber. Orientation of the anisotropy director in nth fiber is determined by angle

$$\theta_j = 2\pi p\, n/N + \theta_0, \tag{2}$$

where p is a semi-integral parameter of anisotropy helicity, which in stable modes corresponds to the effective topological charge of the supermode with the index $m = 0$, and θ_0 specifies the initial phase of the entire configuration. This helical structure of anisotropy directly influences the interaction between the fibers.

For anisotropic fibers the coupling matrix $\mathbf{C}_{nm}$, describing this interaction, is

$$\mathbf{C}_{nm} = k\left[\cos^2(\theta_n - \theta_m) + \delta\sin^2(\theta_n - \theta_m)\right], \tag{3}$$

where k — coupling amplitude, δ — dimensionless parameter of coupling anisotropy, depending on the relative orientation of adjacent fiber directors. As it was noted above, β is a constant of main mode propagation in the isolated fiber, and the loose coupling condition is met at $k/\beta \approx 0.01$.

Internal modes of the system (so called supermodes) are found from the solution of the stationary equation

$$\beta\Psi_n = \Sigma_m \mathbf{C}_{nm}\Psi_m + \Delta\beta_n\Psi_n. \tag{4}$$

For a configuration with chiral anisotropy the solutions demonstrate the structure of type

$$\Psi_n^{(m)} = A_m \exp(i2\pi m\, n/N) f(\theta_n), \tag{5}$$

where m is an integer index of the mode, and function $f(\theta_n)$ describes the modulation of the field amplitude in nth fiber, provided for by the local orientation of anisotropy. For the weak anisotropy model $(\Delta\eta \ll \eta_0)$ its appearance may be approximated as $f(\theta_n) \approx 1 + \varepsilon\cos(2\theta_n + \xi)$, where $\varepsilon \ll 1$, and ξ is the phase defined by polarization. This is exactly the function that jointly with a phase multiplier shapes the profile of the complex amplitude of supermode $\Psi_n^{(m)}$. The effective topological charge l_{eff} of such supermode determined by the expression

$$l_{eff} = \left(\frac{1}{2\pi}\right)\oint L\boldsymbol{\nabla}\varphi(r)\cdot d\mathbf{l} = p + m\Delta\theta/2\pi, \tag{6}$$

where $\varphi(r)$ — field phase in point r, integration is carried out in the closed circuit L, covering the axis of the system, $d\mathbf{l}$ — element of this circuit length, and $\Delta\theta$ is the full rotation of anisotropy along the circuit, consists of the contribution of helical configuration of anisotropy (p) and phase incursion related to the mode index (m).

Stable existence of HOFTC in the system requires meeting three key conditions. First, the quantization condition must be met, according to which the parameter of anisotropy p is a semi-integral number, i. e.

$$p = (2s + 1)/2, \quad s \in \mathbf{Z}. \tag{7}$$

Second, the condition of phase synchronization is necessary

$$\beta(l_{eff}) = \beta(l_{eff} + 1), \tag{8}$$

providing for the agreement of mode propagation. Third, the stability condition must be met

$$\partial^2\beta/\partial l^2\big|_{\{l=l_{eff}\}} > 0, \tag{9}$$

responsible for the mode stability in respect to small disturbances. Accounting for non-linear effects provided for by the dependence of the refractive index on intensity causes modification of the effective topological charge that takes the form

$$l_{eff}^{nonlin} = l_{eff} + \Delta l(I). \tag{10}$$

Correction Δl to the value of the linear effective charge l_{eff} is related to the non-linear distortion of the mode profile, I is the optical field intensity. The correction value is determined from the solution of the non-linear equation

$$\Delta l = (\gamma I/2k)\partial|\Psi|^2/\partial l, \tag{11}$$

which demonstrates the ability to manage the topological properties of the system by changing the light intensity.

To demonstrate that the fields $\Psi_n^{(m)}$ contain FTC, let us consider the behavior of the $\Psi_n^{(m=0)}$ mode phase in R radius circuit covering the array of fibers. In the case of configuration with $p = 3/2$ when bypassing along the closed circuit covering the array, the field phase demonstrates a specific pattern that differs from the case of $p = 1/2$.

In Fig. 1 the distribution of the interference of the studied supermode with the reference plane (a) and spherical (b) waves and phase (c) demonstrates a specific three-beam vortex structure formed by three radial breaks (forks). The number of such forks $(N_v = 3)$ is the direct visual indicator of the fractional topological charge $p = N_v/2 = 3/2$. Interference with the spherical wave (Fig. 1, b) identifies the triple of singularities (optical vortices), arranged along the ring, which also unambiguously indicates the 3/2 charge. Invariance of the pattern along axis z for the interference with the plane wave follows from the stationary nature of the supermode (Fig. 1, a). It is important to note that in case of interference with the spherical wave (Fig. 1, b) the pattern is influenced by Gouy phase shift, which the spherical wave acquires when converging to the focus. However, this shift is global (identical in the entire wave front) and causes not the change in the pattern structure, but to its constant shift by π when transitioning through the focal plane, which does not influence the observed quantity and location of singularities — key signs of the topological charge.

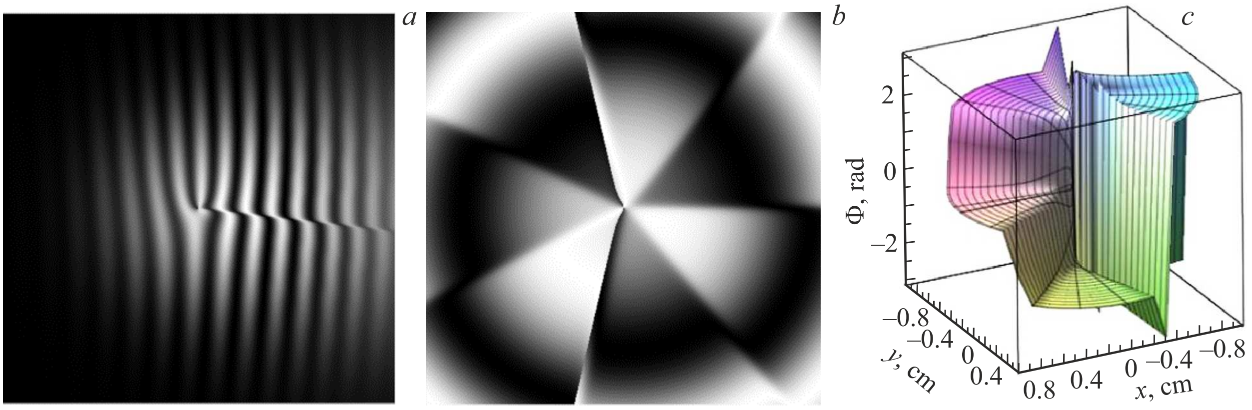

Figure 1. Interference with plane (a) and spherical (b) waves and phase (c) of optical vortex with fractional topological charge $p = 3/2$ in circular array of fibers $N = 7$, $\rho = 30\,\mu$m (core radius), $r_0 = 100\,\mu$m (specific scale of the distance).

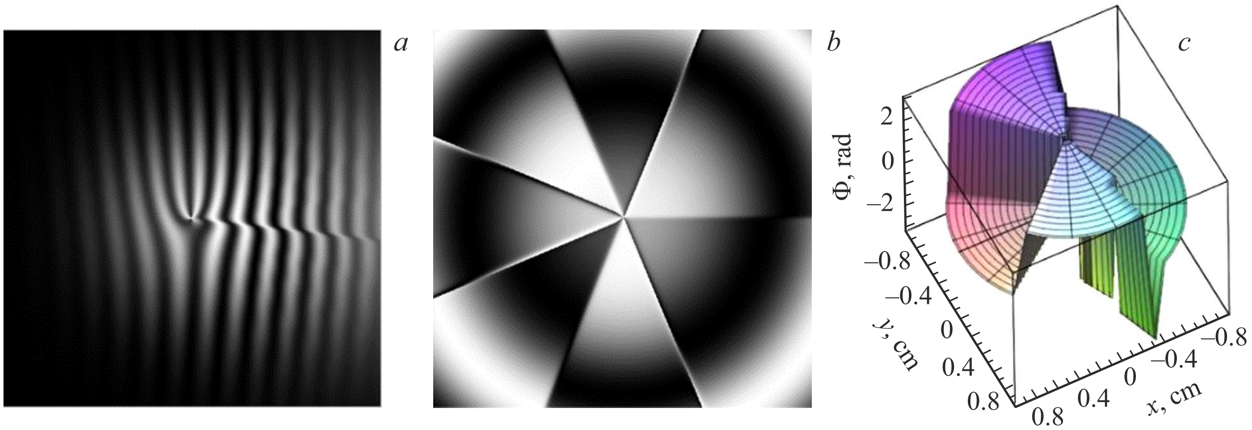

Figure 2. Interference with plane (a) and spherical (b) waves and phase (c) of optical vortex with fractional topological charge $p = 5/2$ in circular array of fibers $N = 7$, $\rho = 30\,\mu$m (core radius), $r_0 = 100\,\mu$m (specific scale of the distance).

The effective topological charge of such supermode according to the model is determined as $l_{eff} = p + m = 3/2 + m$, where m is the integer transverse mode index. The conditions for stable existence of the modes with FTC, including conditions of $p = 3/2$ quantization, phase synchronization and stability, remain fulfilled, providing for the ability to observe this state in the system.

For visual confirmation of FTC presence in stationary supermodes $\Psi_n^{(m)}$ with topological parameter $p = 5/2$, the distribution of interference (Fig. 2) with plane (a) and spherical (b) waves and phase (c) was analyzed. This interferogram is stationary and invariant in respect to the longitudinal coordinate z, and its structure differs from the case of $p = 3/2$ by complication of the topological configuration, which demonstrates the scalability of the approach and opens the prospects for control of multicharge topological states in non-linear modes.

It is shown that as radius R increases, the phase distribution acquires a pronounced three-step structure (Fig. 3). Three drastic phase jumps are observed with the value of π each, separated with the sections of continuous linear phase change. The total growth of the phase is 3π. Therefore, this phase behavior qualitatively corresponds to the field with topological singularity.

The paper theoretically and numerically studied the possibility to stabilize the optical vortices with high-order fractional topological charges ($p = 3/2$ and 5/2) in the discrete photon system — circular array of anisotropic optic fibers. Three key conditions for existence of such stable states have been formulated and justified: quantization of helicity parameter, phase synchronization and positivity of the second derivative of the dispersion ratio. The methods of computer modeling were used to demonstrate formation of the corresponding supermodes. The stationary nature and topological structure of modes are visually confirmed by the analysis of interferograms with the plane and spherical waves and the study of the evolution of the phase profile from the short-range to the long-range area that demonstrated preservation of phase singularities. The

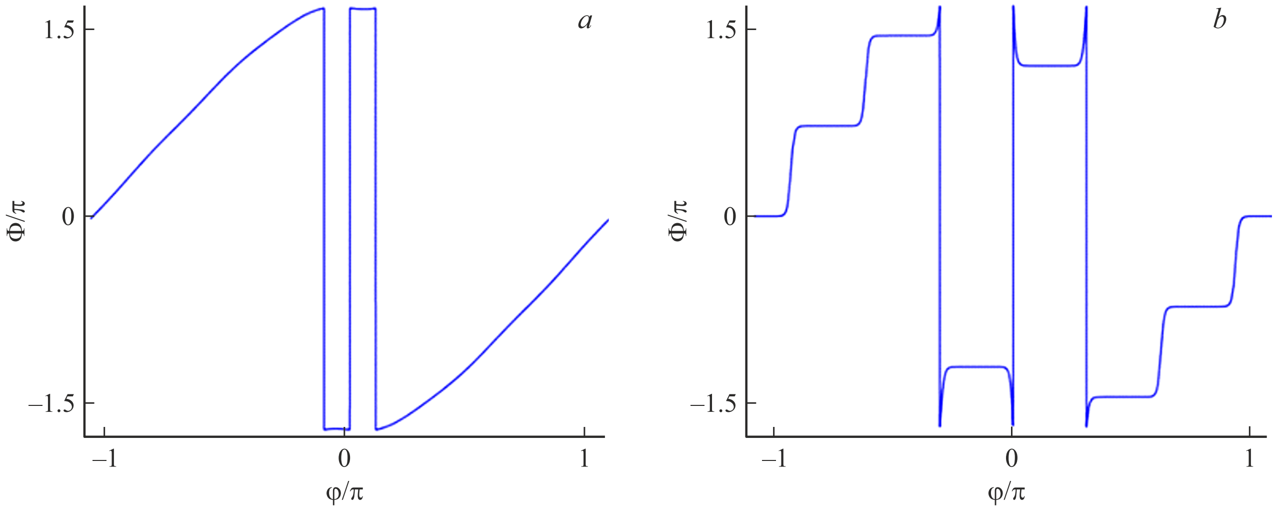

Figure 3. Dependence of phase of Φ field of FTC supermode $p = 3/2$ on azimuthal coordinate φ, the center of which matches the center of the entire array of optical fibers in the circuit $r = R$, at $R = 0.3r_0$ (a) and $R = 10r_0$ (b), $N = 7$, $\rho = 30\,\mu$m (core radius), $r_0 = 100\,\mu$m (specific distance scale).

obtained results open the path to development of compact photon devices using topologically protected complex light fields to process and transmit information.

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Conflict of interest

The authors declare that they have no conflict of interest.

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