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Normal resistance of $SN-N-NS$ Josephson structures

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The normal resistance of $SN-N-NS$ Josephson contacts has been calculated in the case of setting a measurement current in the N film of the structure. Expressions have been obtained for the characteristic lengths of current redistribution in the SN electrodes at temperatures greater than and less than the critical temperature of the structure's transition to the superconducting state. A method for experimentally determining these lengths has been proposed. It is based on measuring the dependence of the contact's normal resistance on the width of its S electrode.

Keywords: $SN-N-NS$ Josephson junction, normal resistance, conversion lengths.

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Superconducting electronics road map [1] emphasizes that „variable thickness bridge" type $SN-N-NS$ Josephson structures [2,3] are a match for $SNIS$ tunnel contacts that are widely used in modern low-current superconducting electronics. Such structures (see Figure, a) consist of superconducting (S) electrodes with a critical superconducting transition temperature T_C in contact with a connecting thin and narrow bridge film made of normal (N) metal.

Calculation of normal resistance of such structures essentially depends on the method used to set current in them and on the arrangement of electrodes detecting the measured difference of potentials. The calculation is substantially hindered by the existence of finite transparency of their SN-interfaces [4–6] and possible modification of transport and superconducting properties in some region of the S electrodes adjoining the N film. In this C region (see Figure, b) having the effective thickness d_c, both conductivity σ_c and critical temperature T_C^* can differ substantially from the conductivity and T_C of the σ_s S material.

The objective of this study is to calculate the normal resistance of the R_n $SN-N-NS$ bridge structure for various temperature intervals in case when the bias current I_b is injected into the structure through one of its outer $N-NS$ interfaces.

Suppose the SN electrode spacing is equal to L, the length of overlap region between the N and S parts is equal to W, the width of the N film is w, and the S and N materials satisfy the diffusive-limit conditions. Then I_b injected into the N film of the composite SN electrode is able to spread both through a part of the S film covering the N layer and through the electrode ends, flowing deep into the film through the end interface with the N film. In the latter case, the current spread region size is on the order of $W \gg w$, d, where d is the S film thickness. Therefore, for resistance calculation, we limit ourselves to only current exchange between the end areas of the SN interface, taking the S film and N film thicknesses equal (see Figure, b).

The $0z$ axis is placed perpendicular to the end of the SN interface, and the $0x$ axis is placed along the N film, and the origin of coordinates is aligned with the center of the N film on one of the outer $N-SN$ interfaces of the structure (see the figure). R_n can be measured by applying the measurement current I_b to the contact N film. Difference of potentials is measured between the SN electrodes (points *2*, *3* in the figure) and points *1* ($x = 0$) and *4* ($x = L + 2W$), respectively.

Due to the existing symmetry of the problem, R_n is sum of the SN electrode resistance R_{ns} and bridge film resistance $R_M = L/wd\sigma_n$, where σ_n is the conductivity of the N material. To calculate R_{ns}, the Laplace equation shall be solved

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial z^2} = 0 \qquad (1)$$

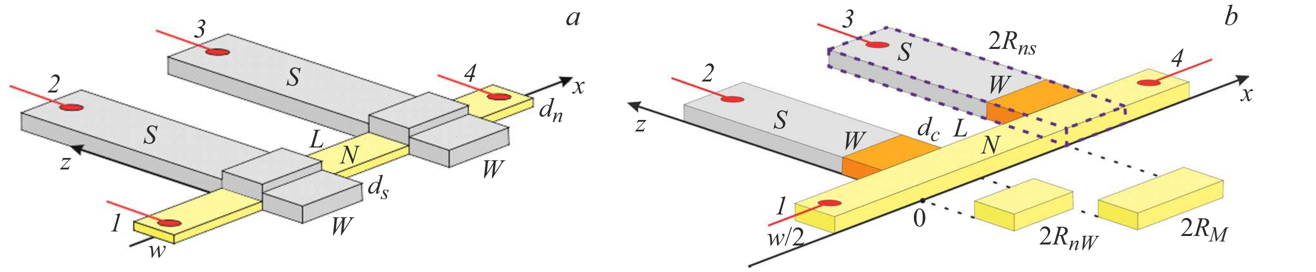
for the scalar potential $U(x, z)$ in one of the SN electrodes. Symmetry of the problem about the $0x$ axis allows us to limit ourselves to search for solution within $0 \leqslant x \leqslant W$, $z \geqslant 0$. Equation (1) shall be supplemented by the following boundary conditions:

$$\frac{\partial U}{\partial z} = 0, \quad z = 0, \quad z \to \infty, \quad 0 \leqslant x \leqslant W, \qquad (2)$$

$$U(x, d_c + w/2 + 0) = U(x, d_c + w/2 - 0),$$
$$z = d_c + w/2, \qquad 0 \leqslant x \leqslant W, \qquad (3)$$

$$\sigma_s \frac{\partial}{\partial z} U(x, d_c + w/2 + 0) = \sigma_c \frac{\partial}{\partial z} U(x, d_c + w/2 - 0),$$
$$z = d_c + w/2, \qquad 0 \leqslant x \leqslant W, \qquad (4)$$

$$\frac{\partial U}{\partial x} = \frac{I_b}{wd\sigma_n}, \qquad x = 0, W, \qquad 0 \leqslant z \leqslant \frac{w}{2}. \qquad (5)$$



a – schematic drawing of a typical *SN–N–NS* structure fabricated via electron-beam lithography techniques; *b* — schematic drawing of the *SCN–N–NCS* structure for description of physical electron transport processes. Normal metal strip *N*, superconducting electrode *S* and suppressed superconductivity region *C* are shown in yellow, gray and orange, respectively. I_b is applied to the *N* film of the structure, voltage is measured between points *1* and *4* or between the *S* electrodes (points *2*, *3*). For comparison, Figure *b* shows components having the normalizing resistances $2R_{nW}$ and $2R_M$, an electrode region with effective resistance $2R_{ns}$ is shown dashed. A color version of the figure is provided in the online version of the paper.

Boundary conditions at the *SN* interface $z = w/2$ and $0 \leqslant x \leqslant W$ follow from the equality of currents flowing through the interface

$$\sigma_n \frac{\partial}{\partial z} U\left(x, \frac{w-0}{2}\right) = \frac{U(x, w/2+0) - U(x, w/2-0)}{R_b},$$

$$\sigma_n \frac{\partial}{\partial z} U\left(x, \frac{w-0}{2}\right) = \sigma_c \frac{\partial}{\partial z} U\left(x, \frac{w+0}{2}\right). \tag{6}$$

Here, R_b is the resistivity of the *SN* interface.

It is convenient to seek for solution of boundary value problem (1)–(6) as stated above in the form of the Fourier series

$$U(x, z) = \sum_{n=0}^{\infty} U_n(z) \cos \frac{\pi n (x - W)}{W},$$

$$U_n(z) = \frac{2}{W} \int_0^W U(x, z) \cos \frac{\pi n (x - W)}{W} dx, \tag{7}$$

coefficients $U_n(z)$ of which satisfy

$$\frac{\partial^2}{\partial z^2} U_n(z) - \left(\frac{\pi n}{W}\right)^2 U_n(z) = \frac{2}{W} F. \tag{8}$$

The right-hand side of equation (8) $F = 0$ with $z > w/2$ and $F = \big((-1)^n - 1\big) I_b / \sigma_n d w$ within $0 \leqslant z \leqslant w/2$.

Solution of boundary value problem (1)–(6) can be written as

$$\frac{U_n(z)}{I_b R_{nW}} = \frac{w^2}{W^2 \theta_n^2} \left[1 - \frac{\sigma_c p_n}{g_n p_n + \sigma_n p q_n \sinh\theta_n}\right] \cosh \frac{2\theta_n z}{w},$$

$$0 \leqslant z \leqslant \frac{w}{2}, \tag{9}$$

$$\frac{U_n(z)}{I_b R_{nW}} = C_n \left[\cosh \frac{2\theta_n (z - d_c - w/2)}{w} - \frac{\sigma_s}{\sigma_c} \sinh \frac{2\theta_n (z - d_c - w/2)}{w}\right],$$

$$\frac{w}{2} \leqslant z \leqslant \frac{w}{2} + d_c, \tag{10}$$

$$\frac{U_n(z)}{I_b R_{nW}} = \frac{w^2}{W^2 \theta_n^2} \frac{\sigma_c \sinh\theta_n}{g_n p_n + \sigma_n p q_n \sinh\theta_n} \exp\left\{\frac{2\theta_n (w/2 + d_c - z)}{w}\right\},$$

$$z \geqslant \frac{w}{2} + d_c, \tag{11}$$

where the following notations are introduced:

$$p_n = \sigma_s \cosh \frac{2\theta_n d_c}{w} + \sigma_c \sinh \frac{2\theta_n d_c}{w},$$

$$q_n = \sigma_s \sinh \frac{2\theta_n d_c}{w} + \sigma_c \cosh \frac{2\theta_n d_c}{w}, \quad \theta_n = \frac{w\pi(2n+1)}{2W},$$

$$g_n = R_b \sigma_c \sigma_n \frac{2\theta_n}{w} \sinh\theta_n + \sigma_c \cosh\theta_n,$$

$$C_n = \frac{w^2}{W^2 \theta_n^2} \frac{\sigma_n \sigma_c p_n}{g_n p_n + \sigma_n p q_n \sinh\theta_n}, \quad R_{nW} = \frac{W}{\sigma_n d w}.$$

From solution of (11), it follows that the perpendicular cross-section potential of the *S* electrode far from the *N* bridge is constant $U(x, \infty) = U(W/2, 0)$. Therefore, a voltmeter connected between terminals *2* and *3*, besides the potential drop in the region between the electrodes will measure a doubled difference of potentials between the positions $x = W/2$ and $x = 0$.

In the experimentally implemented *SN–N–NS* structures, the width of the *SN* electrodes *W* is generally much greater than the width of the bridge's *N* film *w*. Taking into account this condition and the fact that the sums that define $U(x, z)$ in (7) converge at $n \ll w/W$, we get that $U(x, z)$ within $0 \leqslant z \leqslant w/2$ actually doesn't depend on the *z* coordinate. As a result, when the voltmeter is connected between points *1* and *4* , for the *SN* electrode resistance R_{ns} at temperatures above the critical temperature $(T > T_C)$ we get

$$\frac{R_{ns}}{R_{nW}} = \frac{U(W) - U(0)}{I_b R_{nW}} = 2 \frac{U(W)}{I_b R_{nW}}$$

$$= \sum_{n=0}^{\infty} \frac{2w^2}{W^2 \theta_n^2} \left[1 - \frac{\sigma_c p_n}{g_n p_n + q_n \sigma_n \sinh\theta_n}\right]. \tag{12}$$

Summation of the first of summands in (12) gives unity, i. e. leads to $R_{ns} = R_{nW}$. The second summand in (12) is the spreading resistance, shunting R_{nW}. It can be easily seen that the voltmeter connected between S films *2* and *3* will display the voltage drop, contribution of the SN electrodes to which will be twice as low as $U(W) - U(0)$ included in (12).

In the neighborhood of the critical temperature ($T \lesssim T_C$), the S electrodes start switching to the superconducting state. Assuming that in this case $\sigma_s \gg \sigma_c$, and considering that generally $W \gg d_c$, from (12) we have

$$\frac{R_{ns}}{R_{nW}} = \frac{2\xi}{W}\tanh\frac{W}{2\xi}, \quad \xi = \sqrt{\frac{w\sigma_n}{2}R_{bef}}, \quad R_{bef} = R_b + \frac{d_c}{\sigma_c}, \tag{13}$$

where ξ is the typical scale of current flow from the N part to the S part of the composite SN electrode. It follows from (13) that the region of S electrodes with modified transport properties existing at $w/2 \leqslant z \leqslant w/2 + d_c$ can be treated as an increase in the SN-interface resistivity R_b to R_{bef}.

As the temperature further decreases, the Josephson contact completely switches to the superconducting state. Then R_n can be measured experimentally only in the region of bias currents considerably exceeding the contact's critical current I_C, i. e. in that region of currents and voltages where the current-voltage curve of the structure reaches Ohm's law. In this case, the current in the N film is purely quasiparticle. Injection of the current into the effective SN electrode leads to three simultaneous processes: redistribution of the normal current between the S and N parts of the SN electrodes, normal to supercurrent conversion in the S film [7–9] and redistribution of the superconducting component of the total current between the S and N parts of the electrodes. Whereas due to conversion processes, the effective resistance of the interface depends on temperature as $(1 - T/T_C)^{-\alpha/2}$, where α is of the order of unity [10]. Following the phenomenological model proposed in [11,12], we also chose to introduce a single typical scale of normal to superconducting current transformation λ and to describe this process in the model of two interacting current flowing channels. The first of them is fully superconducting and has zero resistivity, and the potential in this line corresponds to the electrochemical potential of the Cooper pairs. Quasi-particle current I_N flows via the second channel. It is characterized by resistance per unit length of the N film in the normal state $r = R_{nW}/W$. Conversion of I_N to I_S in the S film is described by the specific conductivity G, and the conversion process itself is described by the telegrapher's equations

$$\frac{d^2}{dx^2}I_N - \frac{1}{\lambda^2}I_N = 0, \quad \frac{d^2}{dx^2}I_S + \frac{1}{\lambda^2}I_N = 0, \quad \lambda^2 = \frac{1}{rG}. \tag{14}$$

In the given case, $I_N(0)$ and $I_N(W)$ applied to the SN electrode are equal to I_b. Solution of equations (14) with such boundary conditions is defined by the following expressions:

$$I_N(x) = \frac{I_b}{\cosh(W/2\lambda)}\cosh\left(\frac{x - W/2}{\lambda}\right),$$

$$U(x) = \frac{1}{G}\frac{dI_N}{dx} = \frac{I_b r\lambda}{\cosh(W/2\lambda)}\sinh\left(\frac{x - W/2}{\lambda}\right), \tag{15}$$

where $U(x)$ denotes the electrochemical potential of quasiparticles. This solution reflects the spatially symmetric mode of current and potential distribution relative to the midpoint $x = W/2$ of the SN contact region. From (15) it follows that for the voltmeter connected between points *1* and *4*, resistance of the SN electrode is equal to

$$\frac{R_{ns}}{R_{nW}} = \frac{U(W) - U(0)}{I_b R_{nW}} = \frac{2\lambda}{W}\tanh\frac{W}{2\lambda}. \tag{16}$$

To measure resistance, when the voltmeter is connected between points *2* and *3*, potential in the S film region shall be determined. This potential is independent of the x coordinate and from symmetry considerations is equal to the potential in the middle of the subelectrode part of the N film $U(W/2) = 0$. Thus, when the voltmeter is connected between points *2* and *3*, SN electrode resistance is twice as low as that in the previous measurement case.

As can be seen, the derived expressions for the SN electrode resistance (13) and (16) have the same form up to the replacement of the characteristic length ξ by λ, even though the former was derived via limiting transition from the two-dimensional problem solution for resistive case, and the latter was derived from the phenomenological current conversion model. This means that the typical length and resistance of the interface can be directly calculated from experimental measurements of resistance of a series of Josephson contacts (for example, as described in [13,14]) with different electrode widths using the calculated equations. Comparison of the calculated value with the measured R_{ns} at a temperature above T_C will allow one to determine the mechanism of transformation of the superconducting and normal components of the total current in SN structures at different temperatures and voltages.

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Conflict of interest

The authors declare no conflict of interest.

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