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On losses of the electro-osmotic emitter in a fluid and in air

© B.P. Sharfarets, V.E. Kurochkin

Institute of Analytical Instrument Making, Russian Academy of Sciences,
198095 St. Petersburg, Russia
e-mail: lavrovas@yandex.ru

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The study addresses viscous and Joule losses that accompany an electro-osmotic process in water and in air. A physical model was presented by a cylindrical capillary with open ends, which was placed in water or air and filled with the fluid that surrounds it. We have considered a model of the viscous incompressible fluid. Validity of such an approximation is noted. It is demonstrated that the electro-osmotic losses have a number of specific features as compared to other processes of motion of a fluid in porous media. The said losses in both the media have been calculated.

Keywords: electrokinetic emitters, a fluid-filled capillary, a viscous incompressible fluid, viscous losses, Joule losses, comparative estimation of losses in air and the liquid medium.

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Introduction

The previous study of the authors [1] has addressed an issue of comparative estimation of efficiency of electro-acoustic transformation in various media (in particular, in water and in air). However, it did not take into account dissipative losses that inevitably accompany such transformations. The present study also addresses functioning of the electrokinetic emitter in the viscous, air and liquid media. But dissipative losses are taken into account herein.

The present study again takes a model of the emitter designed as a cylindrical capillary with open ends, which is submersed into the fluid and filled with the same fluid. End faces of the capillary are energized with an electric field that consists of a sum of constant and harmonic electric potentials. But this already addresses energy dissipation that accompanies the process of electro-acoustic transformation in the electrokinetic emitter in the air and liquid media. At the same time, the Rayleigh dissipative function is used to take into account viscous dissipation as well as dissipation caused by losses that are typical for the electrokinetic converters and related to the Joule–Lenz law when an electric current flows through an electrically-conductive medium.

1. Basic equations

In order to obtain the required results, we provide equations that are necessary hereinafter. The law of conservation of energy for the viscous fluid is taken into account by the differential energy equation [2]. In particular, for the viscous compressible fluid, this equation is written as

$$\rho \frac{du}{dt} = \varepsilon + \nabla(\lambda \nabla T) - p\nabla \mathbf{v} + \Phi, \qquad (1)$$

where u is specific internal energy; ρ, T, p is a density, temperature and pressure of the fluid, respectively; $\mathbf{v} = (v_x, v_y, v_z)$ is a fluid velocity field determined by the Navier–Stokes system of equations; Φ is a Rayleigh dissipative function; λ — a thermal conductivity coefficient; ε is an amount of heat entering a fluid unit volume per unit time due to radiation or any other causes, except for thermal conductivity, for example, due to chemical reactions (do not confuse with relative dielectric permittivity ε). We note that the magnitudes ε and Φ have dimension $[\varepsilon] = [\Phi] = \mathrm{W/m^3}$, i.e. power per a unit volume, and characterize respective losses.

Since further on, when substantiation of such a transition is followed by consideration of the incompressible fluid, below is given a version of the energy equation (1) for the incompressible fluid [2]:

$$c\rho \frac{dT}{dt} = \varepsilon + \nabla(\lambda \nabla T) + \Phi, \qquad (2)$$

where c — specific heat capacity of the fluid; ρ — the density of the incompressible fluid; $\frac{dT}{dt}$ — a convective derivative of the temperature.

Hereinafter, the present study intensely uses the Rayleigh dissipative function Φ. Since only the incompressible fluid is considered here, we proved its respective version of the dissipative function Φ in the Cartesian coordinates [3]:

$$\Phi = \eta\left[\varepsilon_{11}^2 + \varepsilon_{22}^2 + \varepsilon_{33}^2 + 2(\varepsilon_{12}^2 + \varepsilon_{23}^2 + \varepsilon_{31}^2)\right].$$

Here, ε_{ik} are components of a strain-rate tensor; η — dynamic viscosity of the fluid. In the Cartesian coordinates, the components ε_{ik} of the tensor are determined by the expression

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right).$$

We designate the strain-rate tensor with the components (ε_{ik}) by $\mathbf{D}$. After an operation of double convolution of the tensor $\mathbf{D}$ (we obtain its scalar square $\mathbf{D}:\mathbf{D}$):

$$\mathbf{D}:\mathbf{D}=\sum_{i,k}\varepsilon_{ik}\varepsilon_{ik}=\left[\varepsilon_{11}^2+\varepsilon_{22}^2+\varepsilon_{33}^2+2(\varepsilon_{12}^2+\varepsilon_{23}^2+\varepsilon_{31}^2)\right],$$

and comparing this expression with the above-given dissipative function Φ, we obtain, in an operator form, an expression for the Rayleigh dissipative function in terms of the strain-rate tensor in the Cartesian coordinates

$$\Phi=\eta\,\mathbf{D}:\mathbf{D}.$$

Further on, we go over to solving the announced problem in the cylindrical coordinates. For this purpose, we take into account the following note that if the magnitude Φ would depend on a system of coordinates, it would mean that the amount heat released in a point varies on a method of recording of the equation, which contradicts the physical meaning.

Due to cylindrical symmetry of the problem under study, a version of the strain-rate tensor in the cylindrical system of coordinates will be required, which is presented, for example, in the study [4] (the formulas are adapted to the cylindrical system of coordinates (r,φ,z)):

$$\varepsilon_{rr}=\frac{\partial v_r}{\partial r},\quad \varepsilon_{\varphi\varphi}=\frac{v_r}{r}+\frac{1}{r}\frac{\partial v_\varphi}{\partial\varphi},\quad \varepsilon_{zz}=\frac{\partial v_z}{\partial z},$$

$$\varepsilon_{r\varphi}=\frac{r}{2}\frac{\partial}{\partial r}\left(\frac{v_\varphi}{r}\right)+\frac{1}{2r}\frac{\partial v_r}{\partial\varphi},\quad \varepsilon_{zr}=\frac{1}{2}\left(\frac{\partial v_r}{\partial z}+\frac{\partial v_z}{\partial r}\right),$$

$$\varepsilon_{\varphi z}=\frac{1}{2}\left(\frac{\partial v_\varphi}{\partial z}+\frac{1}{r}\frac{\partial v_z}{\partial\varphi}\right).$$

After the operation of double convolution on the components of the strain-rate tensor, we obtain the Rayleigh dissipative function in the cylindrical system of coordinates:

$$\Phi=\eta\begin{bmatrix}\left(\frac{\partial v_r}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial v_\varphi}{\partial\varphi}+v_r\right)^2+\left(\frac{\partial v_z}{\partial z}\right)^2\\ +2\left(\frac{1}{2}\left(\frac{1}{r}\frac{\partial v_r}{\partial\varphi}+\frac{\partial v_\varphi}{\partial r}-\frac{v_\varphi}{r}\right)\right)^2\\ +2\left(\frac{1}{2}\left(\frac{\partial v_r}{\partial z}+\frac{\partial v_z}{\partial r}\right)\right)^2+2\left(\frac{1}{2}\left(\frac{1}{r}\frac{\partial v_z}{\partial\varphi}+\frac{\partial v_\varphi}{\partial z}\right)\right)^2\end{bmatrix}. \quad (3)$$

In (1) and (2), attention should be paid to a summand ε with dimension W/m^3. The magnitude implies any energy losses that differ from Rayleigh viscous losses. Such losses that take place in the present study will include losses by Joule heating of the fluid in the capillary, which occur due to applying the variable and constant electric fields to the end faces of the capillary. Thus, the present study will consider two kinds of losses: losses due to Joule heating and viscous losses.

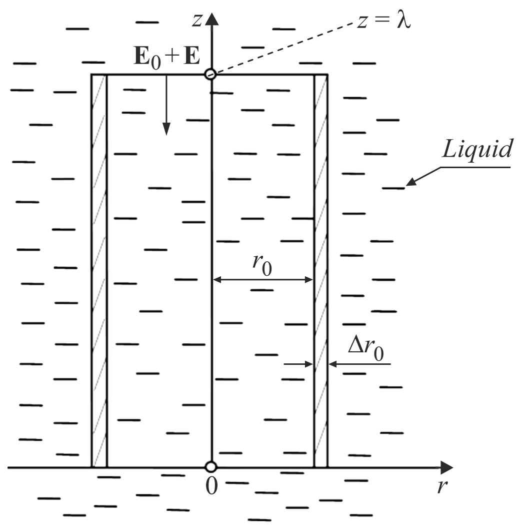

Image of the process of electro-osmosis in the open cylindrical capillary that is located in the infinite fluid.

2. Physical model of the process

Again, as in [1], we consider the cylindrical capillary filled with the viscous incompressible fluid. The infinite homogeneous viscous incompressible fluid (water or air) holds the cylindrical glass capillary with open end faces, which has a length l, a radius r_0 and a wall thickness Δr_0 and is also filled with the surrounding fluid. The end faces of the capillary are energized with the electric field that consists of the constant electric field of strength $\mathbf{E}_0$ and an amplitude E_0 and the variable harmonic field of strength $\mathbf{E}$ and an amplitude E. Under effect of the summed electric field, the interior of the capillary has the undergoing electro-osmotic process that is previously described in the study [1] and other studies of the authors. A diagram of the physical process is shown in the figure.

3. Hydrodynamic model of the electro-osmotic emitter

The study [1] provides arguments for describability of the processes of electro-acoustic transformation under the model of the viscous incompressible fluid. Herein, we write the respective Navier–Stokes equations and the continuity equation as applied to the cylindrical system of coordinates [5–9].

As will be demonstrated below, for the incompressible fluid, in case of electro-osmotic flow at high values of the electrokinetic radius, along the cylindrical capillary with an axis oriented along the Oz axis, which has the constant velocity $\mathbf{v}_0=(0,0,v_{0z})$ and the variable velocity $\mathbf{v}=(0,0,v_z)$ as well as constant and variable pressure $p_\Sigma=p_0+p$, the only non-zero z-component of the Navier-

Stokes equation is written as [6,7]:

$$\frac{\partial v_z}{\partial t} + (v_z + v_{0z})\frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + v\left(\frac{\partial^2 v_z}{\partial z^2}\right) + F_z/\rho. \quad (4)$$

Besides, the continuity equation shall be fulfilled, which in this case is written as [8]:

$$\frac{\partial v_z}{\partial z} = 0. \quad (5)$$

In (4) the magnitude $v = \eta/\rho$ is kinematic viscosity of the fluid; $\rho = \text{const}$ — the density of the incompressible fluid; F_z is a ponderomotive force that is equal to $F_z = \rho_{el}\mathbf{E} = \rho_{el}E_z$ in case of homogeneous electrical parameters of the fluid [8,10]. Here, ρ_{el} is a density of the electric charge. Taking into account that the electric field has two components: $E_z = E_{z0} + E_{z\sim}$, the right part in (1) is written as

$$\frac{F_z}{\rho} = \frac{E_{z0}}{\rho} + \frac{E_{z\sim}}{\rho}.$$

Moreover, the equation (4) is non-linear, thereby resulting in the so-called pumping mode that is previously described in [11].

It is clear from the equation (4) that with the constant flow along the z axis there is the pumping mode that consists of influence of the constant flow velocity v_0 on the variable flow velocity and variable pressure [11].

The equations (4) and (5) shall be supplemented with edge conditions. The study [6] writes the edge conditions on side boundaries of the capillary $r = r_0$ and on its axis $r = 0$ as follows:

$$v_z(r = r_0) = 0, \quad \left.\frac{\partial v_z}{\partial r}\right|_{r=0} = 0.$$

However, here, it is more appropriate to set other edge conditions. We are talking about an electro-osmotic phenomenon, which consists of the following. With the high values of the electrokinetic radius $r_0/\lambda \gg 1$, where r_0 is a capillary radius, λ is a thickness of the double electric layer (a Debye length), during origination of electro-osmosis the capillary has almost piston motion of the fluid generated along the capillary axis, i.e. the components of the fluid velocity $v_r = 0$ and $v_\varphi = 0$, and only the velocity v_z is non-zero and it depends on the z axial coordinate (i.e. $v_z = v_z(z)$). It will be discussed below in more detail. Based on this note, the following edge condition of slip on the side surface of the capillary can be set:

$$v_{0z}(r_0) = U_{eo} = \frac{\varepsilon\varepsilon_0\xi}{\eta}E_0, \quad v_z(r_0) = u_{eo} = \frac{\varepsilon\varepsilon_0\xi}{\eta}E. \quad (6)$$

Here, U_{eo} is an electro-osmotic velocity when applying the constant field of electric strength E_0; u_{eo} is an electro-osmotic velocity when applying the variable field of electric strength E; ε and ε_0 — relative dielectric permittivity and the electrical constant, respectively; ξ — Zeta-potential.

4. Joule losses in the capillary

The electro-osmotic process is accompanied by Joule heating due to flowing of an electrically-ionic current in the fluid, which is caused by the applied external electric field. In its differential description, the Joule–Lenz law is formulated as follows [12]: „Heat power per a unit volume q ($[q] = \text{W/m}^3$) is proportional to a square of the electric current density $\mathbf{j}$ and reversely proportional to specific conductivity of the medium“. Or, formally

$$q = \frac{1}{\sigma}\mathbf{j}^2.$$

(it is this magnitude that is featured in the equation (1) and (2) under the symbol ε, i.e. $\varepsilon = q$).

In this form, the Joule–Lenz law is general, i.e. it does not depend on a nature of forces that excite the electric current. Besides, it is true both for the alternating as well as direct currents [3]. We note that q has dimension of heat power in the unit volume $[q] = \text{W/m}^3$.

Further on, in order to solve tasks of the present study, we rewrite the Joule–Lenz law in a somewhat different form. For this purpose, we will use the Ohm's law [12], which says that a vector of the electric current density $\mathbf{j}$ is proportional to electric field's strength $\mathbf{E}$:

$$\mathbf{j} = \sigma\mathbf{E}.$$

By combining the Joule–Lenz law and the Ohm's law, we obtain the Joule–Lenz law in the following form:

$$q = \frac{1}{\sigma}\mathbf{j}^2 = \sigma\mathbf{E}^2. \quad (7)$$

The vector $\mathbf{E}$ can be both constant and variable. As noted above, in the first case it will be designated as $\mathbf{E}_0$, so will as $\mathbf{E}$ in the second case. Hereinafter, we will use the Joule–Lenz law in the form (7).

First, we consider the constant electric field $\mathbf{E}_0$. Then (7) provides for power q_0 of heat caused by the constant electric field:

$$q_0 = \sigma E_0^2 = \sigma\frac{U_0^2}{l^2}, \quad (8)$$

where U_0 is constant pumping voltage applied to the end faces of the capillary; l — the capillary length.

If a case of a harmonic dependence of the electric field $E\cos\omega t$, $E = \text{const}$ is considered, then (8) is transformed into the following form

$$q_\sim = \sigma E^2\cos^2\omega t = \sigma\frac{U^2}{l^2}\cos^2\omega t. \quad (9)$$

Taking into account the identity

$$\cos^2(\omega t) = \frac{\cos(2\omega t) + 1}{2},$$

we bring (9) to the form

$$q_\sim = \sigma\frac{(U/l)^2\big(\cos(2\omega t) + 1\big)}{2}.$$

It is clear from the last equality that the average value of heat power per the unit volume during the period of oscillations is

$$\overline{q}_\sim = \sigma\,\frac{(U/l)^2}{2}, \tag{10}$$

while its amplitude $\overline{q}_\sim$ in case of harmonic voltage when $U = U_0$ is twice les than the respective value for the direct current (8).

After this, we can write the total power of Joule losses Q in the capillary volume, which is caused by flowing of the direct and alternating currents in the capillary fluid, with taking into account the magnitude of power of losses of the direct (7) and harmonic (10) currents:

$$Q_0 + Q_\sim = \sigma\pi r_0^2 l\left(\frac{U_0^2}{l^2} + \frac{1}{2}\,\frac{U^2}{l^2}\right) = \frac{\sigma\pi r_0^2}{l}\left(U_0^2 + \frac{1}{2}U^2\right). \tag{11}$$

Here, $\pi r_0^2 l$ is a volume of the capillary.

According to generally available data, specific electric conductivity of common running water (with salts and minerals): $\sigma_{\text{water}} = 10^{-4}{-}10^{-3}$ S/m, specific electric conductivity of humid air $\sigma_{\text{air}} = 10^{-13}{-}10^{-10}$ S/m (common room conditions).

Thus, according to (11), the losses for Joule heat in air are by 7−9 orders smaller than in water.

5. Rayleigh dissipation. On viscous dissipation in the cylindrical capillary during electro-osmotic flowing

According to [10], the electro-osmotic velocity (both U_0 as well as u_0) in the cylindrical capillary oriented along the z axis, with the large electrokinetic radius $r_0/\lambda_D \gg 1$, has only one non-zero component that depends on the current radius r

$$v_z(r) = \left[1 - \frac{I_0(r/\lambda_D)}{I_0(r_0/\lambda_D)}\right] V_{eo}, \tag{12}$$

where r_0 is a radius of the capillary; I_0 is a modified first-kine zero-order Bessel function; λ_D is a thickness of the double electric layer (the Debye length);

$$V_{eo} = U_{eo} = \frac{\varepsilon\varepsilon_0\,\xi}{\eta}\,E_0$$

— an amplitude of the constant electro-osmotic velocity caused by the constant electric field E_0 and

$$V_{eo} = u_{eo} = \frac{\varepsilon\varepsilon_0\,\xi}{\eta}\,E$$

— an amplitude of the variable electro-osmotic velocity caused by the variable electric field E [10].

However, we note that at the quite high values of the electrokinetic radius r_0/λ_D according to [10], specifically when $r_0/\lambda_D \geq 100$ the equality (12) can be replaced by an approximate equality $v_z(r) \approx V_{eo}$ when $0 \leq r \leq r_0$. Thus, electro-osmotic flowing both with the variable and the constant velocity of the flow takes a piston type, i.e. it does not depend on a current radius of the capillary and a current polar angle across the entire cross section. A typical value of the Debye radius λ_D for water is of an order of several nanometers; for example, [10] provides the value $\lambda_D \approx 10^{-8}$ m. In the previous subject-similar study [1] by the authors, the radius of the capillary was $r_0 = 10^{-5}$ m, then $r_0/\lambda_D = 1000$. Thus, the condition $r_0 > 100\lambda_D$ is obviously fulfilled.

In this conditions, the constant flow velocity $\mathbf{U}_0$ has almost only the longitudinal z-component $\mathbf{U}_0 = (0, 0, U_{eo})$ (see Fig. 9.2 (a) in [10]). The other components of the velocity are equal to zero. A similar conclusion can also be made in relation to the variable velocity $\mathbf{u} = \big(0, 0, u_{eo}(z)\big)$, where $u_{eo} = \frac{\varepsilon\varepsilon_0\,\xi}{\eta}\,E$, E — strength of the variable electric field. It reasonably makes it possible to assume that under these conditions the capillary has both the constant electro-osmotic velocity $\mathbf{U}_0$ as well as the variable electro-osmotic velocity $\mathbf{u} = \big(0, 0, u_{eo}(z)\big)$ with the only non-zero z-component $u_{eo}(z)$, which are almost constant across the cross section.

Below we will consider a type of viscous losses that originate in the capillary when the fluid with the velocities $\mathbf{U}_0 = (0, 0, U_{eo})$ and

$$\tilde{u}_{eo} = \frac{\varepsilon\varepsilon_0\,\xi}{\eta}\,E\cos(\omega t - kz) = u_{eo}\cos(\omega t - kz),$$

flows therein, i.e. when there are the constant flow with the velocity $\mathbf{U}_0$ and the variable harmonic flow with the velocity $\tilde{u}_{eo} = u_{eo}\cos(\omega t - kz)$. For this purpose, we will consider the Rayleigh dissipative function (3) for the considered case.

6. Dissipation caused by viscous friction in the incompressible fluid in the capillary

We will consider dissipation of energy in the capillary, which is caused by viscous friction. For this purpose, we will use the respective Rayleigh dissipative function (3).[1]

As can be seen from the expression for the dissipative function (3), dissipation at the constant velocity $\mathbf{U}_0 = \text{const}$ is zero, as all the summands in (3) are zeroed when

[1] We note that the study [8] also addresses dissipation of energy in the incompressible fluid. The following expression is presented for a rate of decrease of kinetic energy in a certain volume V of the incompressible fluid:

$$\frac{\partial E_{kin}}{\partial t} = -\frac{\eta}{2}\int_V \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)^2 dV, \ \ i, k = 1, 2, 3..$$

Here $(\partial E_{kin})/\partial t$ is a rate of decrease of kinetic energy per time unit in the volume V; η — dynamic viscosity of the fluid. Thus, the expression

$$\Phi_E = \frac{\eta}{2}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)^2$$

is in the Cartesian coordinates a Rayleigh dissipative function for viscous losses in the incompressible fluid with dimension [W/m^3].

substituting $\mathbf{U}_0 = \text{const}$ into them. Hence, there is an important conclusion that the constant electro-osmotic flow in the considered cylindrical symmetrical case moves without viscous losses.

Further on, let us consider the variable velocity of the fluid in the capillary $\tilde{u}_{eo} = u_{eo}\sin(\omega t - kz)$. In this case the dissipative function is (see (3))

$$\Phi_R = \eta\left(\frac{\partial v_z}{\partial z}\right)^2.$$

. By substituting the value of the velocity $\tilde{u}_{eo}$ into it, we obtain a final expression for the dissipative function

$$\Phi_R = \eta\left(\frac{\partial v_z}{\partial z}\right)^2 = \frac{\eta}{2}u_{eo}^2k^2\cos^2(\omega t - kz). \quad (13)$$

As can be seen from (13), the energy losses are quasi-harmonic; dissipation power depends on fluid viscosity η, on a square of the amplitude of the variable electro-osmotic velocity u_{eo}, on a square of the angular frequency ω (via the wave number $k = \omega/c$, where ω is an angular frequency) and, in a reverse proportion, on a square of the speed of sound in the medium c. Maximum dissipation takes place when the wave passes through an equilibrium position ($\omega t = kz$). The function $\cos^2(\omega t - kz)$ at the fixed value of kz is a periodic function of time t with a period of π. An average value of the integral of the function $\cos^2(\omega t - kz)$ over the magnitude of the initial oscillation period

$$T = \frac{2\pi}{\omega} \quad \overline{\cos^2(\omega t - kz)} = \frac{1}{2}.$$

Therefore, in a time-averaged option the expression (13) can be rewritten as

$$\overline{\Phi}_R = \frac{\eta}{4}\left(\frac{\partial v_z}{\partial z}\right)^2 = \frac{\eta}{4}u_{eo}^2k^2. \quad (14)$$

As known, the dissipative function has dimension [W/m^3] Therefore, when the losses in the volume of the cylindrical capillary are considered as a whole, then total power of losses is determined by integration (14) across the entire volume of the capillary

$$\overline{\Phi}_{R\Sigma} = \frac{\eta}{4}u_{eo}^2k^2\pi r_0^2L. \quad (15)$$

The function $\overline{\Phi}_{R\Sigma}$ will already have [W], i.e. this function characterizes power of losses in the capillary as a whole. Let us write out the electro-osmotic velocity's amplitude u_{eo} in the last formula:

$$u_{eo} = \frac{\varepsilon\varepsilon_0\xi E}{\eta}.$$

Here u_{eo} is an amplitude of the electro-osmotic velocity; ε is dielectric permittivity of the medium; ε_0 is the electrical constant. η — dynamic viscosity of the fluid. Then (15) is rewritten as

$$\Phi_{R\Sigma} = \frac{\eta}{4}\left(\frac{\varepsilon\varepsilon_0\xi E}{\eta}\right)^2k^2\pi r_0^2L = \frac{1}{4\eta}(\varepsilon\varepsilon_0\xi E)^2k^2\pi r_0^2L. \quad (16)$$

Let us find a ratio of power of viscous losses (16) in water Φ_{water} in relation to air Φ_{air}:

$$\frac{\Phi_{\text{water}}}{\Phi_{\text{air}}} = \frac{\frac{1}{4\eta_{\text{water}}}(\varepsilon_{\text{water}}\varepsilon_0\xi E)^2\frac{\omega^2}{c_{\text{water}}^2}\pi r_0^2L}{\frac{1}{4\eta_{\text{air}}}(\varepsilon_{\text{air}}\varepsilon_0\xi E)^2\frac{\omega^2}{c_{\text{air}}^2}\pi r_0^2L} = \frac{\eta_{\text{air}}}{\eta_{\text{water}}}\left(\frac{\varepsilon_{\text{water}}\varepsilon_0\xi E}{\varepsilon_{\text{air}}\varepsilon_0\xi E}\right)^2\frac{c_{\text{air}}^2}{c_{\text{water}}^2} = \frac{\eta_{\text{air}}}{\eta_{\text{water}}}\left(\frac{\varepsilon_{\text{water}}}{\varepsilon_{\text{air}}}\right)^2\frac{c_{\text{air}}^2}{c_{\text{water}}^2}. \quad (17)$$

Substituting the values of

$$\eta_{\text{air}} = 18.2\cdot10^{-6}\,\text{Pa}\cdot\text{s}, \quad \eta_{\text{water}} = 0.00101\,\text{Pa}\cdot\text{s},$$

into (17)

$$\varepsilon_{\text{air}} = 1.0006, \quad \varepsilon_{\text{water}} = 80.4,$$

$$c_{\text{air}} = 340\,\text{m/s}, \quad c_{\text{water}} = 1500\,\text{m/s},$$

we obtain $\Phi_{\text{water}}/\Phi_{\text{air}} = 5.913$.

Thus, when the electrokinetic process is implemented, power of viscous losses in water is almost in 6 times higher than in air.

Conclusions

The study has provided all the necessary equations for comparatively estimating the main losses that accompany operation of the electro-osmotic emitter in water and in air. Losses due to viscous friction and Joule losses are considered. As a result of the calculations, the following results are obtained. It is demonstrated that at the high values of the electrokinetic radius there are not viscous losses during uniform (under impact of the constant electric field) electro-osmotic motion of the fluid along the capillary. Motion of the fluid along the capillary is of a special piston type both for the variable velocity of motion as well as for its constant velocity. The ratio of the Joule losses in water and air is considered. It is found that the losses for Joule heat in air are by 7–9 orders smaller than in water. Besides, it included comparison of the values of viscous losses in water and in air. It is also demonstrated that power of viscous losses in water is almost in 6 times higher than in air. The results obtained shall be taken into account when designing the electro-osmotic converters.

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Conflict of interest

The authors declare that they have no conflict of interest.

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