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Autowave nature of plastic flow process

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The paper discusses the nature of autowave characteristics of plastic flow, determining the current state of the autowave theory of plasticity. Analysis of dispersion relations for different stages of strain hardening allowed us to establish a correspondence between the structure of the medium and the form of the dispersion law. The mechanism of occurrence of spatial scales of plastic flow and the relationship of the laws of strain hardening with the dispersion of autowaves are explained.

Keywords: plasticity, dislocations, localization, autowaves, dispersion, strain hardening, stages of plastic flow.

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Introduction

A fundamental idea proposed in 1987 by Seger and Franck [1], which stipulates that a plastic flow is a process of structuring in deformed media far away from the balance, was supported by Nikolis and Prigozhin [2]. In 1989 they claimed that „plasticity and flowability could not be studied on a purely mechanical bases, but should be considered as a part of the problems of non-linear dynamic systems, operating far away from the balance". These studies have paved the way for applying notions and an apparatus of synergetics [2,3] to explanation of the plasticity phenomenon. An interest of researchers to these problems was caused by understanding that a deformed body is an open system that is far away from the equilibrium and irreversibly developing under laws that are different from thermodynamic laws of closed systems [4].

Unfortunately, the first attempts of using the synergetics ideas in plasticity physics did not result in a significant progress. As it is clear now, a failure thereof was caused by unclear indication of an ordering item that obviously could not be claimed by dislocations. There was still a serious problem of matching scales of a deformed sample and strain carriers — dislocations. The situation changed when Haken [3] formulated a principle, according to which a behavior of complex systems shall be described in synergetics by means of macroscopic observed magnitudes to be followed by transiting to clarification of a microscopic structure of processes that generate the macroscopic structure or the macroscopic behavior.

By successively applying this principle to the problem of plasticity, a new approach could be developed to form autowave physics of plasticity [5,6]. It was based on a hypothesis formulated in 1990: „Self-organization of deformation is observed as a pattern of macroscale plastic flow localization. Localization is an indispensable informative feature of a plastic deformation process that accompanies it from an elastic-plastic transition to fracture and takes various forms along the way". The validity of the hypothesis was experimentally checked at 50 various materials by means of a speckle-photo technique updated for studying large plastic deformations [5].

An object under study in autowave physics of plasticity is a macroscopic localized plasticity pattern in the form of distributions of local longitudinal deformations on the working surface of the sample $\varepsilon_{xx}(x,y)$ (Fig. 1), which is characterized by a spatial period (an autowave length) $\lambda \approx 10^{-2}$ m, frequency $\omega_{aw} \approx 10^{-2}$ Hz and a velocity of localized plasticity fronts $V_{aw} = \lambda\omega_{aw} \approx 10^{-4}$ m/s [5,6]. The pattern is a projection of autowave deformation processes to

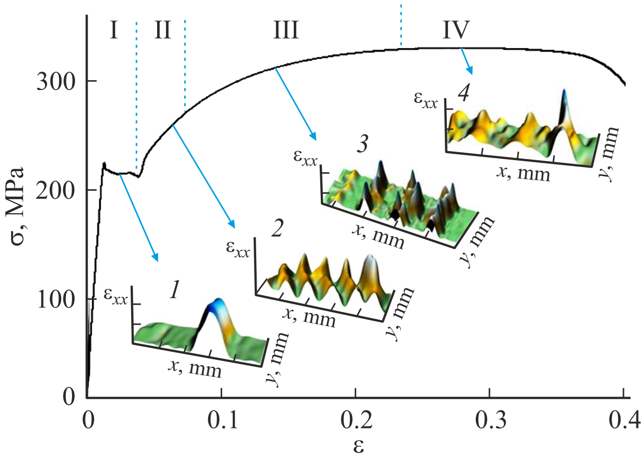

Figure 1. Multi-stage curve of the plastic flow: I — the yield plateau, II — linear strain hardening, III — parabolic strain hardening, IV — pre-fracture. Localized plasticity patterns in the form of distributions of local elongations on the working surface of the sample $\varepsilon_{xx}(x,y)$: *1* — the yield plateau (the Luders band), *2* — linear strain hardening (the phase autowave), *3* — parabolic strain hardening (a stationary dissipative structure), *4* — pre-fracture (collapse of the localized plasticity autowave.

the observed surface of the deformed object. The autowaves of the localized plastic flow are described by a system of parabolic differential equations [7] for the velocities of plastic strain $d\varepsilon/dt$ and stress relaxation $d\sigma/dt$ [5,6]:

$$\begin{cases} \dfrac{\partial \varepsilon}{\partial t} = f(\varepsilon) + D_{\varepsilon\varepsilon}\dfrac{\partial^2 \varepsilon}{\partial x^2}, & (1) \\ \dfrac{\partial \sigma}{\partial t} = g(\sigma) + D_{\sigma\sigma}\dfrac{\partial^2 \sigma}{\partial x^2}, & (2) \end{cases}$$

wherein deformation is an activator of the plastic flow, while the stresses are a damper thereof. The equations shaped in this way are basic for analyzing self-organization processes in media of a various physical nature. Any of these media consists of elementary full-mixing volumes, wherein each of them contains an open metastable system with N-shaped kinetics of disintegration $f(\varepsilon)$ and $g(\sigma)$. Interaction of the volumes is controlled by transfer processes, whose transport coefficients $D_{\varepsilon\varepsilon}$ and $D_{\sigma\sigma} \gg D_{zz}$ with kinematic viscosity dimension m^2/s are diagonal elements of the matrix

$$D_{ij} = \begin{Vmatrix} D_{\varepsilon\varepsilon} & D_{\varepsilon\sigma} \\ D_{\sigma\varepsilon} & D_{\sigma\sigma} \end{Vmatrix}, \qquad (3)$$

which relate elastic (reversible) and plastic (irreversible) shifts during deformation. Due to an Onsager principle [8] off-diagonal elements of the matrix (3) are equal $D_{\varepsilon\sigma} = D_{\sigma\varepsilon}$.

Presently, the autowave approach is quite well experimentally [5,6] and theoretically [9] justified. It has been demonstrated by the studies on more than fifty various materials that the localization phenomenon and formation of the localized plasticity autowaves are inherent in all the materials without exception. In particular, it is demonstrated that each stage of strain hardening of the material is matched with a certain autowave mode and main characteristics of these modes are specified. The present study is aimed at clarifying a nature of some magnitudes of the equations (1) and (2) and describing their role in formation of the localized plasticity pattern.

1. Plastic deformation as a process in the active medium

According to the main principle of synergetics [3,10], self-organization of the complex system consists of separation of a set of its possible degrees of freedom into an infinite number of microscopic (diffusion) degrees and a small number of macroscopic (hydrodynamic) degrees. This separation is reflected in the structure of the equations (1) and (2), in which the terms $f(\varepsilon)$ and $g(\sigma)$ describe a hydrodynamic mode, while the terms with the derivatives ε'' and σ'' describe a diffusion mode. In this case, it is of primary importance to understand what are mechanisms of origination of the hydrodynamic modes in the deformed medium.

1.1. Structure of the active deformed medium

The general theory of the autowave processes [7] requires that the medium generating the autowaves would be active, non-equilibrium and nonlinear. These qualities are not obvious for the deformed medium and require special justification. As said above, activity of the medium suggests that its structure contains a volume-distributed energy source and the set of the elementary full-mixing volumes, wherein each of them contains the open non-equilibrium point system. Let us demonstrate that the above said fully refers to the deformed medium that contains dislocations or assemblies thereof [11,12].

We use a theory of thermally assisted plastic flow, which is designed in [11], and we will believe that plastic deformation is based on an elementary plasticity event that consists of relaxation of the stress concentrator with generation of new dislocation shifts. A set of these events provides plastic deformation of the medium, while a role of the concentrators in this model is attributed to decelerated flat dislocation clusters that are distributed across the volume of the medium and created by Franck–Read sources [12] (Fig. 2). Elastic fields of the concentrators act as sources of potential energy by creating medium activity, wherein an area of stresses near the concentrator corresponds to the full-mixing volume. In this model, the plastically-deformed medium is a mosaic of mutually-balanced elastically-stressed volumes. The energy of the concentrator [11] $W \approx nGb^2(\ln 4R/\xi + 1/2) \approx 7.7 \cdot 10^{-9}\,\text{J} \approx 5 \cdot 10^{10}\,\text{eV} \approx$ $\approx 1\,\text{eV}/b$ creates non-equilibrium (metastability) of each of these volumes. The energy W has been numerically estimated for Al for the number of dislocations $n = 10$, the concentrator length $\xi = 10^{-5}\,\text{m}$, the crystal size $R = 10^{-2}\,\text{m}$, the Burgers vector $b = 0.286\,\text{nm}$ and the shear modulus $G = 26\,\text{GPa}$.

Going over to a problem of generation of the localized plasticity autowaves, we note that a cause of their creation can be qualitatively justified based on dislocation models of plastic flow. They are usually based on the Taylor–Orovan

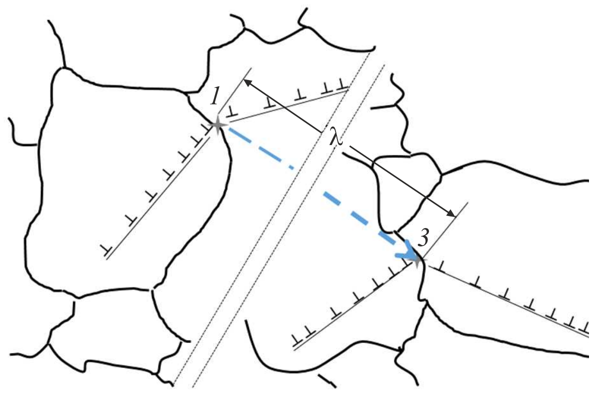

Figure 2. Dislocation-origin elastic stress concentrators (*1*) and (*3*) in the active deformed medium.

equation [12]:

$$\varepsilon \approx b\rho_{mob}L$$

or

$$\frac{d\varepsilon}{dt} \approx b\rho_{mob}V_{disl}, \quad (4)$$

which relates macroscopic strain ε or its rate $\dot{\varepsilon} = d\varepsilon dt$ to microscopic characteristics of the dislocation structure: the path length L and the dislocation velocity $V_{disl}(\sigma)$ that depends on the stress σ as well as the density of movable dislocations $\rho_{mob}(\varepsilon)$. The condition of active loading $\dot{\varepsilon} = \text{const}$ is fulfilled when $\rho_{mob}\varepsilon = \text{const}$. If during the plastic flow the density of movable dislocations and their rate decrease, then at stresses that are insufficient for fracture, in order to retain the condition $\dot{\varepsilon} = \text{const}$ the medium shall generate a new seat of deformation, while the right-hand side of the equation (4) shall include an additional summand of strain rate's dimension s^{-1}. At the same time, the localization pattern becomes coherent.

Formally, it follows from the equation for the strain rate [13]:

$$\frac{\partial\varepsilon}{\partial t} = \frac{\partial}{\partial x}\cdot\left(D_{\varepsilon\varepsilon}\frac{\partial\varepsilon}{\partial x}\right), \quad (5)$$

in which $D_{\varepsilon\varepsilon}(x)$. Then

$$\frac{\partial\varepsilon}{\partial t} = \frac{\partial\varepsilon}{\partial x}\cdot\frac{\partial}{\partial x}D_{\varepsilon\varepsilon} + D_{\varepsilon\varepsilon}\frac{\partial^2\varepsilon}{dx^2} = f(\varepsilon) + D_{\varepsilon\varepsilon}\frac{\partial^2\varepsilon}{\partial x^2}, \quad (6)$$

where $f(\varepsilon) = (\partial\varepsilon/\partial x))\cdot(\partial D_{\varepsilon\varepsilon}/\partial x)$ is a non-linear rate of local strain. Along with non-linear kinetics of the relaxation event and its respective N-like shape of the functions $f(\varepsilon)$ and $g(\sigma)$ in the equations (1) and (2) it provides nonlinearity of the deformed medium. Thus, analysis of the structure and the properties of the deformed medium make it possible to conclude that the deformed medium is really active, non-equilibrium and nonlinear and is suitable for self-organization and generation of the localized plasticity autowaves in terms of synergetics principles [3,7,10].

The equation (6) is obviously equivalent to the autowave equation (1). As demonstrated in [5], at the low density of dislocations $f(\varepsilon) \approx b\rho_{mob}V_{disl}$. Therefore, it can be assumed that the dislocation equation (4) is a limiting case of the equation (1), which is true for small deformations. Thereby, it establishes a relation between the dislocation approach and the autowave approach in physics of plasticity. A decisive issue of the mechanism of origination of a diffusion-like term $D_{\varepsilon\varepsilon}\varepsilon''$ in the equation (1) will be considered below.

1.2. Two-component model of deformation and a macroscale of the plastic flow

It may be done as a result of the transition to the two-component model of autowave plasticity [6], which develops the theory of thermally assisted deformation [11]. This model complements the theory [11], taking into account a role of local elastic fields of the stress concentrators, i.e. taking into account activity of the deformed medium. It is assumed that energy released in relaxation of the stress concentrator is spent for implementing two effects. Its one part forms new shifts in a local neighborhood of the concentrator, while the other is transformed into acoustic emission signals, transmitted over large distances [14] and synchronizes relaxing concentrators. As a result, the behavior of the relaxation events becomes correlated (coherent) in an area with the size of about a coherence radius $\approx \lambda$ [15].

The mechanism of origination of such correlation is as follows. Let the time of expectation of the thermally assisted event of relaxation at the temperature T be [11]

$$\tau \approx \omega_D^{-1}\exp\left(\frac{U-\gamma\sigma_\Sigma}{k_BT}\right) = \omega_D^{-1}\exp\left[\frac{H(\sigma)}{k_BT}\right], \quad (7)$$

where $H(\sigma)$ is activation enthalpy, k_B is the Boltzmann constant, ω_D is a Debye frequency, U is a height of the potential barrier, $\gamma \approx 10^3b^3$ is an activation volume [12], while $\sigma_\Sigma = \sigma_{mech} + \sigma_{ac}$ is a sum of applied stresses σ_{mech} and stresses in an acoustic pulse σ_{ac}. It has been numerically demonstrated [5] that effect of the acoustic pulse reduces the time τ in more than 50 times. The correlation mechanism consists of exchange of the local concentrators with the acoustic emission pulses, which, by affecting the stress concentrator being in a waiting mode, initiate its relaxation according to the mechanism of the acoustoplastic effect [14].

Generation of the macroscopic radius of coherence of the concentrators in the two-component model is explained by Fig. 3. We assume that the concentrator *1* relaxes, generating new dislocations in its neighborhood and emitting the acoustic emission pulse. The new dislocations activate a nearby concentrator *2*, inducing accommodative plastic deformation at the plasticity front and ensuring continuous or jump-like motion of the front, which is described by the function $f(\varepsilon)$ in the equation (1).

The acoustic pulse emitted during relaxation of the concentrator *1* causes relaxation of the concentrator *3* that is at the distance $\sim\lambda$ to the concentrator *1* when a difference of the height of the potential barrier U and work of acting stresses, including stresses in the acoustic pulse,

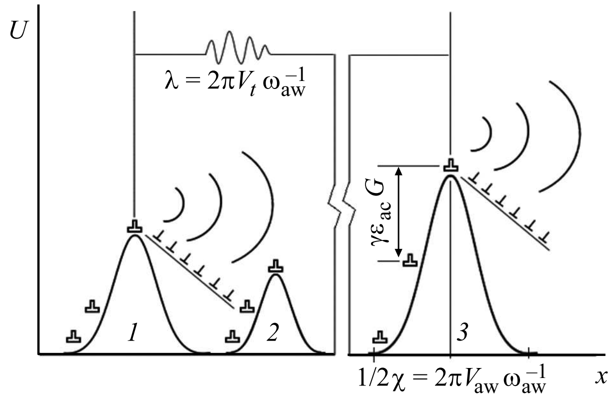

Figure 3. Diagram of excitation of the stress concentrator with the acoustic emission signal.

$(\gamma(\sigma + \delta\sigma_{ac}))$, is zero, i.e. when

$$U - \gamma(\sigma + \delta\sigma_{ac}) = U = bl\frac{\chi}{2}(\sigma + \varepsilon_{ac}G) \approx 0. \quad (8)$$

At this condition, the plasticity front detaches from a local barrier of the width $\chi \approx b$. For this purpose, the concentrator 3 shall be imparted with energy $(bl\chi/2)\varepsilon_{ac}G \approx (bl\chi/2)\delta\sigma_{ac}$, where l is a distance between adjacent barriers, while ε_{ac} is an amplitude of deformation in the pulse. It is clear from Fig. 3 that the condition (8) can be fulfilled if the acoustic pulse emitted during relaxation of the concentrator *1* reaches the concentrator *3* when $U - \gamma\sigma \approx (bl\chi/2)\varepsilon_{ac}$. This requirement is satisfied when there is an equality of times of flight $\vartheta \approx \lambda/V_i$ of the acoustic pulse to the concentrator *3* for the distance $\sim \lambda$ and of a shift of the autowave front $\vartheta = \chi/2V_{av}$ along a slope of the local barrier for a half of its width $\sim \chi/2$:

$$\frac{\lambda}{V_t} \approx \vartheta \approx \frac{\chi}{2V_{aw}}. \quad (9)$$

The velocities $V_{aw} \approx (2\pi)^{-1}\lambda\omega_{aw} \approx 10^{-4}$ m/s and $V_t \approx (2\pi)^{-1}\chi\omega_D \approx 10^3$ m/s in the equation (9) are determined by spatial scales λ and χ and frequencies ω_{aw} for the autowave and the Debye frequency ω_D for the elastic wave. Finally, relaxation of the concentrator *1* turns out to be correlated to relaxation of the concentrator *3* placed at the distance $\lambda \gg \chi$ thereto, thereby explaining origination of the macroscopic scale and generation of the localized plasticity autowaves. An estimation by the equation (9) provides $\lambda \approx \chi \cdot V_t/2V_{aw} \approx 10^{-2}$ m, which coincides with the experimental data.

A plastic-flow scenario presented by a sequence „stress concentrator disintegration → dislocation formation → formation of new concentrator“ agrees with the notion of activity of the deformed medium. It can be assumed that the latter has all the qualities required for generating the localized plasticity autowaves, so that application of the basic provisions of synergetics [2,3,10] to explanation of the nature of the plastic flow is expedient, while the theory of plasticity shall be constructed as a theory of structuring during deformation.

2. On parameters of the plastic flow's autowave processes

It is well known that plastic flow includes serial implementation of pronounced stages of strain hardening [12], which can be distinguished on the flow curves $\sigma(\varepsilon)$ approximated by the Ludwik formula [16] $\sigma(\varepsilon) = \sigma_0 + K\varepsilon^n$, where K is a hardening coefficient, while $\sigma_0 = \text{const}$. For each stage of the process, a hardening index $n = \text{const}$, i.e. $n = 0$ at the stage of the yield plateau, $n = 1$ at the stage of linear hardening, $n = 1/2$ at the stage of parabolic hardening and $n < 1/2$ at the stage of pre-fracture.

A cause of stage changing and a nature of the strain hardening mechanisms are still unclear under the dislocation theory of plasticity. Certain perspectives of solving this problem are related to the autowave model of plasticity [5,6].

2.1. Strain hardening and the autowave dispersion laws

One of the most important provisions to base the autowave model of plasticity [5,6] thereon is a correspondence rule, which stipulates that each stage of strain hardening is uniquely matched with its ow autowave mode of localized plastic flow (Fig. 4). There are grounds to assume that a regular change of the autowave modes during the deformation process is inextricably linked to evolution of the microscopic mechanisms of strain hardening. In this case, it is useful to compare the detailed macroscopic characteristics of the plastic flow's autowave processes [5,6] with rich information about a morphology of dislocation assemblies of the microscopic scale, which are typical for various stages of plastic flow [17]. It seems that it would be of the highest interest of comparing dislocation substructures with dispersion laws of the localized plasticity autowaves, i.e. with dependences of the frequency on the wavenumber $k = 2\pi$ at the various stages of hardening.

The experimentally found dispersion laws $\omega_{aw(k)}$ for each of the stages of the deformation process are shown in Fig. 5. They are described by a parabola $\omega_{aw}(k) \sim k^\beta$, where β discretely varies during an interstage transition [18] (in the same way as the hardening index n in the Ludwik equation). The dimensions may be analyzed to find a dispersion relationship

$$\omega_{aw}(k) \sim \left(\frac{\Lambda^\beta}{\vartheta}\right) k^\beta, \quad (10)$$

in which the coefficient Λ^β/ϑ for each stage of the deformation process includes a structural scale Λ that is typical for it.In the considered approximation, the relaxation

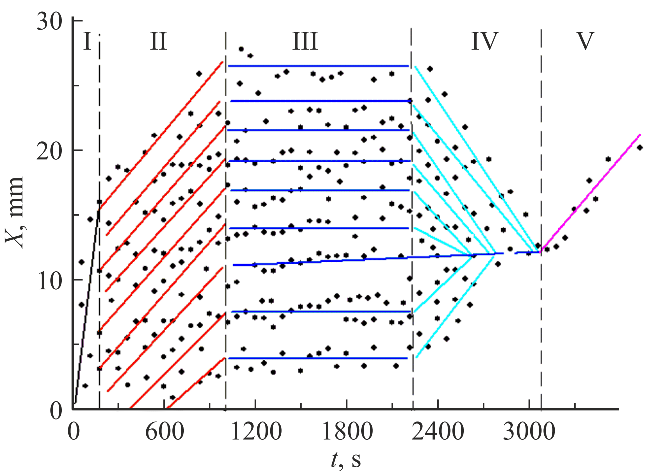

Figure 4. Serial implementation of the plastic flow's autowave modes as exemplified by $X-t$ diagrams for a hydrogenated chromium-nickel steel. The stages are designated as in Fig. 1. V — motion of the fracture neck formed.

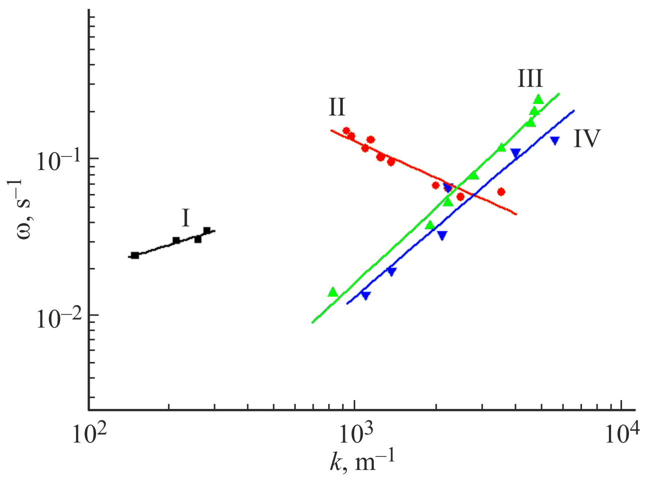

Figure 5. Dispersion dependences for the localized plastic flow's autowaves. The stages are designated as in Fig. 1.

time ϑ is assumed to be the same for all the stages of the plastic flow process.

The equation (10) is analyzed for the distinguished stages of plastic flow to demonstrate that for Luders deformation ($n = 0$; $\beta = 1$) there is linear dispersion $\omega_{aw}(k) \sim (\Lambda/\vartheta)k$, for the stage of linear strain hardening ($n = 1$; $\beta = 2$) the dispersion gets a quadratic type $\omega_{aw}(k) \sim (\Lambda^2/\vartheta)k^2$, and for the stage of pre-fracture (the autowave collapse) ($n < 1/2$; $\beta = 3$) the dispersion becomes cubic $\omega_{aw}(k) \sim (\Lambda^3/\vartheta)k^2$. At the stage of parabolic strain hardening ($n = 1/2$; $2 < \beta = 5/2 < 3$) and $\omega_{aw}(k) \sim (\Lambda^{5/2}/\vartheta)k^{5/2}$. While discussing the nature of the dispersion relationship (10) and the considered consequences therefrom, we assume that variations of the dispersion law are caused by restructuring of the deformed medium and we will try to relate these variations with data about the dislocation mechanisms that act on relative stages of strain hardening.

Thus, during the elastic-plastic transition, at the stage of the yield plateau (Luders deformation) the elastic medium becomes plastically deformed due to mass release of dislocations from stoppers with formation of an avalanche of movable dislocations [16,19,20]. These events are implemented at the Luders band front that moves at the constant velocity. Phase and group velocities of the Luders front

$$V_{aw}^{(ph)} = \frac{\omega}{k} \quad \text{and} \quad V_{aw}^{(gr)} = \frac{d\omega}{dk} \tag{11}$$

are equal, i.e. $V_{aw}^{(ph)} = V_{aw}^{(gr)} = V_{aw}$. By multiplying the right and left parts of the equations (11), we obtain

$$\frac{\omega_{aw} d\omega_{aw}}{k dk} = \frac{\int \omega_{aw} d\omega_{aw}}{\int k dk} = \frac{1/2\omega_{aw}^2 + c_1}{1/2k^2 + c_2} = V_{aw}^2, \tag{12}$$

where c_1 and c_2 are integration constants. Hence, from the equation (12), when $c_2 = 0$, the dispersion law of the kind $\omega_{aw}^2 \sim 1 + k^2$, which is relevant for this stage, follows and when $k^2 \gg 1$ it results in $\omega_{aw} \sim k$.

The Luders front has all the signs of a switching autowave [21] in a bistable medium that consists of interrelated elements with two stable states. Elements of this medium are dislocations that transit from a blocked into a mobile state. The found form of the dispersion relationship for thee autowave at this stage $\omega_{aw}^2 \sim 1 + k^2$ corresponds to the Klein–Gordon equation (KG) [22]:

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + u = 0 \tag{13}$$

for shifts u. At the steady-state mode of deformation $k \gg 1$ and the equation (13) is reduced to the wave equation $\ddot{u} - u'' = 0$ with the linear dispersion $\omega_{aw} = V_{aw}^{(gr)} k = V_{aw}^{(ph)} k$.

At the stage of linear strain hardening, the characteristics of elastic deformation and plastic flow are related by an elastic-plastic invariant $2\lambda V_{aw} \approx \chi V_t$ [6], in which the plasticity autowaves are specified by the length λ and the velocity V_{aw}, while the elastic waves are related by an interplanar spacing χ and a transverse-sound speed V_t. When introducing the Hartree scales [23] for the speed of sound $V_s \approx e^2/\hbar(m/2M)^{1/2}$ and the length $a_0 = \hbar^2/me^2$, where $\hbar = h/2\pi$ is a reduced Planck constant, e and m are the charge and the mass of the electron, respectively and M is an atomic weight, then after replacement $\chi \to a_0$ and $V_t \to V_s$ we have [24]:

$$\lambda V_{aw} = \frac{\chi V_t}{2} \approx \frac{\hbar}{2(mM)^{1/2}}. \tag{14}$$

Here the magnitude $D_{\min} \approx \lambda V_{aw} \approx D_{\varepsilon\varepsilon}$ means a minimum value of kinematic viscosity of the deformed medium. It is calculated for Al investigated in the present study that $D_{\min} \approx 5 \cdot 10^{-7}\,\text{m}^2/\text{s}$, which is close to the experimentally found values [6].

Setting that $\lambda V_{aw} \approx \Lambda^2/\vartheta$ we obtain

$$\lambda V_{aw} = \frac{\Lambda^2}{\vartheta} = \frac{(2\pi/k)^2}{2\pi/\omega_{aw}} = 2\pi\frac{\omega_{aw}}{k^2} \approx \frac{\hbar}{\sqrt{mM}} \approx \text{const}, \tag{15}$$

hence, the quadratic dispersion equation for the linear stage

$$\omega_{aw} = \frac{\hbar}{2\pi\sqrt{mM}} \cdot k^2 = \frac{D_{\min}}{2\pi} \cdot k^2 \sim k^2. \tag{16}$$

The autowave frequency is estimated by the equation (16) and it results in $\omega_{aw} \approx 10^{-3}$ Hz when $k = 10^2$ m, which is close to the experimentally-determined autowave frequency [5]. The quadratic dispersion $\omega_{aw}(k) \approx 1 + k^2$ corresponds to solutions of the nonlinear Schrödinger equation (NSE) [22]:

$$i\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + 2|u|^2 u = 0, \tag{17}$$

where $i = \sqrt{-1}$. This equation describes self-organization of the medium, which serially implements the thermally-assisted plasticity events [11]. An auto-model nature of the deformation structures underlines an auto-oscillation type of elements of the medium at this stage, while an originating autowave mode is considered as a phase autowave [21].

A cubic law of dispersion $\omega_{aw}(k) \sim (\Lambda^3/\vartheta)k^3$ for the stage of collapse of the localized plasticity autowaves corresponds to the Korteweg–de Vries equation (KDV) [22],

$$\frac{\partial u}{\partial t} - \frac{\partial^3 u}{\partial x^3} - u\frac{\partial u}{\partial x} = 0, \tag{18}$$

which describes propagation of solitary excitation pulses of a soliton type in the active excitable media. This effect can be exemplified on the macroscopic level by motion of the originated solitary deformation front at the final stage of pre-fracture just before formation of the fracture neck, as shown in Fig.4 (the stage V).

At the stage of parabolic strain hardening the deformed material forms a stationary dissipative structure [2], which is characterized by the autowave length λ and its velocity $V_{aw} = 0$. The dispersion law $\omega_{aw} \approx k^{5/2}$ for this stage was found by varying deformation conditions and changing the autowave length. Due to an intermediate value of the index $\beta = 5/2$, the stage of parabolic strain hardening can be considered to be a stage of transition from the stage of linear strain hardening, wherein $\omega_{aw} \sim k^2$, to the stage of collapse of the autowave function, for which $\omega_{aw} \sim k^3$.

This point of view is confirmed by the data [12,17], according to which a cellular dislocation structure formed at the beginning of the stage of parabolic strain hardening is replaced by a coil one. It is another argument in favor of a transitional type of the stage of parabolic strain hardening.

It is logical to relate the change of the dispersion relationship to changes of the scale and shape of dislocation assemblies during deformation [17]. We will assume that $\Lambda^2 = \Sigma$ is an area of the surface of dislocation cells at the stage of linear strain hardening and $\Lambda^3 = \Omega$ is a volume of dislocation coils or fragments that originate at the stage of pre-fracture [17]. It gives a geometrical meaning to this interpretation.

2.2. Active deformed medium as the autowave generator

The deformed medium is unique in that it can generate a regular sequence of the autowave modes of the various type when the samples are deformed at the constant rate. This deformed medium is different from chemical media, in which implementation of each of the autowave modes requires special-type reactors, precise control of a reagent concentration and temperature modes of chemical reactions [7]. The deformed sample is a universal generator of the autowave processes, which operates in all temperature-force conditions, a composition and structure of the material. The process of generation of the localized plasticity autowaves is sensitive to loading conditions. Thus, when varying power of the energy flux into the deformed sample $\sim \sigma V_{mach}$, when increasing a tensioning rate V_{mach} the velocity of motion of the localized plasticity autowaves varies proportional to this magnitude [9].

Kinetics of an interstage transition is very important for understanding the nature of the process of deformation in these conditions. It was studied on the example of the transition from the parabolic stage of hardening to the stage of collapse of the autowave during deformation of polycrystalline Al with the average grain size $\sim 40\,\mu$m. Mechanical tests were supplemented with simultaneous recording of the velocity of ultrasound Rayleigh waves V_R. This characteristic is used due to experimental indications that the velocity of the ultrasound waves in the deformed medium is sensitive exactly to movable dislocations [25]. Flat samples cut out of Al sheets, which have working part's sizes $50 \times 5 \times 2$ mm were tensioned at the rate of $3.3 \cdot 10^{-4}\,\mathrm{s}^{-1}$ within the temperature interval $211 \le T \le 350$ K. The velocity of ultrasound Rayleigh waves at the frequency of 3 MHz was measured by a method of autocirculation of an ultrasound pulse. The deformation-coordinated dependences $\sigma(\varepsilon)$ and $V_R(\varepsilon)$ are analyzed below.

We introduce a deformation-dependent magnitude with dimension of a length

$$L^* = \frac{D_{\min}}{\theta^* V_R}, \tag{19}$$

where the value of $D_{\min}$ is determined above, while $\theta^* = G^{-1}d\sigma/d\varepsilon$ is a dimensionless coefficient of strain hardening. For $\theta^* \approx 10^{-4}$ calculation by the formula (19) provides $L^* \approx 8 \cdot 10^{-8}$ m. By assigning this magnitude with a meaning of the path length of dislocations, which is included in the Taylor–Orovan equation (4), we write that $L = \varepsilon/b\rho_{mob}$. If assuming that $L^* \equiv L$ then

$$\frac{D_{\min}}{\theta^* V_R} = \frac{\varepsilon}{b\rho_{mob}}, \tag{20}$$

hence, the equation

$$\rho_{mob} \approx \frac{\theta^* V_R}{bD_{\min}}\varepsilon = \Psi\varepsilon, \tag{21}$$

that is suitable for estimating the density of movable dislocations according to [26]. Results of calculations by the equation (21), which are shown in Fig. 6, *a*, agree with data of the studies [27–29], thereby confirming applicability of the method [26] for estimating the magnitude ρ_{mob}.

It follows from Fig. 6,a that the dependence $\rho_{mob}(\varepsilon)$ has two portions, wherein within each of therm linearity $\rho_{mob} \approx \Psi_\varepsilon$ is fulfilled according to the equation (21). Slope coefficients of these portions $\Psi_1 > 0$ when $\varepsilon < \varepsilon_c$ and when $\varepsilon > \varepsilon_c$, i.e. with deformation $\varepsilon_c \approx 0.02$ the dependence $\rho_{mob}(\varepsilon)$ has a maximum. The temperature dependence of the coefficients Ψ_1 and Ψ_2 (Fig. 6, *b*) was analyzed to determine enthalpy of activation of the respective processes $H_1 = U_1 - \gamma_1\sigma_1 \approx 0.09$ eV and $H_2 = U_2 - \gamma_2\sigma_2 \approx -0.08$ eV. Smallness of strain hardening of Al at large deformations makes it possible to consider that $\gamma_1\sigma_1 \approx \gamma_2\sigma_2 = \gamma\sigma \approx 0.03$eV [6]. At the

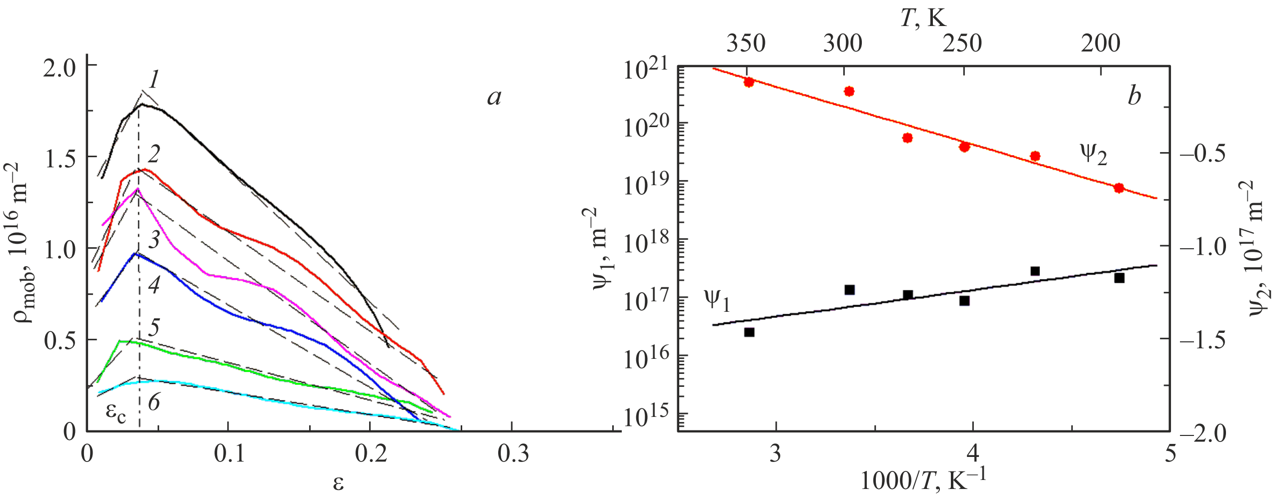

Figure 6. Density of dislocations as a function of deformation for the various temperatures; the dashed line marks linear approximation of the dependences (*a*); the temperature dependence of the coefficients in the relationship (21), too (*b*).

same time, for deformations $\varepsilon < \varepsilon_c$ a height of the barrier $U_1 = H_1 + \gamma\sigma \approx 0.12\,\text{eV} > 0$ and it was overcome by thermal activation. For deformations $\varepsilon > \varepsilon_c$, $U_2 = H_2 + \gamma\sigma \approx -0.05\,\text{eV} < 0$ and motion of dislocations is controlled by their capture with potential wells (traps) when a new dislocation assembly is formed, which is marked as a drop of the density of movable dislocations. Thus, with critical deformation there is a significant change of the behavior of dislocations and the mechanism of strain hardening. It is confirmed by data of electron-microscopic analysis in [28], according to which with deformation $\varepsilon_c \approx 0.02$ a dislocation chaos is replaced by formation of dislocation cells in the dislocation substructure of deformed Al.

Taking into account results of the study [19], the coefficient of proportionality of Ψ in the equation (21) can be written as

$$\Psi \approx \frac{2\theta V_R (mM)^{1/2}}{\hbar b}, \qquad (22)$$

hence, $10^{17} \geq |\Psi| \geq 10^{16}\,\text{m}^{-2}$. This value is close to the experimentally-determined values of Ψ_1 and Ψ_2 and a difference of moduli can be explained by different values of the coefficient of strain hardening $\theta^* > 0$ at the various stages of plastic flow.

A change in the sign of the coefficient Ψ in the dependence $\rho_{mob}(\varepsilon)$ during plastic flow has a deep physical meaning. Formally, this fact is explained by two signs of the magnitude $\pm(mM)^{1/2}$ in the equation (22), but it is explicitly insufficient to understand its physical nature. If $D_{\min}$ is kinematic viscosity of the medium [19], then when $\Psi_2 < 0$ the magnitude becomes negative, which can be puzzling.

However, it is known that negative viscosity is not physically meaningless [30]. Its origination is related to a reverse of energy transfer at certain conditions, i.e. to its transmission from orderless (thermal) motion to ordered one, which complies to excitation of the hydrodynamic modes [3,10], i.e. self-organization of the medium [31]. This situation is typical for the open systems, in which irreversible processes [2,3,10] underlying physics of plasticity are developed. Instead of energy dissipation that is common for isolated systems, the synergistic approach addresses spontaneous formation of dissipative structures [2] (the localized plasticity autowaves) and in this case viscosity of the medium shall be negative. Moreover, the author [10] believed that his condition was necessary for self-organization of the deformed medium and generation of the autowave mode therein.

This explanation agrees with the above-listed data about the dislocation nature of critical deformation ε_c, at which the weakly-ordered substructure of the dislocation chaos is replaced by the cellular structure with the higher order degree that is specified by a geometrical configuration of the dislocation cells [15].

Conclusion

It has been demonstrated by discussion of the obtained results that using the basic representations of the autowave theory of plastic flow can be fruitful for studying details and patterns of the complex phenomenon of plastic flow of solid bodies.It was possible to relate the dislocation and autowave representations about plastic flow and to demonstrate that its staging is based on formation of active media that are specific to each stage of the process. The dispersion law for each stage of the process is also related to a dislocation structure that is typical for this stage. The autowave theory makes it possible to explain time evolution of patterns of plastic flow localization. Serial application of the autowave representations made it possible:

— to explain the nature of generation of the autowave processes of plastic flow during plastic deformation and origination of macroscale nonuniformities of the deformation structure of the deformed solid body;

— to relate the stages of strain hardening during plastic flow to the dispersion laws of the localized plasticity autowaves and to the dislocation structure of the deformed medium, to propose mechanisms of serial generation of the localized plasticity autowaves and to obtain quantitative estimates of their autowave length and frequency;

— to demonstrate that each stage of strain hardening can be matched with one of the nonlinear evolution equations (KG, NSE, KDV) that describe collective excitations of this medium;

— to propose a mechanism of the transition between the stages of strain hardening, which is related to changes of dislocation characteristics of the deformed medium and results in self-organization of the deformed medium.

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Conflict of interest

The authors declare that they have no conflict of interest.

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