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Selection of an efficiency criterion of gas mixing based on the data on a transverse jet in a supersonic flow in the presence of spark discharges

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Four gas mixing criteria have been compared by using data obtained by numerical modeling of a transverse transonic gas jet in a supersonic air crossflow in the presence of spark discharges. The relationship between the criteria and their behavior have been examined in dependence on the flow structure. The paper shows that instantaneous values of the criteria are not directly interrelated. Nevertheless, the criteria averaged over time are consistent with each other in most cases. Selection of an acceptable criterion for experimental studies is discussed.

Keywords: supersonic flow, injection, mixing, spark discharge.

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A transverse secondary transonic jet injected into a supersonic air flow through an opening in the flat wall is one of the standard flow configurations used as examples to study methods for intensifying gas mixing under the supersonic flow conditions [1]. Investigation of new methods for rapid mixing of gases (e. g., fuel with air) is a pressing issue in view of improving combustion chambers with supersonic flows at the inlet [2]. This is evidenced by a wide variety of proposed mixing strategies, e. g., those based on selecting optimal jet parameters for different geometric configurations of the supersonic tract [3,4], on modulating the gas flowrate [5], and on other active methods [6].

At present, research calculations involve, as a rule, the following integral criterion for the efficiency of jet-flow mixing [1]:

$$\eta_m(\rho, U, W) = \frac{\int W_r \rho U dA}{\int W \rho U dA}, \ W_r = \min\left\{W, W_{st}\frac{1-W}{1-W_{st}}\right\}. \tag{1}$$

The integrals are taken over a certain plane perpendicular to the flow, W is the jet gas mass fraction, U is the velocity longitudinal component, ρ is the density, W_{st} is the jet gas mass fraction in the stoichiometric mixture, W_r is the mass fraction of fuel able to react with air within a certain elementary volume.

$\eta_m = 0$ means the absence of mixing, while $\eta_m = 1$ means the absence of areas containing a rich fuel-air mixture (which, however, does not exclude the presence of areas with a lean mixture). Paper [8] shows that in the case of combustion η_m correlates with the fuel combustion coefficient. In [8] it is noted that it is important to substitute into (1) instantaneous values rather than average ones, since the combustion rate is determined by instantaneous substance concentrations at each point.

However, η_m is not used in experiments because it requires measuring the velocity, density and chemical composition of the mixture at a large number of points. Papers devoted to jets in supersonic and subsonic flows present indirect methods for quantitatively describing the process of mixing. For instance, in [9] the cross-section area of a seeded jet illuminated by a laser sheet was measured. In the figures of [10,11], the jet boundary in the given planes was visualized and presented explicitly in the same way, which made it possible to find the boundary length. In [12], the method of planar laser-induced fluorescence (PLIF) was used to examine instantaneous distribution of the jet substance concentration; concentration fields and profiles were constructed. A decrease in concentration indirectly evidences that mixing is improved.

By now, the relationship between integral criterion (1) and simpler criteria potentially applicable in experiments remains unstudied.

The goal of this study was to compare several criteria for gas mixing using as an example specific non-stationary data obtained by numerical simulations, and thus to determine which criteria best match relation (1) and what advantages and disadvantages they have for their application in experimental investigations. Two techniques of comparing were used. First, the criteria values as functions of time were compared for one set of non-stationary data. Then, average criteria values calculated from several data sets were compared.

Data for the analysis was obtained by numerical simulation of the flow formed during transverse injection of a carbon dioxide (CO_2) transonic jet into a supersonic air flow in the presence of spark discharges at the injector's leading edge. The computational problem definition is determined by the continuity with respect to previous studies [13,14] devoted to intensification of mixing a supersonic flow with transverse gas jet by using spark discharges inducing disturbances in the shear layer at the jet boundary.

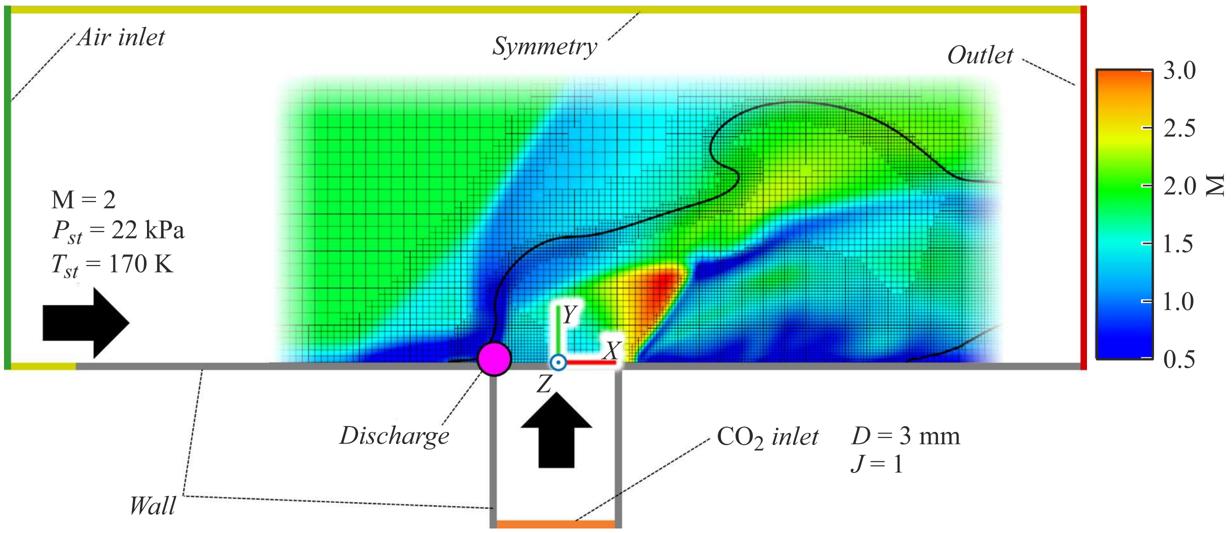

Figure 1. Computational domain, boundary conditions and location of the heat source simulating the discharge. The computational grid cross-section, instantaneous distribution of the Mach number, and conditional boundary of the jet (30 % of the CO_2 mass fraction, black curve) are shown.

The computational domain is shown in Fig. 1. Since the problem is symmetrical, only the $Z > 0$ half-space was modeled. The supersonic flow parameters are as follows: Mach number $\mathrm{M} = 2$, static pressure $P_{st} = 22$ kPa, static temperature $T_{st} = 170$ K, velocity $U_\infty = 500$ m/s. The jet parameters are: injector diameter $D = 3$ mm, jet-to-crossflow momentum flux ratio $J = 1$, velocity at the center of injector (in the absence of discharges) $U_{jet} = 270$ m/s. The discharges were simulated as pulsed heating in a certain limited area at the edge of injector (Fig. 1). Each discharge emitted energy $E = 25$ mJ during $2\,\mu$s. Discharge frequency f was set to a value from the range of 8 to 50 kHz.

Numerical simulation was performed in software package FlowVision by the URANS method. Study [13] has demonstrated the grid convergence of the solution and validated the method on the example of a particular case of transverse jet in supersonic flow. Comparison of the simulation results with experimental data performed in [14] has shown that the selected calculation method correctly determines the shape and frequency of transverse vortices in the shear layer at the jet boundary, and also correctly determines the flow character in the presence of spark discharges. In this study we used a grid with dynamic adaptations to the gradient of CO_2 density and concentration (Fig. 1). The time increment ranged from 30 ns (during the discharge) to 100 ns (between discharges).

Four criteria for the mixing efficiency were calculated from the data obtained in the $X/D = 5$ plane, i. e., in the plane perpendicular to the free supersonic flow and located $5D$ downstream from the injector center. Fig. 1 shows the directions of the X, Y, Z axes. The reference zero (origin of coordinates) is assumed to be in the center of the injector opening. The first criterion is integral η_m (1). Since the air/CO_2 mixture is non-reactive, W_{st} was arbitrarily set to 0.3. Second criterion L is the length of isoline $W = 0.3$ in the given plane. Third criterion $c_{\max}$ is the maximum CO_2 mass concentration in the same plane. Fourth criterion S is the area of the region bounded by isoline $W = 0.3$, i. e. the jet cross-section area.

Fig. 2 presents the plots of instantaneous values of η_m, L, $c_{\max}$. Criterion S is not shown since its behavior is similar to that of L. In addition, images obtained by integrating the CO_2 concentration along the Z axis are presented for two time points (A and B) (Fig. 3). Axis Z is oriented perpendicular to the figure plane (Fig. 1). Its zero point (origin of coordinates) is in the center of the injector opening.

Fig. 2 presents also the W distribution in the $X/D = 5$ plane for the discharge frequencies of $\mathrm{Sh} = 0.24$ and 0.3 at the time points when the next large disturbance in the jet-boundary shear layer (transverse vortex) passes through this plane. Here the characteristic time is $\tau = D/U_\infty$ and dimensionless frequency is $\mathrm{Sh} = fD/U_\infty$. Velocity of the free supersonic flow is $U_\infty = 500$ m/s.

Let us consider temporal interrelation of the criteria. At time point A, minimum η_m coincides with maximum $c_{\max}$. This means that the passage of a portion of mixture with high CO_2 content through cross-section $X/D = 5$ results in a decrease in the mixing quality. Indeed, Fig. 3, a shows that at this moment a large disturbance (transverse vortex) passes through this cross-section. At the same time, criterion L reaches a plateau, i. e., despite the passage of a mixture portion with high CO_2 concentration through the mentioned section, the length of the conditional jet boundary remains approximately the same. The relatively high value of L at this moment may be explained by that the W isosurface gets bent in the region of transverse vortex. This is, for instance, shown in Fig. 1 where the upper conditional boundary of

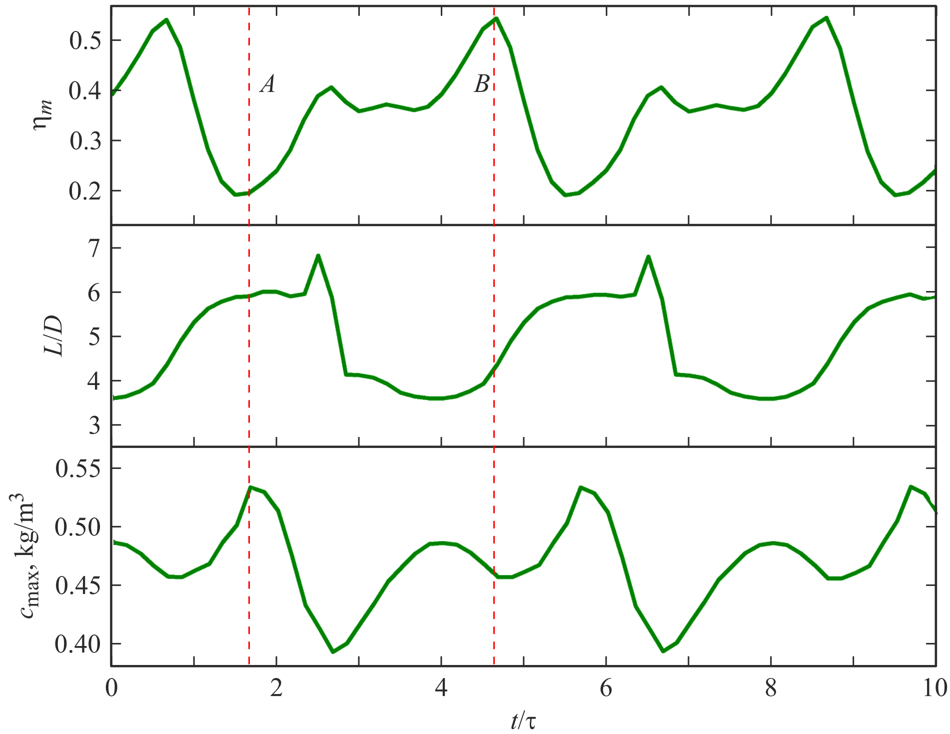

Figure 2. Plots of the mixing criteria in the absence of discharges. For these data, the coefficient of correlation between $c_{\max}$ and η_m is -0.57, that for L and η_m is -0.49.

the jet is S-shaped and three times intersects the $X = \text{const}$ vertical plane.

In the interval between time moments A and B, criterion L decreases sharply; at the same time, η_m reaches the plateau and $c_{\max}$ gets minimized. The decrease in L is caused by the fact that plane $X/D = 5$ stops intersecting the next transverse vortex. This also explains the $c_{\max}$ minimization: probably, the highest CO_2 concentration is achieved in the transverse vortex region. Thereat, the η_m exit to the plateau corresponds to passing through plane $X/D = 5$ of the jet's homogeneous section between two transverse vortices.

At moment B, criterion η_m reaches its maximum, while criterion $c_{\max}$ reaches neither maximum nor minimum. Indeed, as per the sense of criterion η_m, this situation means that the jet and flow are best mixed, i. e. the CO_2 spatial distribution is most uniform. At this moment, criterion L increases, which may be associated with the start of the next transverse vortex passage through plane $X/D = 5$. This is consistent with Fig. 3, *b* which shows that the expanding jet section approaches plane $X/D = 5$.

Thus, there is no strong correlation between instantaneous values of different criteria (Fig. 2). Now consider their averaged values in the $X/D = 5$ plane for several data sets. Fig. 4 presents a plot of averaged criteria values for several discharge frequencies. For convenience, the criteria are presented in the dimensionless form: for each frequency there is indicated the ratio of averaged criteria at given frequency (η_m, L, $c_{\max}$, S) to criteria values in the absence of discharges ($\eta_{m,0}$, L_0, $c_{\max,0}$, S_0).

All the criteria show that the best mixing gets achieved at discharge frequencies close to $\mathrm{Sh} = 0.24$. This „optimal" frequency is close to natural frequency $\mathrm{Sh} = 0.24$ of transverse vortices formation in the shear layer on the jet windward side (or $\mathrm{Sh}_j = fD/U_j = 0.44$, if the dimensionless frequency is expressed through the jet velocity [14]). Criteria L and η_m exhibit a well pronounced maximum in the range of about $\mathrm{Sh} = 0.24$. Vice versa, criteria $c_{\max}$ and S distinguish only slightly between the cases in the discharge frequency range of 0.18 to 0.27. In transition from $\mathrm{Sh} = 0.27$ to $\mathrm{Sh} = 0.3$, the S criterion increases sharply, while η_m, L and $c_{\max}$ manifest worsening of mixing. At the same time, η_m becomes even lower than in the case without discharges (note that this is not reflected in the L and $c_{\max}$ behavior). This is due to a significantly smaller transverse vortex size compared to that in other cases. Therefore, when the next transverse vortex crosses the $X/D = 5$ plane (Figs. 3, *c*, *d*), then, in the case of $\mathrm{Sh} = 0.3$, instead of two large regions with $W > 0.3$ there remains only one such region with a large area but short boundary.

Thus, the study has shown that instantaneous criteria values do not correlate with each other in time, since they respond differently to large disturbances at the jet boundary. Nevertheless, time-averaged η_m, L, $c_{\max}$ indicate the existence of the „optimal" discharge frequency range corresponding to the best mixing. Criterion $c_{\max}$ may

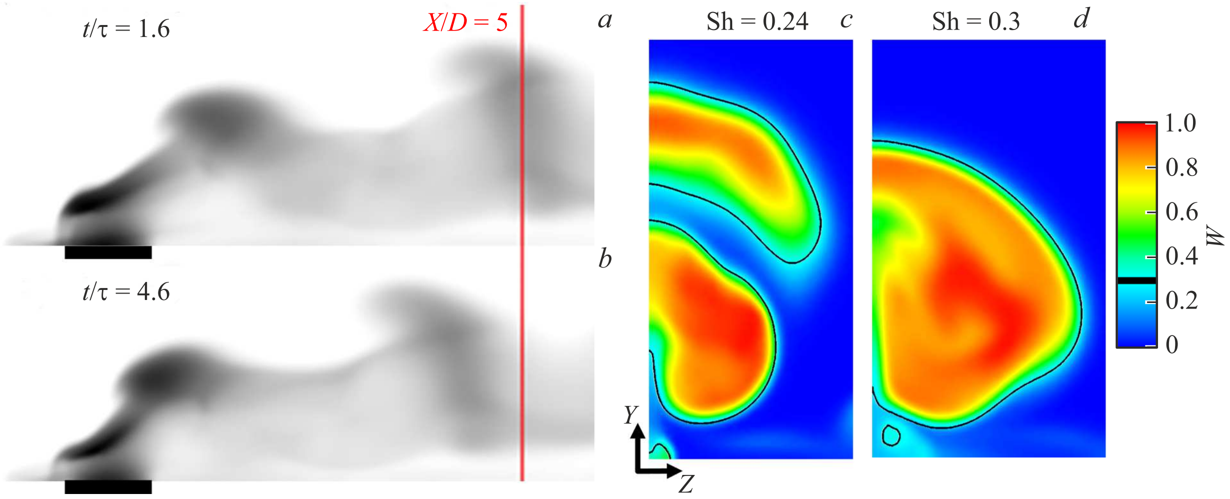

Figure 3. The flow under study. *a*, *b* —an integral over Z of the CO_2 concentration for two time poins (A and B in Fig. 2); *c*, *d* — typical W distribution in plane $X/D = 5$ for the cases with $\mathrm{Sh} = 0.24$ and 0.3, respectively, at the moment of the transverse vortex passage.

be diagnosed by the PLIF method in the same way as in [12]. In this case it is necessary to take into account non-uniformity of laser radiation; however, this criterion is the most direct one and does not require additional assumptions and calculations. Criteria S and L may be determined, e. g., by visualizing the jet by seeding, as in [9,10]. Seeding should be uniform to ensure correct determination of the concentration level lines, while calculation of L and S needs the use of image processing algorithms. Criteria S and $c_{\max}$ are least selective. Criterion S is able to exhibit values contradicting η_m. Hence, in experiments it is preferable to record jet boundary length L, since this criterion is most selective.

The results of this study may be useful in planning and conducting experiments aimed at studying different methods for accelerating gas mixing in high-velocity flows. Detailed data on the relationship between several mixing criteria will allow for more reasonable selection of diagnostic methods and analysis of experimental data. In addition, the presented results are important for studying the method for mixing gases with the aid of spark discharges and physical mechanisms responsible for the mixing intensification.

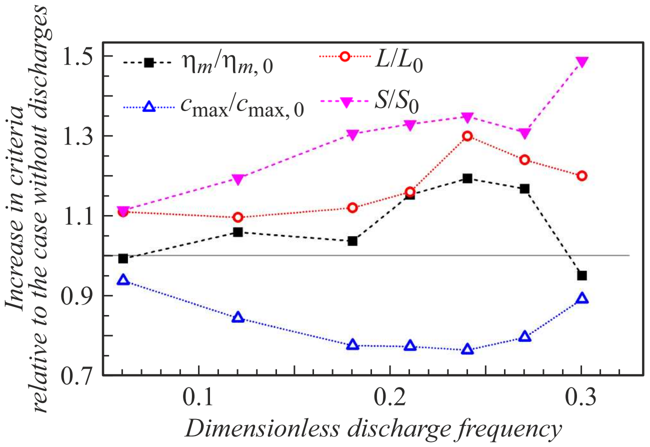

Figure 4. Averaged mixing criteria versus the discharge frequency.

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Conflict of interests

The authors declare that they have no conflict of interests.

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