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Band–pass filter for Cherenkov radiation

© S.N. Galyamin, I.A. Klimov, S.G. Grigoriev, A.V. Tyukhtin

St. Petersburg State University,
199034 St. Petersburg, Russia
e-mail: s.galyamin@spbu.ru

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The influence of a simple nonlocal surface, a thin-wire fine-periodic grid of longitudinal wires, on the characteristics of the electromagnetic field generated by a charged particle bunch moving along the axis of a cylindrical vacuum channel in an unbounded dielectric is investigated. The grid is located in vacuum at a distance of the order of the structure period from the interface. To solve the problem, the averaged boundary conditions are used, taking into account corrections describing the influence of the interface on the averaged surface current excited in the grid plane. It is shown that this configuration works as a band–pass filter, making it possible to distinguish a rather narrow band of frequencies in the Cherenkov radiation spectrum.

Keywords: Cherenkov radiation, bunch form factor, fine–periodic grid, thin-wire grid, averaged boundary conditions, band-pass filter.

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Introduction

A problem of generation of Cherenkov radiation (CR) in a flight of a charged particle bunch through a channel in a dielectric medium is a key problem for several promising fields of modern electrodynamics, such as developing new methods of noninvasive diagnostics of particle beams in accelerators [1–4] and designing new types of radiation sources [5–12].

A solution of this problem in the simplest formulation is well known [13]. In particular, with a quite narrow channel CR is generated almost in the same way as in an unbounded dielectric and the influence of the channel can be generally compared to a low-frequency filter, since high frequencies or short wavelengths (of the order of a channel size and less) are significantly suppressed in the CR spectrum [14,15].

As known, CR can be generated not only in the dielectric, but in any other slow-wave structure as well. Therefore, radiation of moving charged particle bunches in presence of fine-periodic structures made of the so-called wires (i.e., elongated conductors of a various cross section) has been quite actively studied. In particular, large interest was attracted to the so-called wire medium, a three-dimensional lattice of thin wires (a cross size of the conductors is small as compared to a distance between them), which is considered in a long-wavelength approximation (in this case, a more precise term will be a meta-medium) [16–18]. Thus, some studies have analyzed CR both in unbounded [19–21] and semi-bounded wire meta-media [22] mainly in a context of developing methods of diagnostics of the charged particle bunches. The wire medium was also invariably present in theoretical and experimental studies of left-handed media [23,24], including studies on the subject of radiation of moving charges [25–27].

Generation of radiation by the moving charges in presence of the two-dimensional fine-periodic structures (grids) also attracted significant attention of researchers [28,29]. In the long-wavelength approximation (i.e. when the wavelength significantly exceeds the distances between the conductors; as applied to the grids, a term „fine-periodic“ is synonymous to this approximation) and assuming that the transverse sizes of the conductors are small as compared to a period, these grids can be described by means of Kontorovich averaged boundary conditions (ABC) [30–32], thereby significantly simplifying an analysis. In this formulation, the study [33] has investigated CR generation in an open round waveguide (in vacuum) with a fine-wire grid wall made of crossing wires. In particular, it was shown in the study [33] that the wire waveguide made only of longitudinal wires does not support Cherenkov-type radiation generation.

We also note that one-dimensional grids made of thick conductors can also be averagedly described in the long-wavelength approximation by means of the so-called Weinstein-Sivov equivalent boundary conditions [34], but this situation is not considered in the present study. Nevertheless, in this regard, one should mention, for example, the study [35], which has investigated a wakefield dielectric acceleration structure (the round waveguide filled with the dielectric and provided with an axial vacuum channel), whose external wall was not a solid metal housing, but was formed by a dense one-dimensional grid of longitudinal conductors. Besides, the study [36] has analyzed a multilayer frequency-tunable wakefield structure, in which a layer of control electrodes (the constant voltage supplied to them changes permittivity of an adjacent ferroelectric layer) was a one-dimensional grid of longitudinal conducting tapes. In both cases, such azimuthal segmentation of the conducting cylindrical layers results in suppression of azimuthal surface currents and prevents excitation of spurious nonsymmetrical modes.

In the present study, we deal with the key problem (mentioned in the very beginning of the present review) on CR generation in the flight of the charged particle bunch through the dielectric channel and study the influence of the fine-periodic grid made of parallel wires, which is arranged in the channel near the dielectric surface, on the radiation characteristics. It is necessary to note that the discussed grid is the simplest example of the so-called nonlocal surface, since the respective ABCs include a tangential derivative of the averaged surface current (in a Fourier representation it means a dependence of ABC coefficients on components of a wave vector). In general, the nonlocal surfaces have attracted close attention of the researchers in recent years, since they provide wide capabilities for controlling characteristics of electromagnetic radiation [37–39]. However, the literature we know has not investigated this issue in a context of control of the structure of the CR field. Moreover, boundary problems with ABC application in case when the grid is close to the dielectric boundary (in this situation the ABC coefficients are significantly modified [40,41]) and a field source is a uniformly moving charged particle bunch, have not been analyzed in the literature we know. The main physical result of the study is detection and description of a new physical effect that is called a band-pass filter effect for CR.

1. Formulation of problem. Field of the charge bunch in the unbounded dielectric medium

Let an homogeneous isotropic dielectric with $\varepsilon > 1$ ($\mu = 1$) have a vacuum cylindrical channel of the radius a, along whose axis the charge bunch uniformly moves at the velocity $\upsilon = c\beta$ (c is a speed of light in vacuum) (Fig. 1) (we use a cylindrical system of coordinates r, φ, z), wherein $\varepsilon > \beta^{-2}$, i.e. the CR condition is fulfilled in the dielectric. We will assume that the studied bunch is an infinitely-thin uniformly-charged disk of the radius of b (the influence of longitudinal smearing is discussed at the end of Section 1).

At the distance h from the vacuum-dielectric interface, there is the wire waveguide made of thin wires with the cross-section radius r_0, which are uniformly arranged at the distance $p = (a - h)\Delta\varphi$ from each other (p is counted along an arc, $\Delta\varphi = 2\pi/N$, $N \gg 1$ is a total number of the conductors) and oriented along the channel axis (along the axis z). It is easy to realize the described structure in practice by placing a layer of a material with near-unit permittivity between the conductors and the dielectric. This material can be, for example, a hydrophobic aerogel, which is specially designed for beam applications, whose refractive index n may be within a wide range of near-unit values: $1.0026 < n < 1.26$ [42].

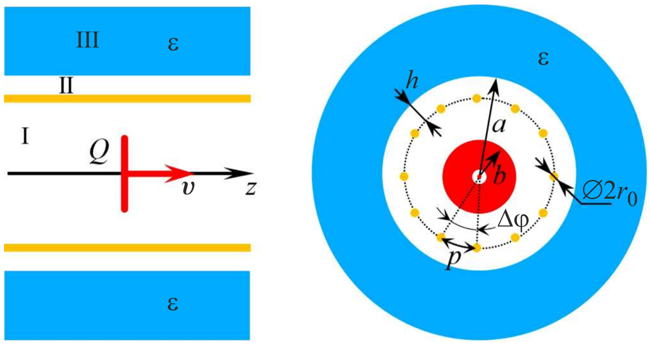

Figure 1. Geometry of the problem and main designations

For certainty, the wires are deemed to be round in the present study, but this circumstance is not fundamental: the wires of another cross section just require to replace r_0 with an effective radius [30]. It is assumed that $b < a - h$ (the bunch is totally inside the wire waveguide) and the following inequalities are fulfilled:

$$r_0 \ll p \ll \lambda, \tag{1}$$

where λ is a wavelength. It is required to determine the electromagnetic field within the entire space, but the main interest lies in a dielectric domain III ($r > a$).

In the used system of coordinates, a charge density ρ and a current density $\mathbf{j} = j\mathbf{e}_z$ of the bunch in question are written as

$$\rho = \frac{Q}{\pi b^2}\theta(b - r)\delta(z - \upsilon t), \quad j = \upsilon\rho, \tag{2}$$

where θ is a unit step function (a Heaviside function), δ — a delta function, Q is a full charge of the disk.

For further discussion, it is expedient to first consider the electromagnetic field that is created by a source (2) in the unbounded isotropic nonmagnetic medium with arbitrary permittivity $\tilde{\varepsilon}$ (the further discussion will include cases when $\tilde{\varepsilon} = \varepsilon$, 1). We will briefly provide respective calculations and make some remarks about a result. We assume that the field components and sources are decomposed into a Fourier frequency integral written as

$$H_\varphi = \int_{-\infty}^{+\infty} H_{\omega\varphi} e^{-i\omega t}\, d\omega = 2\,\mathrm{Re}\int_{0}^{+\infty} H_{\omega\varphi} e^{-i\omega t}\, d\omega, \tag{3}$$

(due to he second equation it is enough to consider only positive frequencies in (3), $\omega > 0$). In particular, for a Fourier transform of the charge density (2) we have

$$\rho_\omega = \frac{Q}{2(\pi b)^2}\frac{\theta(b - r)}{\upsilon} e^{i\frac{\omega}{\upsilon}z}. \tag{4}$$

A system of the Maxwell's equations for Fourier harmonics is written as follows

$$\begin{cases} \mathrm{rot}\vec{E}_\omega = ik_0\vec{H}_\omega, \\ \mathrm{div}\vec{E}_\omega = 4\pi\tilde{\varepsilon}^{-1}\rho_\omega, \\ \mathrm{rot}\vec{H}_\omega = -k_0\tilde{\varepsilon}\vec{E}_\omega + 4\pi c^{-1}\vec{j}_\omega, \\ \mathrm{div}\vec{H}_\omega = 0, \end{cases} \tag{5}$$

where $k_0 = \omega/c = 2\pi/\lambda$, and the following material relationships are taken into account: $\vec{B}_\omega = \vec{H}_\omega$, $\vec{D}_\omega = \tilde{\varepsilon}\vec{E}_\omega$. It is convenient to introduce a single-component vector potential $\vec{A}_\omega = A_\omega\vec{e}_z$ and a scalar potential Φ_ω, through which the field vectors are expressed in a standard way: $\vec{B}_\omega = \mathrm{rot}\vec{A}_\omega$, $\vec{E}_\omega = -\nabla\Phi_\omega + ik_0\vec{A}_\omega$. By applying a Lorentz calibration condition $\partial A_\omega/\partial z - ik_0\tilde{\varepsilon}\Phi_\omega = 0$, in the standard way we will obtain two heterogeneous Helmholtz equations for the introduced potentials, whence a relation between them, $A_{\omega z} = \tilde{\varepsilon}\beta\Phi_\omega$, which makes it possible to further search for, say, only the potential Φ_ω. The source (4) imposes its dependence on z on all the magnitudes, therefore, we factorize the required potential as follows: $\Phi_\omega(r, z) = \phi_\omega(r)\exp(ik_0 z/\beta)$. As a result, we obtain a heterogeneous zero-order Bessel equation for a radial part of the potential ϕ_ω:

$$\begin{aligned}(\Delta_r + \tilde{s}^2)\,\phi_\omega(r) &= -\frac{2Q}{\tilde{\varepsilon}\pi b^2}\frac{\theta(b-r)}{\upsilon} \\ &= -\frac{2Q}{\tilde{\varepsilon}\pi\upsilon b}\int_0^\infty J_0(\eta r)J_1(b\eta)d\eta, \end{aligned} \tag{6}$$

where $J_{0,1}$ — Bessel functions, $\Delta_r = \partial^2/\partial r^2 + r^{-1}\partial/\partial r$, $\tilde{s}^2 = k_0^2\beta^{-2}(\tilde{\varepsilon}\beta^2 - 1)$. An incident field that is of interest to us is a particular solution of the equation (6), which can be found by various known methods. In particular, the right-hand part side can be decomposed into a zero-order Fourier–Bessel integral (see. (6), the second equation) [43] and to search for $\phi_\omega(r)$ as a respective reciprocal transformation, thereby leading us to the following result:

$$\phi_\omega(r) = \frac{2Q}{\tilde{\varepsilon}\pi b\upsilon}\int_0^\infty \frac{J_1(b\eta)J_0(\eta r)}{\eta^2 - \tilde{s}^2}\,d\eta = \frac{iQ}{2\upsilon\tilde{\varepsilon}}\,2\,\frac{J_1(b\tilde{s})}{b\tilde{s}}\,H_0^{(1)}(r\tilde{s}),$$

$$r > b, \tag{7}$$

where $H_0^{(1)}$ — a Hankel function, $\tilde{s} = \sqrt{\tilde{s}^2}$, $\mathrm{Im}\,\tilde{s} > 0$ (as usual, small absorption is taken into account in the medium). It is also convenient to include a magnitude $\tilde{\sigma} = \sqrt{-\tilde{s}^2}$, $\mathrm{Re}\,\tilde{\sigma} > 0$, wherein $\tilde{s} = i\tilde{\sigma}$. As a result, the potential of the charged disk Φ_ω can be written as follows:

$$\Phi_\omega = F_b\Phi_\omega^{\tilde{\varepsilon}}, \tag{8}$$

where magnitudes in the right-hand part side of the equation (8) can be written in two equivalent forms:

$$\Phi_\omega^{\tilde{\varepsilon}} = e^{i\frac{\omega}{\upsilon}z}\begin{bmatrix} \dfrac{iQ}{2\upsilon\tilde{\varepsilon}}H_0^{(1)}(r\tilde{s}), \\ \dfrac{Q}{\pi\upsilon\tilde{\varepsilon}}K_0(r\tilde{\sigma}), \end{bmatrix} \quad F_b = \begin{bmatrix} \dfrac{2J_1(b\tilde{s})}{b\tilde{s}}, \\ \dfrac{2I_1(b\tilde{\sigma})}{b\tilde{\sigma}}. \end{bmatrix} \tag{9}$$

Here, K_0 — a Macdonald function, I_1 is a modified Bessel function, $\Phi_\omega^{\tilde{\varepsilon}}$ — a field potential of a point charge Q that uniformly moves in the medium with permittivity $\tilde{\varepsilon}$, and F_b is the so-called transverse form factor of the disk, i.e. a multiplier, by which the potential of the moving charged disk of the radius b differs from the potential of the moving point charge. The same form factor will differentiate field components of the point charge and the bunch, while the latter are calculated as follows:

$$E_{\omega z} = i\beta k_0^{-1}\tilde{s}^2\Phi_\omega, \quad H_{\omega\varphi} = \tilde{\varepsilon}\beta E_{\omega r} = -\tilde{\varepsilon}\beta\partial\Phi_\omega/\partial r. \tag{10}$$

In other words, in this case TM polarization is excited relative to a z-coordinate [44], wherein the components $E_{\omega\varphi}$ and $H_{\omega r}$ are zero due to axial symmetry, $\partial/\partial\varphi = 0$. Finally, the magnetic field has the only component $H_{\omega\varphi}$ and the electric field lies within the plane (r, φ), as usual for the field of an axisymmetric bunch in the isotropic medium [14].

A record according to the upper line in (9) is convenient in a case (i), when $\tilde{\varepsilon} > \beta^{-2}$, i.e. when the bunch moves in the medium at the velocity that exceeds a CR threshold. According to the formulation of the initial problem (Fig. 1), the case (i) would occur if the bunch in question moves in the unbounded medium with permittivity of the domain III, i.e. when $\tilde{\varepsilon} = \varepsilon$. At the same time, $\tilde{s}$ is real and written as $\tilde{s} = s \equiv k_0\beta^{-1}\sqrt{\varepsilon\beta^2 - 1}$.

The lower line in (9) is convenient in the case (ii), when $\tilde{\varepsilon} < \beta^{-2}$, i.e. when the bunch moves more slowly than the Cherenkov threshold. This case is exactly realized in the considered problem (Fig. 1), since the bunch moves in the vacuum domain I, $\tilde{\varepsilon} = 1$ and the CR condition can not be fulfilled, wherein $\tilde{\sigma} = \sigma \equiv k_0\beta^{-1}\sqrt{1 - \beta^2} = k_0/(\beta\gamma)$, where γ is a Lorentz factor of the bunch.

Fig. 2 show typical dependences of the form factor on an argument (which is proportional to the frequency ω). It is clear that in both the cases $F_b \to 1$ when $b \to 0$. However, in the case (i) the form factor oscillates with a decrease and turns into zero at the values of sb, which are equal to zeros of the Bessel function J_1 (the field of the moving charged disk is absent at these parameters due to destructive interference). An upper limit of the CR spectrum can be assumed to be a frequency that satisfies the relationship $sb = j_{11} \approx 3.832$, where j_{11} is the first zero of the Bessel function J_1. In other words, in this case transverse charge smearing is equivalent to an effect of the low-frequency filter with a cut-off frequency $\omega_0 = cb^{-1}\beta j_{11}/\sqrt{\varepsilon\beta^2 - 1}$.

On the contrary, in the case (ii) the form factor exponentially increases, since this case includes summing nonoscillating evanescent waves, whose amplitudes have the same sign, i.e. interference is totally constructive. An exponential growth of F_b does not result in a physical contradiction, since the field potential of the point charge $\Phi_\omega^{\tilde{\varepsilon}}$ is itself exponentially small and decreases with an increase of σ faster than the form factor ($r > b$): $\sim\exp(-\sigma r)$ vs $\sim\exp(\sigma b)$. A frequency cut-off induced by transverse smearing of the bunch is absent in this case.

We note that it is easy to take into account a charge distribution along the longitudinal coordinate z, which is

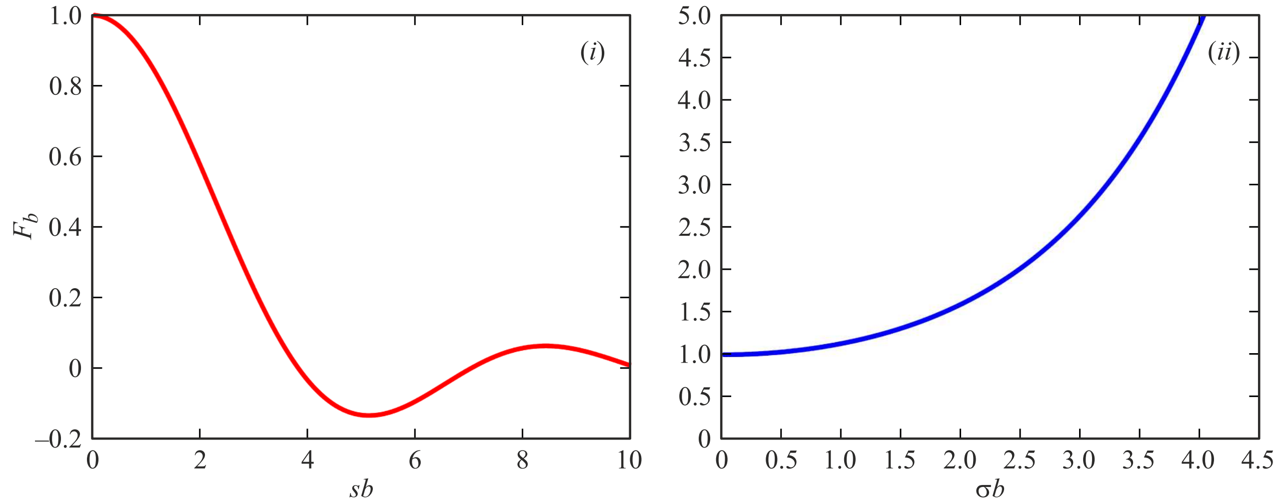

Figure 2. Dependence of the form factor F_b on sb when the CR condition is fulfilled, the case (i) (on the left), and its dependence on σb when this condition is not fulfilled, the case (ii) (on the right).

different from a delta-like one, we designate it as $\eta(z - \upsilon t)$. For the longitudinally-smeared charged disk, instead of (2) the charge density is written as

$$\rho_l = \frac{Q}{\pi b^2}\theta(b - r)\eta(z - \upsilon t). \quad (11)$$

The Fourier transform of the density (11) is written as $\rho_{l\omega} = \rho_\omega F_l$, where ρ_ω is given by the formula (4) and

$$F_l = \int\limits_{-\infty}^{+\infty} \eta(\xi) e^{-i\xi\frac{\omega}{\upsilon}}\, d\xi, \quad (12)$$

i.e. with accuracy to the multiplier 2π is a Fourier transform of function of longitudinal charge distribution along a difference coordinate $\xi = z - \upsilon t$, which is calculated in the point ω/υ. Physically, F_l is the so-called longitudinal form factor, i.e. a multiplier, by which the field of the longitudinally-smeared disk differs from the field of the infinitely-thin (delta-like) disk (2). In particular, in case of a Gaussian distribution with root mean squared half-width τ,

$$\eta(\xi) = (\sqrt{2\pi}\tau)^{-1} \exp\left[-\xi^2/(2\tau^2)\right],$$

and the longitudinal form factor is written as

$$F_l = \exp\left[-\omega^2\tau^2/(2\upsilon^2)\right].$$

Since the main focus in the present study is set on the influence of the transverse effects, we will not take into account F_l in further derivations (specific inequalities, at which it can be approximately set that $F_l \approx 1$, will be written out below) and hereinafter the form factor will be the transverse form factor.

2. ABCs on the grid and solution of the problem

Now we start to deal with a boundary problem shown in Fig. 1. A boundary of the domains II and III has standard conditions of continuity of the components $E_{\omega z}$ and $H_{\omega\varphi}$ set. Then the problem is solved in the approximation (1), which makes it possible to use averaged boundary conditions (ABCs) for describing the wire grid [30,32,41]. Using these conditions, we actually calculate the averaged field (it is this field that is of practical interest), which coincides with the real one at the distance from the grid for about a period p. In case of wires codirectional with the axis z (with ideal conductivity) the ABCs are reduced to continuity of $E_{\omega z}$ at the boundary $r = a - h$ and a jump of the magnetic field $H_{\omega\varphi}$ in this layer, which is related to $E_{\omega z}$ in the following way (a dependence of all the magnitudes on z is taken into account as $\exp(ik_0\beta^{-1}z)$):

$$E^{\mathrm{II}}_{\omega z}\big|_{r=a-h} = E^{\mathrm{I}}_{\omega z}\big|_{r=a-h} = \frac{-ipk_0}{2\pi}\,\Pi\,\left(H^{\mathrm{II}}_{\omega\varphi} - H^{\mathrm{I}}_{\omega\varphi}\right)\big|_{r=a-h}. \quad (13)$$

The parameter Π depends on geometrical parameters r_0, p and medium permittivity ε as follows:

$$\Pi = \ln\left(\frac{p}{2\pi r_0}\right) - \delta_* - \frac{1}{\beta^2}\left[\ln\left(\frac{p}{2\pi r_0}\right) - \delta\,\frac{\varepsilon - 1}{\varepsilon + 1}\right], \quad (14)$$

where correction coefficients δ_* and δ (a designation assumed in the studies [30,40,41] is preserved; this should not result in a confusion with delta function are induced by proximity of the dielectric to the grid and describe its influence on distribution of the average surface current in the layer $r = a - h$, and they are written as follows [40,41]:

$$\delta = \frac{1}{2}\ln\left(\frac{p^2 + 4h^2}{r_0^2 + 4h^2}\right) + \frac{p^2}{6\,(p^2 + 4h^2)} + \frac{4h}{p}\arctan\left(\frac{p}{2h}\right) - 2, \quad (15)$$

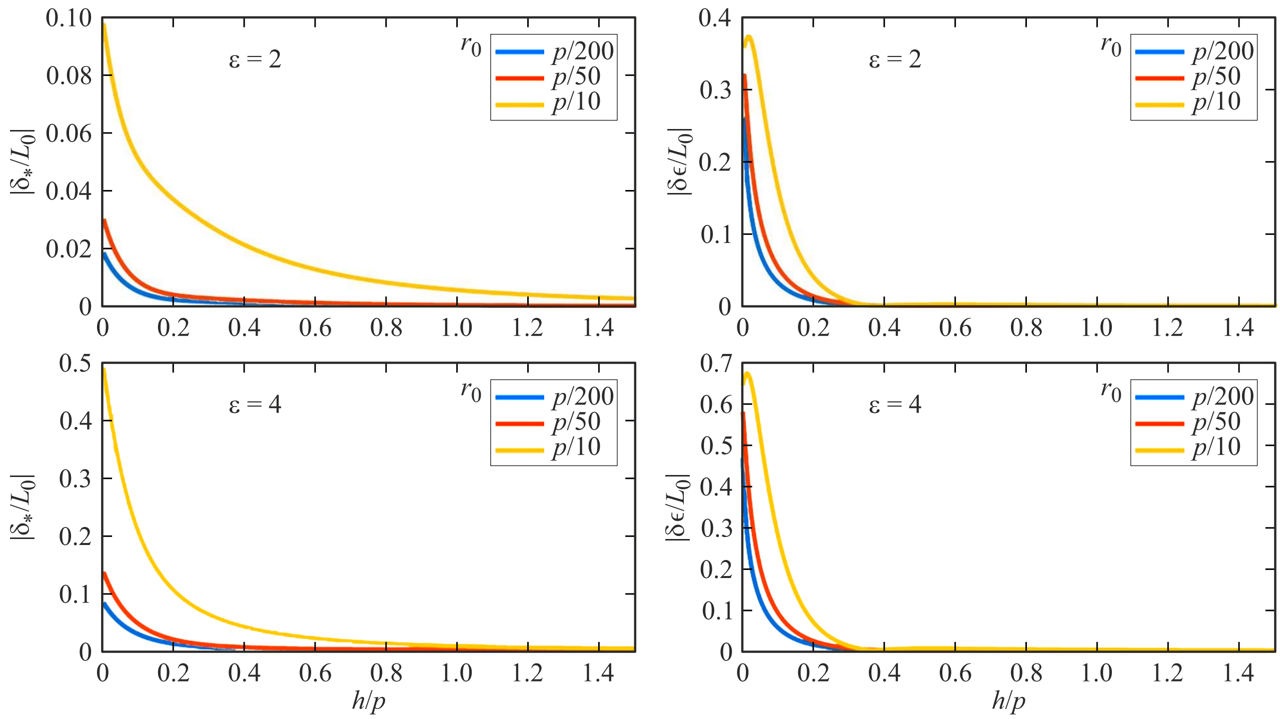

Figure 3. Dependence of the magnitudes $\left|\delta_* L_0^{-1}\right|$ (the left column) and $\left|\delta\epsilon L_0^{-1}\right|$ (the right column) on the relationship h/p at the various values of r_0 (specified in the legend), $\varepsilon = 2$ (the upper row) and $\varepsilon = 4$ (the lower row), $p = 1$ cm, $\lambda = 3p$. The following designations are introduced: $L_0 \equiv \ln\left(\frac{p}{2\pi r_0}\right)$, $\epsilon \equiv \frac{\varepsilon-1}{\varepsilon+1}$.

$$\delta_* = \int_0^\infty \frac{2J_2(\xi)}{\xi}\left[\ln\left(\frac{p^2+(2h+\xi\ell)^2}{r_0^2+(2h+\xi\ell)^2}\right) + \frac{p^2}{3(p^2+(2h+\xi\ell)^2)} + 4\,\frac{2h+\xi\ell}{p}\arctan\left(\frac{p}{2h+\xi\ell}\right) - 4\right]d\xi, \tag{16}$$

where $\ell = ik_0^{-1}/\sqrt{\varepsilon-1}$. The integral (16) can be easily calculated numerically.

We note that without taking into account the influence of the dielectric the ABCs (13) for a cylindrical grid considered herein are directly derived in the study [32] and they also follow from a more general result for the grid made of nonparallel wires, which is arranged on a nonflat surface [31], wherein their kind is not different from a kind of common „flat“ ABCs. Further on, as noted in the study [30,41], the corrections (15) and (16) are equivalent to the influence of certain reflection grids. Therefore, the kind of these corrections for the cylindrical grid is not different from the respective corrections in the flat case [40,41].

In order to illustrate the influence of the corrections (15) an (16) on the ABCs, Fig. 3 shows typical graphs of relative magnitudes

$$\left|\delta_* \Big/ \ln\left(\frac{p}{2\pi r_0}\right)\right| \text{ and } \left|\delta\,\frac{\varepsilon-1}{\varepsilon+1} \Big/ \ln\left(\frac{p}{2\pi r_0}\right)\right|$$

in a dependence on a ratio h/p. It is clear that at the quite small h ($h \ll p$) the influence of a vacuum–dielectric interface turns out to be significant, but already when $h = p$ it is almost negligible. When $h \gg p$, the formulas (15) and (16) will also go over into relationships that determine the ABCs in vacuum [32]. Thus, Fig. 3 confirms a known fact that the influence of the dielectric that is close to the grid on the ABC kind is significant only when $h \lesssim p$ [30,40,41]. In the future, during numerical calculations, we will consider this inequality fulfilled.

A formal solution for the potential in the domain I is taken as a sum of the disk potential in unbounded vacuum (the formula (9), the lower line, $\tilde{\varepsilon} = 1$) and a solution of the homogeneous equation (6), which has not a singularity for $r = 0$. In the domain II, the potential is taken as a sum of two linearly-independent solutions of the homogeneous equation (6), so is in the domain III as one linearly-independent solution that satisfies a requirement of the exponential decrease when $r \to \infty$ (small absorption is taken into account in the medium). Taking this into account, we have

$$\Phi_\omega^{(\mathrm{I})} = \frac{Q}{\pi v}\, e^{i\frac{\omega}{v}z} F_b\left[K_0(r\sigma) + C^{\mathrm{I}} I_0(r\sigma)\right],$$

$$\Phi_\omega^{(\mathrm{II})} = \frac{Q}{\pi v}\, e^{i\frac{\omega}{v}z} F_b\left[C^{\mathrm{II}} I_0(r\sigma) + D^{\mathrm{II}} K_0(r\sigma)\right],$$

$$\Phi_\omega^{(\mathrm{III})} = \frac{iQ}{2v\varepsilon}\, e^{i\frac{\omega}{v}z} F_b D^{\mathrm{III}} H_0^{(1)}(rs), \tag{17}$$

where $C^{\mathrm{I,II}}$, $D^{\mathrm{II,III}}$ are required constants. Application of the above-described boundary conditions when $r = a$ and $r = a - h$ results in a linear system 4×4, which is solved

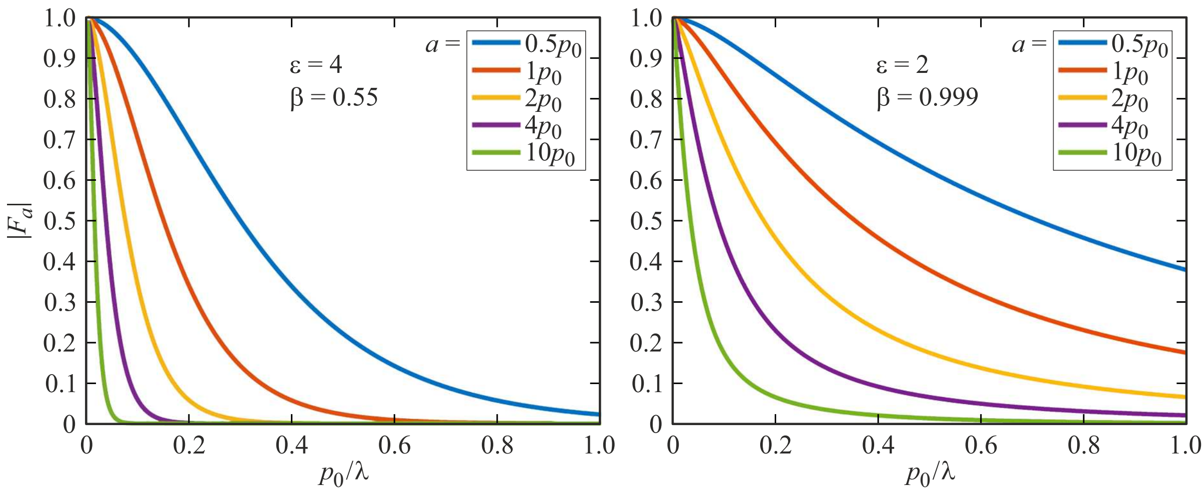

Figure 4. Dependence of the form factor F_a of the non-grid vacuum channel on the relationship $p_0/\lambda = k_0/(2\pi/p_0)$ at the various ε and β (the CR condition is considered to be fulfilled) and the various values of the channel radius a (specified in the legend), $p_0 = 1$ cm.

by standard methods. For brevity, we provide only a final result for the most interesting domain III and write it as follows:

$$\Phi_\omega^{(\mathrm{III})} = F_b F_{ag} \Phi_\omega^\varepsilon, \tag{18}$$

where Φ_ω^ε is a potential of the point charge Q moving in the unbounded dielectric ε (the formula (9), the case (i), $\tilde{\varepsilon} = \varepsilon$) and F_{ag} is a form factor of the grid-modified vacuum channel of the radius a:

$$F_{ag} = \frac{pk_0 \Pi \varepsilon}{i\pi^2 s k_0^{-1}(a-h)s} \Big\{ I_0\left[(a-h)\sigma\right] I_0^{-1}(a\sigma) \big[I_0\left[(a-h)\sigma\right] \times H_0^{(1)}(as) - a\sigma \Delta_a \Delta_h \big] - \frac{pk_0}{2\pi} \frac{k_0}{\sigma} \frac{a}{a-h} \Delta_a \Pi \Big\}^{-1}, \tag{19}$$

where the magnitude Π is determined by the formula (14),

$$\Delta_a = H_0^{(1)}(as) I_1(a\sigma) - \frac{\sigma\varepsilon}{s} H_1^{(1)}(as) I_0(a\sigma), \tag{20}$$

$$\Delta_h = K_0(a\sigma) I_0\left[(a-h)\sigma\right] - I_0(a\sigma) K_0\left[(a-h)\sigma\right], \tag{21}$$

wherein $\Delta_h \to 0$ when $h \to 0$.

It is also expedient for further comparative analysis to provide a known result for the case when the grid is excluded from consideration and we have an empty vacuum channel of the radius a in the dielectric ε (we provide the respective potential with an additional upper index „0") [13]:

$$\Phi_\omega^{(\mathrm{III})0} = F_b F_a \Phi_\omega^\varepsilon, \tag{22}$$

where F_a is a form factor of the grid-unmodified vacuum channel of the radius a (the study [13] designates it as η):

$$F_a = \frac{2i}{\pi a s} \frac{\sigma\varepsilon}{s} \frac{1}{\Delta_a}. \tag{23}$$

3. Numerical results

Fig. 4 shows the dependences of the form factor of the vacuum channel in the dielectric F_a on the normalized wave number k_0 (which is proportional to the frequency ω) at the various radii of the channel. The channel radius is expressed in units of a limit period p_0, whose meaning will be clarified below. Besides, the wave number is normalized by the magnitude $2\pi/p_0$. Pairs of the magnitudes ε and β are selected so that the CR condition is fulfilled in the dielectric, i.e. the case (i) is realized in the dielectric. Fig. 4 illustrates a literature-known fact [13] that introduction of the channel results in limitation of the CR frequency spectrum from above, i.e. it is equivalent to the effect of a certain low-frequency filter. The increase of a results in reduction of the cut-off frequency, while the increase of β results in its increase. We also note that a behavior of the form factor F_a differs from the respective behavior of the form factor of the transversely-smeared bunch (Fig. 2) by absence of strict zeros and an exponential decrease (versus a power one with oscillations) at the high frequencies.

Fig. 5 shows the dependences of the form factor of the system „the vacuum channel in the dielectric — the grid" F_{ag} on the normalized wave number k_0 (which is proportional to the frequency ω) at the various values of the distance h between the grid and the interface. A dashed line in all the graphs corresponds to the non-grid case at the selected a (Fig. 4). Pairs of the magnitudes ε and β are selected so as to fulfil the Cherenkov radiation condition in the dielectric. It is clear that the vacuum channel modified by the simplest fine-wire fine-periodic grid has a form factor, whose behavior is fundamentally different from the behavior of the common form factor of the channel, which is shown in Fig. 4. It is clear in all the listed cases that this structure distinguishes a certain band in the CR frequency spectrum, cutting off both the lower and upper frequencies. In other words, its effect on the CR spectrum is equivalent to an

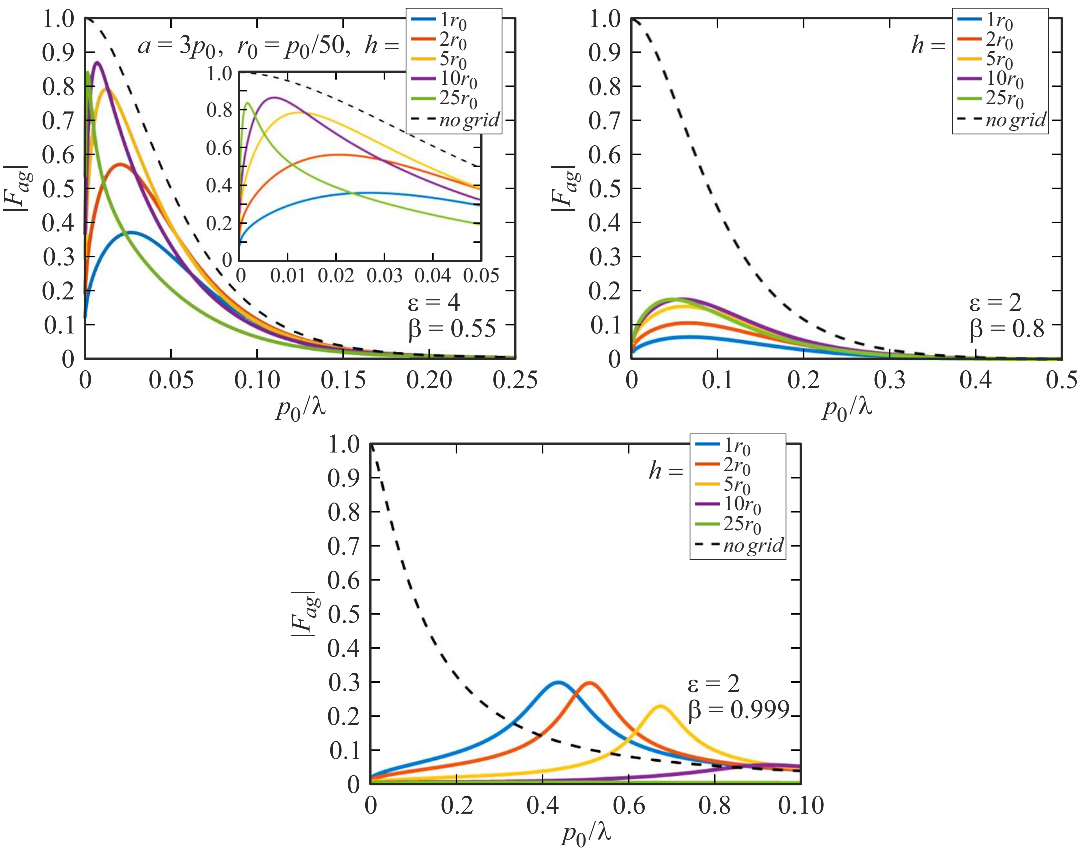

Figure 5. Dependence of the form factor F_{ag} of the z-grid-modified vacuum channel on the relationship $p_0/\lambda = k_0/(2\pi/p_0)$ at the various ε and β (the CR condition is considered to be fulfilled), $a = 3p_0$, $p_0 = 1$ cm and at the various distances h between the grid and the dielectric (specified in the legend). The grid includes $N = 19$ conductors, p varies from $0.99p_0$ to $0.83p_0$ at the said h.

effect of the certain band-pass filter, which is just reflected in a title of the present study. Three additional parameters are available for controlling a position of a transmittance band (as compared to the case of the simple channel) — p, r_0 and h. For certainty, Fig. 5 fixes the radius of the vacuum channel a and the wire thickness r_0 and varies the parameter h. It also fixes a total number of the grid wires $N = 19$ $(\Delta\varphi \approx 1/3)$, wherein the structure period $p = (a - h)\Delta\varphi$ also varies within 15% and when $h \to 0$ it is close to p_0: $p \approx (a - h)/3 \to a/3 = p_0$. Thus, p_0 can be interpreted as a limit (maximum possible) value of the grid period.

With the small velocity of the charge (Fig. 5, the left graph) and a small gap between the grid and the dielectric, a modulus of the form factor F_{ag} in the maximum is in several times smaller as compared to F_a, and the transmittance band is comparable and differs only by absence of near-zero frequencies. But with the increase of h the modulus of the form factor F_{ag} increases and the width of the transmittance band decreases, and when $h = p_0/2$ $(h = 25r_0)$ we have values in the maximum of $|F_{ag}|$, which are about 0.9. With the higher velocity of the bunch (Fig. 5, the middle graph), we have a weaker dependence of the form factor on a gap value, but the general trend is preserved. A value in the maximum of $|F_{ag}|$ is in 5–10 times less than the respective value of $|F_a|$.

The effect of suppression of the near-zero frequencies is explained as follows. As it is clear from the ABCs (13), the wire grid is characterized by certain impedance $\frac{-ipk_0}{2\pi}\Pi \sim k_0\Pi$. At these frequencies $\Pi \approx$ const, therefore, when $k_0 \to 0$ impedance is also close to zero, which at a finite value of $E_{\omega z}$ in the grid plane means excitation of the strong surface current resulting in shielding of the field of the charged particle bunch when $k_0 \to 0$.

Finally, the most interesting situation is realized at a relativistic velocity (Fig. 5, the right graph). In this case, the transmittance band of the considered system is mainly in the frequency range range that is significantly suppressed by the non-grid channel. A curve is shaped almost symmetrically relative to a transmittance carrier frequency. A gain in the potential amplitude at this frequency, whose maximum is realized at small values of the gap h (of the order of r_0) can be about a triple one.

However, in the right graph in Fig. 5 only the curve for $h = r_0$ (more exactly, only its long-wavelength part and a maximum neighborhood) formally falls within an ABC applicability region: as it is known [30], acceptable accuracy of results is already obtained when $p \le \lambda/2$.

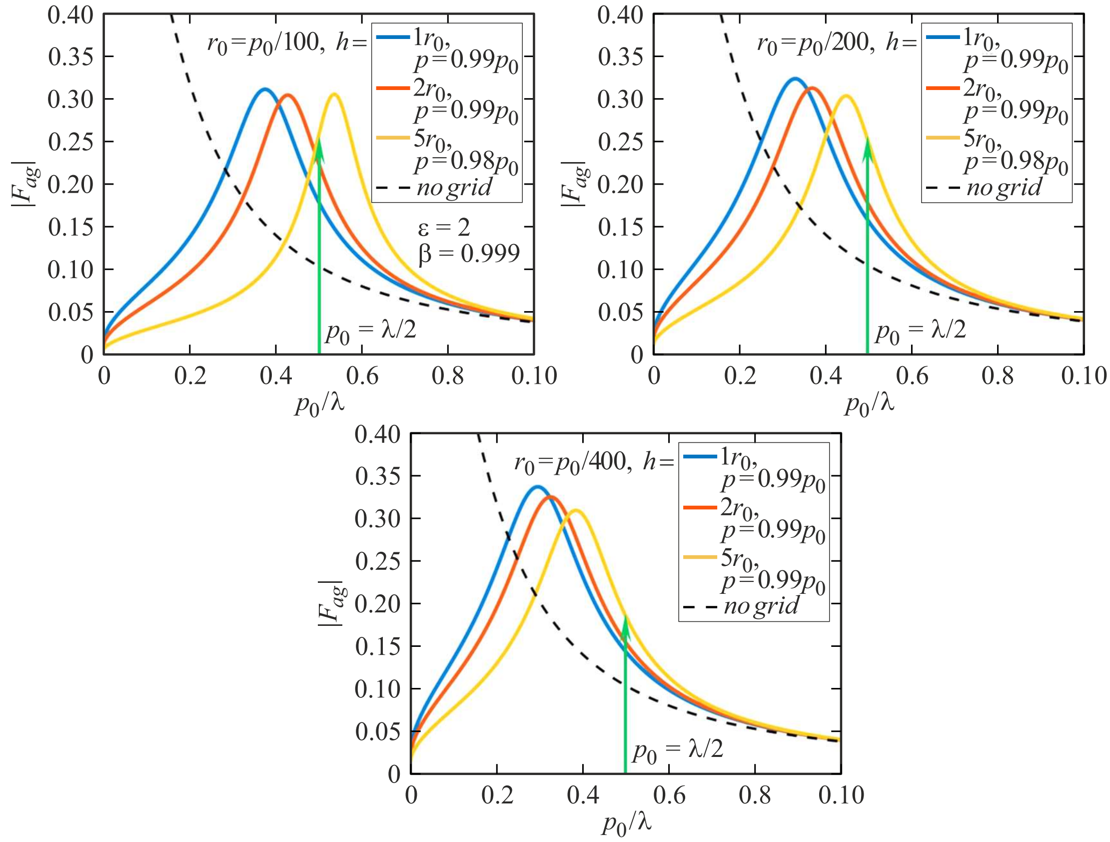

Figure 6. Dependence of the form factor F_{ag} of the z-grid-modified vacuum channel on the ratio $p_0/\lambda = k_0/(2\pi/p_0)$ at the various r_0 and h, the other parameters are the same as in the right graph in Fig. 5.

The transmittance band can be shifted into a more low-frequency range by reducing a cross-section radius r_0 and approximating the grid to the dielectric, which is illustrated in Fig. 6. It is clear that with doubly-thinned wires (Fig. 6, the left graph) the first curve ($h = r_0$, i.e. the grid is directly on the dielectric) definitely falls within a region of allowable wavelengths $\lambda \geq 2p_0$. With reduction of r_0, the curves continue to be shifted to the left and when $r_0 = p_0/400$ all the three curves are within the formal ABC applicability region. With this value of r_0 and the gap $h = 5r_0$ the gain in the potential amplitude is approximately 2.

We remind that for simplicity in the above-given analysis we have not taken into account the longitudinal form factor of the bunch F_l (12). In case of typical Gaussian longitudinal charge distribution, $F_l = \exp\left[-k_0^2\tau^2/(2\beta^2)\right]$, it follows from an explicit expression that longitudinal bunch smearing is equivalent to an additional low-frequency filter with the cut-off frequency $k_{0\tau} = \sqrt{2}\beta/\tau$. In order to neglect this influence, i.e. to assume that $F_l \approx 1$ within the most interesting frequency range, $0 < k_0 < 2\pi/p_0$, it is necessary that the bunch would be quite thin, i.e. $\tau \ll \beta p_0/(\sqrt{2}\pi)$ or $\tau \lesssim p_0$.

Let us discuss physical causes of origination of transmittance peaks in an ultrarelativistic case $\beta \to 1$ (Fig. 6). As it is clear from the formula (14), in this case the magnitude Π is almost completely determined by the corrections δ, δ_* and permittivity ε, while logarithmic summands compensate each other (we note that δ_* is frequency-dependent and, generally speaking, it is complex). If taking into account that $\sigma/k_0 \to 0$ when $\beta \to 1$ as well as assuming that the gap value is small, $h/a \to 0$, then one can obtain the following approximate expressions that are true at least in the range $1/6 < p_0/\lambda < 1/2$, into which the transmittance peaks that are of interest to us fall:

$$F_a \approx \frac{4i}{\pi}\,\frac{\varepsilon}{\varepsilon - 1}\,\frac{1}{H_0^{(1)}(as)}\,\frac{1}{(ak_0)^2 + 2iak_0\frac{\varepsilon}{\sqrt{\varepsilon-1}}}, \tag{24}$$

$$F_{ag} \approx \frac{4i}{\pi}\,\frac{\varepsilon}{\varepsilon - 1}\,\frac{1}{H_0^{(1)}(as)}\,\frac{1}{(ak_0)^2 + 2iak_0\frac{\varepsilon}{\sqrt{\varepsilon-1}} - \frac{4\pi a}{p\Pi}}. \tag{25}$$

The approximate dependences (24) and (25) with a negligible error describe the behavior of the curves shown in Fig. 6. As it is clear, unlike F_a, the form factor F_{ag} has a denominator that is typical for a serial RLC-circuit, since the current J excited in this circuit by an oscillator of voltage U_0 is written as

$$J \sim \frac{U_0}{\omega^2 + 2i\omega R/L - 1/(LC)}.$$

Zero of the real part of the denominator in (25) determines a frequency of the transmittance peak:

$$(ak_0^{\text{res}})^2 = \frac{4\pi a}{p\Pi} \tag{26}$$

(the dependence Π on the frequency in the range in question is quite weak, Π can be assumed to be a real constant). Presence of a resonance frequency means that in the „circuit" (25) has finite inductance and capacitance, while in the „circuit" (24) only inductance is finite and capacitance is infinitely high. A function of finite capacitance obviously belongs to the gap between the wire grid and the dielectric.

Thus, the band-pass filter effect is caused by the fact that for the relativistic charged particle bunch this fine-periodic grid near the dielectric is a resonance structure. The non-grid channel functions as the „circuit" that includes only active resistance and inductance, while the grid introduces a non-zero capacitance load required for resonance formation to the system.

Conclusion

The study has investigated the influence of the thin-wire fine-periodic grid made of the parallel conductors on the spectral characteristics of CR of the charged particle bunch that uniformly moves along the axis of the vacuum channel in the dielectric. The conductors are directed along the channel axis and uniformly distributed along a cylindrical surface near the vacuum–dielectric interface at the distance about the structure period. We have considered a quite low-frequency range (the respective wavelengths are much larger than the structure period) that makes it possible to describe the grid in terms of the Kontorovich ABCs. An essential circumstance in the present study is that the gap between the grid and the channel boundary is small, thereby resulting in necessity of taking into account in the ABCs the corrections induced by the influence of the dielectric medium on distribution of the averaged surface current in the grid layer.

It was shown that the influence of the considered structure on CR is equivalent to the effect of the band-pass filter. In other words, both the low (including near-zero ones) as well as comparatively high frequencies (we remind that consideration is within the framework of ABC applicability, i.e. the upper frequency shall provide a small ratio of the grid period to the wavelength) are suppressed in the generated spectrum. The position of the transmittance band, the transmittance coefficient's maximum and the shape of its profile (as a function of the frequency) depend on the parameters of the grid and the gap value. The most interesting situation seems to occur with the relativistic velocity of the bunch and the small gap (of about the conductor radius), when the transmittance band is quite narrow and falls into the frequency range that is cut off by the non-grid channel (as known, the vacuum channel itself functions as the low-frequency filter). In this case, the transmittance coefficient of the considered structure for the field at the carrier frequency can exceed the respective channel coefficient in 2–3 times, thereby corresponding to the gain in power almost by an order. In other words, ceteris paribus, one can select an emitting structure with a wider channel (it is more easy to pass the charged particle bunch therethrough, since transverse dynamics is less critical, and it is possible to increase its charge, etc.), modify its internal surface with the discussed thin-wire grid and at least keep intensity of Cherenkov radiation. Moreover, a band-like nature of the produced generator is generally deemed to be quite attractive, since it makes it possible to record CR in a mode that is more close to a monochromatic one.

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Conflict of interest

The authors declare that they have no conflict of interest.

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