

# Generalized von Neumann criterion for flows with pulsed energy release and a change in the chemical composition of a gas mixture at a Mach stem

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An analog of the von Neumann criterion is analytically derived and numerically studied for changing the type (regular or Mach) reflection of an oblique steady shock if there is a pulsed release or absorption of energy at the Mach reflection, as well as a change in chemical composition of gas mixture on a strong main (Mach) shock. The influence of those factors on the displacement of the generalized criterion obtained is estimated in comparison with the well-known case of flow without chemical reactions and effects of a real gas.

**Keywords:** regular and Mach reflection, von Neumann criterion, detonation.

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The transition from Mach reflection of oblique shocks (e.g., shock  $j_1$  shown in Fig. 1, *a*) in a steady gas flow to regular reflection is often characterized by the von Neumann criterion, which has been formulated back in the 1940s [1]. With this criterion satisfied, incident ( $j_1$ ), reflected ( $j_2$ ), and main (Mach,  $j_3$ ) shocks form the so-called stationary Mach configuration (SMC) at the triple point ( $T$ ). Mach shock  $j_3$  at the SMC triple point is normal to incoming flow, and its intensity  $J_3$  (the ratio of static pressures behind the shock and in front of it) is written as

$$J_3 = J_m(M) = p_3/p = (1 + \varepsilon)M^2 - \varepsilon. \quad (1)$$

Here,  $M$  and  $p$  are the Mach number and the static pressure of incoming flow;  $\varepsilon = (\gamma - 1)/(\gamma + 1)$ , where  $\gamma$  is the adiabatic index of gas. Next,  $p_i$  and  $T_i$  are the static pressure and the temperature of gas behind shock  $j_i$ , and  $J_i$  is the intensity of the  $i$ th shock (the ratio of static pressures behind the shock and in front of it). Intensity  $J_1 = p_1/p$  of the incident shock satisfying the von Neumann criterion is traditionally determined from the following equation [2,3]:

$$\sum_{n=0}^3 A_n J_1^n = 0, \quad (2)$$

where

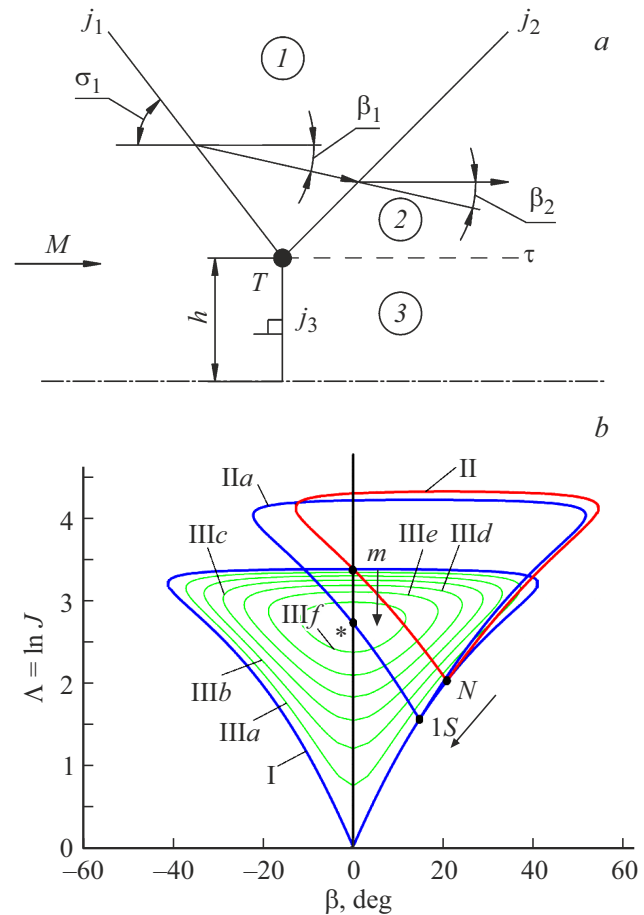
$$A = 1 - \varepsilon,$$

$$A_2 = -[(1 + \varepsilon - \varepsilon^2 + \varepsilon^3)M^2 + (1 - \varepsilon)(1 - \varepsilon + \varepsilon^2)],$$

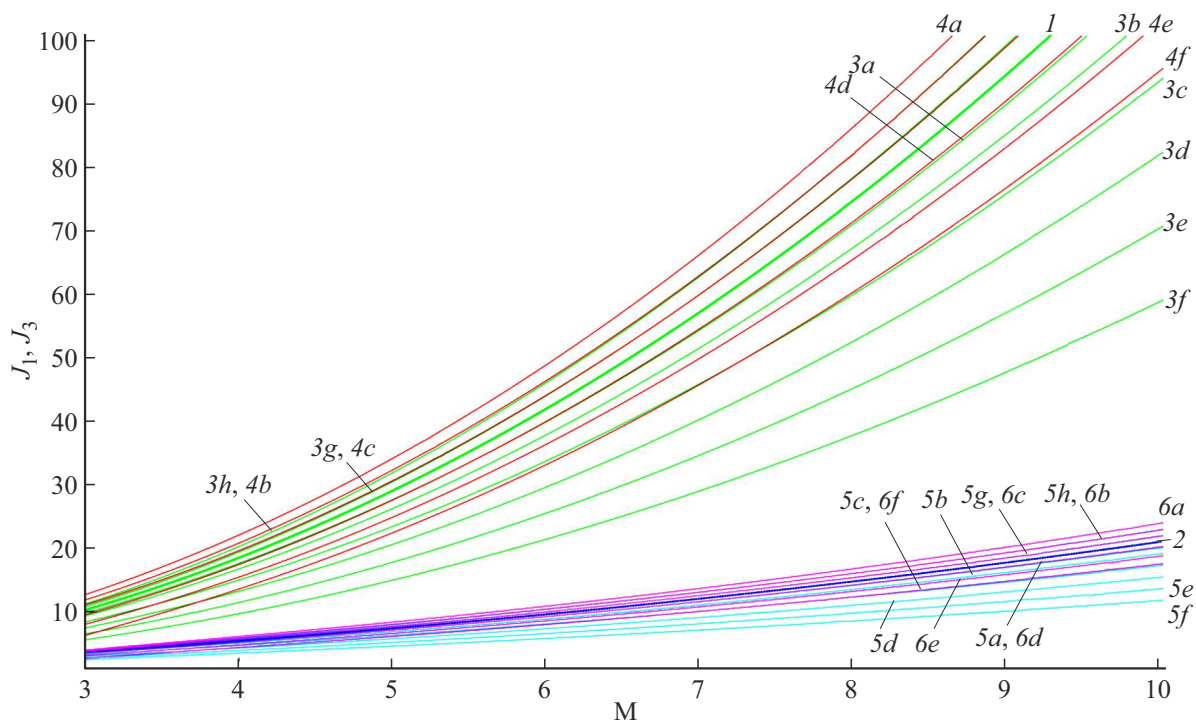
$$A_1 = \varepsilon[(1 + \varepsilon)M^2 + 1 - \varepsilon][(1 - \varepsilon)M^2 - 2 + \varepsilon],$$

$$A_0 = (1 - \varepsilon)(M^2 - 1)((1 + \varepsilon)M^2 - \varepsilon).$$

Relation (2) was derived in [2,3] from the system of equations specifying the conditions of equality of pressures



**Figure 1.** Graphical description of the stationary Mach configuration. *a* – Flow diagram in the vicinity of the triple point; *b* – solution of the problem on the plane of shock polars (Mach number of the incident flow  $M = 5$ ). See explanation in the text.



**Figure 2.** Intensities of main (Mach) and incident shocks at  $\gamma = 1.4$ . Curves *1* and *2* — main and incident shocks in non-reactive gas; *3a–3f* — the main shock at an energy release constituting 0.1, 0.2, 0.4, 0.6, 0.8, and 1.0 of the critical value, respectively; *3g* and *3h* — the main shock at an energy absorption constituting 0.1 and 0.2 of the critical value, respectively; *4a–4f* — the main shock at zero energy release or absorption and  $\gamma_3 = 1.1, 1.2, 1.3, 1.5, 1.7,$  and  $1.9,$  respectively; *5a–5f* — the incident shock at an energy release constituting 0.1, 0.2, 0.4, 0.6, 0.8, and 1.0 of the critical value, respectively; *5g* and *5h* — the incident shock at an energy absorption constituting 0.1 and 0.2 of the critical value, respectively; *6a–6f* — the incident shock at zero energy release or absorption and  $\gamma_3 = 1.1, 1.2, 1.3, 1.5, 1.7,$  and  $1.9,$  respectively.

and co-directionality of flows on the sides of tangential discontinuity  $\tau$  behind the SMC:

$$J_1 J_2 = J_3 = J_m(M), \quad \beta_1 - \beta_2 = 0,$$

which may also be written in the form

$$\Lambda_1 + \Lambda_2 = \Lambda_3, \quad \beta_1 - \beta_2 = 0,$$

that allows for a graphical solution of the problem on the plane of shock polars (Fig. 1, *b*). Here,  $\Lambda_i = \ln J_i$  and  $\beta_i$  is the angle of flow deflection at shock  $j_i$  (Fig. 1, *a*) that is related to the shock intensity and Mach number  $M$  in front of it in the following way [2]:

$$\tan \beta_i = \sqrt{\frac{(1 + \varepsilon)M^2 - \varepsilon - J_i}{J_i + \varepsilon} \frac{(1 - \varepsilon)(J_i - 1)}{(1 + \varepsilon)M^2 - (1 - \varepsilon)(J_i - 1)}}. \quad (3)$$

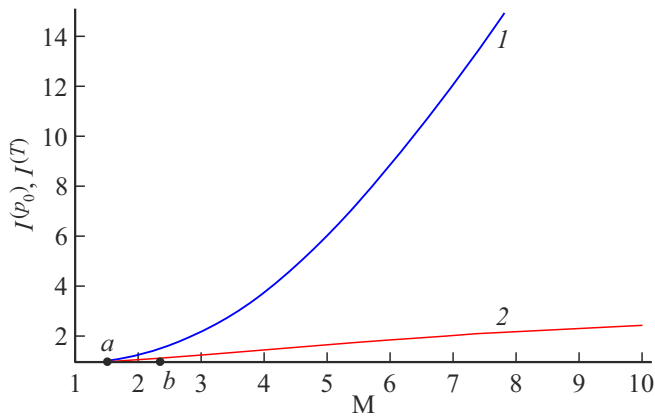
Relation (3) between the intensities of SMC shocks and angles  $\beta_i$  of flow deflection is illustrated on the plane of shock polars at  $\gamma = 1.4$  (Fig. 1, *b*). Heart-shaped curve I represents this dependence for incident shocks, and its top point  $m$  corresponds to main shock  $j_3$  in the flow of non-reactive gas. Curves II and II*a* illustrate similar dependences for reflected shocks at various intensities of the incident shock (and, consequently, flow parameters

behind it) corresponding to points  $N$  and  $1S$  on polar I. Curves III*a–IIIf* correspond to an energy release at the main shock of 15, 30, 45, 60, 75, and 90 % of the maximum value ( $\bar{\varphi}_{crit}$ ), respectively.

Curves *1* and *2* (Fig. 2) represent the intensities of main and incident shocks  $J_3 = J_m(M)$  and  $J_1(M)$ , which satisfy the von Neumann criterion for inert gas flow, obtained from Eq. (2) at  $\gamma = 1.4$  ( $\varepsilon = 1/6$ ). It should be noted that SMCs exist only at  $M \geq M_a$ , where  $M_a = \sqrt{(2 - \varepsilon)/(1 - \varepsilon)} = 1.483$ , and set the boundaries for stable Mach reflection at  $M \geq M_b$ , where Mach number  $M_b = 2.202$  is determined by the fourth-degree algebraic equation known from [3].

According to modern models of supersonic flows with Mach reflection [4–7], the von Neumann criterion (SMC formation) corresponds to vanishing height  $h$  of shock  $j_3$  (Fig. 1, *a*) and, consequently, to a continuous transition from Mach reflection to regular reflection of incident shock  $j_1$ .

A considerable number of flow parameters assume significantly different values in regions 2 and 3 (Fig. 1, *a*) behind the reflected and main shocks, which are separated by tangential discontinuity  $\tau$ . Figure 3 presents the ratios of stagnation pressures  $I^{(p_0)} = p_{02}/p_{03}$  (curve *1*) and temperatures  $I^{(T)} = T_3/T_2$  (curve *2*) on the sides of tangential discontinuity  $\tau$  behind the SMC triple point as functions



**Figure 3.** Ratios of stagnation pressures (curve 1) and flow temperatures (curve 2) at the tangential discontinuity behind the triple point.

of the Mach number of incoming flow of diatomic perfect gas. In the limit case ( $M \rightarrow \infty$ ), the static pressures and temperatures behind the main and reflected SMC shocks differ manyfold [3]:

$$\lim_{M \rightarrow \infty} I_{p_0} = \left[ \frac{1 + 2\varepsilon - 2\varepsilon^3 + \varepsilon^4 + (1 - \varepsilon)D}{2\varepsilon(2 - \varepsilon)} \right]^{\frac{1+\varepsilon}{2\varepsilon}} = 69.72,$$

$$\lim_{M \rightarrow \infty} I_T = \frac{1 + 2\varepsilon - 2\varepsilon^3 + \varepsilon^4 + (1 - \varepsilon)D}{2\varepsilon(2 - \varepsilon)} = 3.363,$$

$$D = \sqrt{(1 + \varepsilon)^2 - \varepsilon(1 - \varepsilon)[2(1 + \varepsilon)(2 - \varepsilon) - \varepsilon^3(1 - \varepsilon)]}.$$

Significant differences in flow parameters behind the triple point may be used, e.g., in jet engine construction [8,9]. Relatively small losses of total pressure provide an opportunity to use a flow stagnated in the system of oblique shocks as in a classic air-breathing jet engine. The total pressure losses at a strong Mach shock are very high, but a much greater increase in temperature may initiate detonation of the reactive mixture, making it possible to use this part of flow as in a ramjet detonation engine.

Effective separation of flows (non-detonating one behind the reflected shock; detonating one behind the Mach shock) requires establishing (numerically or analytically) the height of the triple point, the shape of the outgoing tangential discontinuity, and the conditions of transition from Mach reflection to regular reflection. The first attempts at calculation and applying approximate analytical models in the presence of shocks with energy release [10] revealed that the pulsed energy release on the surface of the main shock does not only increase its height, but also shifts the von Neumann criterion toward weaker incident shocks. Mach reflection of shocks, which are reflected only regularly in flows with zero energy release, is observed. The reverse (toward stronger incident shocks) shift of the von Neumann criterion accompanies energy absorption at the main shock (with energy spent on excitation of vibrational degrees

of freedom or dissociation of molecules). A change in adiabatic index of the mixture also alters the conditions for changing the type of reflection.

In addition to its application in advanced detonation devices, establishing the von Neumann criterion for high-speed flows of reactive gas mixtures will provide an opportunity to avoid Mach reflection (with the accompanying increase in aerodynamic drag, loss of total pressure in the air intakes, and loss of thrust of an air-breathing engine) in a more proper manner. The control of parameters of regular and Mach reflection of strong shock waves also helps optimize the mechanical effect of explosion, the translational effect of satellite flow on the objects carried by it, which differs widely on the sides of emerging tangential and contact discontinuities [11].

According to [12] and [13], the application of the Chapman–Jouguet model to a normal or oblique shock, which is regarded as a stationary detonation wave, yields the following relation between intensity  $J$  of this shock and angle  $\beta$  of flow deflection on its surface:

$$|\beta| = \arctg \left[ \frac{(J - 1)\sqrt{F - 1}}{\gamma M^2 - (J - 1)} \right],$$

$$F = \frac{2\gamma M^2 [(\gamma - \gamma_3) + (\gamma - 1)((J - 1) - (\gamma_3 - 1)\bar{\varphi})]}{(\gamma - 1)(J - 1)[(\gamma_3 + 1)(J - 1) + 2\gamma_3]}, \quad (4)$$

where  $\gamma$  is the adiabatic index of the gas mixture in front of the shock and  $\gamma_3$  is the adiabatic index behind the shock. Pulsed energy release  $\varphi$  per unit mixture mass is characterized by dimensionless quantity  $\bar{\varphi} = \varphi/(p/\rho) = \gamma\varphi/[(\gamma - 1)c_p T]$ , which is finite, determined by the chemical composition of the gas mixture, and does not depend explicitly on the Mach number of flow.

Relation (4) stems from the Chapman–Jouguet conditions in the form [13]

$$[\rho V_n] = 0, \quad [V_\tau] = 0, \quad [\rho V_n^2 + p] = 0, \quad [h_0] = \varphi,$$

applied to shock  $j_3$  as to a stationary detonation wave and is applicable only if this detonation model is adequate to the processes at the shock. Here,  $\rho$ ,  $p$ , and  $h_0$  are the density, pressure, and total enthalpy on the sides of the shock;  $V_n$  and  $V_\tau$  are the normal and tangential (with respect to the shock) components of flow velocity;  $\varphi$  is the pulsed energy release per unit gas mixture mass; and square brackets denote the difference between the corresponding values on the surface in front of the shock and behind it.

Formula (4) applied to normal shock  $j_3$  with energy release initiated by a high temperature behind it specifies its intensity:

$$aJ_3^2 + bJ_3 + c = 0, \quad (5)$$

where

$$a = \varepsilon(1 - \varepsilon), \quad b = -\varepsilon(1 - \varepsilon_3)[(1 + \varepsilon)M^2 + 1 - \varepsilon],$$

$$c = \varepsilon_3[(1 - \varepsilon^2)M^2 - \varepsilon(1 - \varepsilon) + 2\varepsilon(1 + \varepsilon)\bar{\varphi}M^2],$$

$$\varepsilon_3 = (\gamma_3 - 1)/(\gamma_3 + 1).$$

The physically substantiated solution of Eq. (5) has the form

$$J_3 = \frac{(1-\varepsilon_3)[(1+\varepsilon)M^2+1-\varepsilon]}{2(1-\varepsilon)} + \frac{\sqrt{\varepsilon h - 8\varepsilon^2(1-\varepsilon^2)\varepsilon_3 M^2 \bar{\varphi}}}{2\varepsilon(1-\varepsilon)},$$

$$h = \varepsilon[(1+\varepsilon)(1-\varepsilon_3)M^2 + (1-\varepsilon)(1+\varepsilon_3)]^2 - 4\varepsilon_3 M^2(1-\varepsilon^2)(1-\varepsilon\varepsilon_3). \quad (6)$$

Relation (6) illustrates the reduction in intensity of shock  $j_3$  at  $\bar{\varphi} > 0$ . The intensity of shock  $j_1$  decreases monotonically alongside with it, and the von Neumann criterion shifts toward weaker incident shocks.

The degree of weakening of the main shock corresponding to the von Neumann criterion with a positive pulsed energy release is represented by curves  $3a-3f$ , and the degree of its enhancement with a theoretically possible energy absorption is represented by curves  $3g$  and  $3h$  in Fig. 2 (in comparison with curve  $l$  plotted for inert gas). If we assume that  $\gamma = \gamma_3$ , relation (6) is transformed to a form that shows even more clearly the shift of the von Neumann criterion toward weaker shocks upon energy release:

$$J_3 = \frac{(1+\varepsilon)M^2+1-\varepsilon}{2} + \frac{(1+\varepsilon)(M^2-1)}{2} \times \sqrt{1 - \frac{8\varepsilon M^2 \bar{\varphi}}{(1-\varepsilon^2)(M^2-1)^2}}. \quad (7)$$

Curves  $3a-3f$  illustrate the variation of intensity of the main shock at  $\gamma = \gamma_3 = 1.4$  and specific energy release  $\bar{\varphi}$  constituting a certain fraction of the critical value

$$\bar{\varphi}_{crit} = [\varepsilon(1+\varepsilon)^2(1-\varepsilon_3)^2 M^4 + 2(1-\varepsilon^2) \times (\varepsilon - 2\varepsilon_3 + \varepsilon\varepsilon_3)M^2 + \varepsilon(1-\varepsilon)^2(1+\varepsilon_3)^2] \times [8\varepsilon\varepsilon_3(1-\varepsilon^2)M^2]^{-1}.$$

Quantity  $\bar{\varphi}_{crit}$  denotes the maximum dimensionless energy release behind a stationary detonation wave at a given Mach number  $M$  of oncoming flow. If  $\bar{\varphi} > \bar{\varphi}_{crit}$ , detonation wave  $j_3$  starts to move upstream due to the excessively large pulsed energy release behind its surface [14]. The energy release in the process of detonation of reactive mixtures is normally comparable to the critical one. An increase in flight speed (the Mach number of incident flow) raises the critical energy release that allows for the existence of stationary waves.

At  $\gamma = \gamma_3$ , the maximum energy release is written as  $\bar{\varphi}_{crit} = [(1-\varepsilon^2)(M^2-1)^2]/8\varepsilon M^2$ . The intensity of the main shock is given by

$$J_3 = J_{crit} = \frac{(1-\varepsilon_3)[(1+\varepsilon)M^2+1-\varepsilon]}{2(1-\varepsilon)} = \frac{1-\varepsilon_3}{1-\varepsilon} \frac{J_m(M)+1}{2},$$

$$J_3 = J_{crit} = [(1+\varepsilon)M^2+1-\varepsilon]/2$$

when the adiabatic index is changed or preserved, respectively (the latter case is illustrated by curve  $3f$  in Fig. 2).

The variation of intensity  $J_3$  at a negative energy release (energy absorption) of  $-0.1\bar{\varphi}_{crit}$  and  $-0.2\bar{\varphi}_{crit}$  is illustrated by curves  $3g$  and  $3h$  in Fig. 2.

The conclusion regarding the suppression of the main shock in the presence of pulsed energy supply and with an increase in adiabatic index is completely consistent with the solution of the problem on the plane of shock polars (Fig. 1,  $b$ ). Specifically, curves IIIa–III*f*, which correspond to condition (4) at  $\gamma = \gamma_3$  and an energy release constituting a certain fraction of the maximum value  $\bar{\varphi}_{crit}$ , have top points that correspond to direct shocks with energy release and are located significantly lower than point  $m$  on the „ordinary“ polar I. A similar pattern is observed at  $\gamma_3 > \gamma$ . At  $\bar{\varphi} = \bar{\varphi}_{crit}$ , the detonation polar degenerates into a point (\*) and vanishes.

According to (7), the intensity of the main shock (both with and without energy release) is characterized at high Mach numbers by relation  $\lim_{M \rightarrow \infty} (J_3/M^2) = (1+\varepsilon)(1-\varepsilon_3)/(1-\varepsilon)$ , which reflects the influence of the change in adiabatic index of the gas mixture.

The degree of influence of the deviation of adiabatic index of the gas mixture from the initial value  $\gamma = 1.4$  on intensity  $J_3$  of the main shock at  $\bar{\varphi} = 0$  is illustrated in Fig. 2 by curves  $4a-4c$  and  $4d-4f$ , respectively. It is evident from Fig. 2 that a decrease in adiabatic index (accompanying exothermic reactions upon detonation) shifts the  $J_3(M)$  dependence slightly toward higher intensities (curves  $4a-4c$ ), which compensates in part (albeit not completely) for the detonation energy release. A decrease in adiabatic index at  $\bar{\varphi} = 0$  does not only shift the  $J_3(M)$  dependence downward (curves  $4d-4f$ ), but also leads to the lack of solutions at low Mach numbers: a solution for  $J_3(M)$  exists only at  $M > M_{crit}$ , where Mach numbers  $M_{crit}$  are determined by dependence

$$M_{min} =$$

$$\sqrt{[2\varepsilon_3 - \varepsilon - \varepsilon\varepsilon_3^2 + 2\sqrt{\varepsilon_3(\varepsilon_3 - \varepsilon)(1 - \varepsilon\varepsilon_3)}] / [\gamma\varepsilon(1 - \varepsilon_3)^2]}$$

at  $\bar{\varphi} = 0$  or from equation

$$\varepsilon\gamma^2(1-\varepsilon_3)^2 M_{min}^4 - 2\gamma(2\varepsilon_3 - \varepsilon - \varepsilon\varepsilon_3^2 + 4\varepsilon\varepsilon_3\bar{\varphi})M_{min}^2 + \varepsilon(1+\varepsilon_3)^2 = 0$$

at nonzero energy release. Thus, the intensity of main and reflected shocks is affected by two oppositely directed factors. The decrease in intensity  $J_3$  at  $\bar{\varphi} > 0$  (detonation) is compensated in part by its increase due to the accompanying decrease in adiabatic index of the gas mixture. In contrast, high-temperature phenomena with  $\bar{\varphi} < 0$  are often accompanied by an increase in adiabatic index (dissociation of polyatomic molecules).

Intensity  $J_1$  of the incident shock, which corresponds to the SMC formation, is a monotonic function of intensity  $J_3$  (increases or decreases along with it). The generalized von Neumann criterion is defined in the form of dependence

$J_1(M)$  by solving Eq. (5) with respect to  $J_3 = J_1 J_2$  simultaneously with equation

$$\sum_{n=0}^3 C_n J_2^n = 0, \quad (8)$$

where

$$\begin{aligned} C_3 &= [(1 + \varepsilon J_1) J_1 M]^2, & C_2 &= -J_1(1 + \varepsilon J_1) x_2, \\ C_1 &= (1 + \varepsilon)(J_1 + \varepsilon)(1 + (1 + 2\varepsilon) J_1^2) M^4 \\ &\quad - x_1 M^2 + (1 - \varepsilon)(J_1 + \varepsilon)(J_1 + 1)(J_1 - 1)^3, \\ C_0 &= -(1 + \varepsilon)(J_1 + \varepsilon)((1 + 2\varepsilon) J_1 - \varepsilon) M^4 \\ &\quad + x_0 M^2 - (1 - \varepsilon)(J_1 + \varepsilon)^2 (J_1 - 1)^2, \\ x_2 &= (1 + \varepsilon)(J_1 + \varepsilon) M^4 + (\varepsilon(2 - \varepsilon) J_1^2 - \varepsilon(1 - 2\varepsilon) J_1 \\ &\quad + 2(1 - \varepsilon^2)) M^2 - (1 - \varepsilon)(J_1 + \varepsilon)(J_1 - 1)^2, \\ x_1 &= 2(1 + \varepsilon(1 - \varepsilon)) J_1^4 - 2(1 - 2\varepsilon^2) J_1^3 - \varepsilon(4 - \varepsilon - 2\varepsilon^2) J_1^2 \\ &\quad + 2\varepsilon(1 - \varepsilon) J_1 - (1 - 2\varepsilon^2), \\ x_0 &= 2(1 + \varepsilon - \varepsilon^2) J_1^3 - (2 - 6\varepsilon^2 + \varepsilon^3) J_1^2 \\ &\quad - 2\varepsilon(1 - \varepsilon)(1 + 2\varepsilon) J_1 + \varepsilon(1 - 2\varepsilon^2), \end{aligned}$$

which relates the intensities of incident and reflected shocks [15]. Relation (8) follows from the condition of zero total angle of flow deflection on incident and reflected shocks of the SMC

$$\beta_1 - \beta_2 = 0,$$

where the angles of flow deflection at shocks are related to their intensities by equations of the form (3) and Mach number  $M_1$  of the flow in front of the second shock is determined according to [2,3]:

$$M_1 = \sqrt{[(J_1 + \varepsilon) M^2 - (1 - \varepsilon)(J_1^2 - 1)] / [J_1(1 + \varepsilon J_1)]}.$$

The resultant (determinant of the Sylvester matrix) of system (5), (8) obtained by excluding  $J_2$  yields an algebraic equation of the form

$$f(M^2, J_1, \varepsilon, \varepsilon_3, \bar{\varphi}) = 0 \quad (9)$$

that is of the tenth degree in  $J_1$  and of the fifth degree in  $M^2$ . At  $\gamma_3 = \gamma$  and  $\bar{\varphi} = 0$ , Eq. (9) is reduced to form (2) that specifies the „classical“ von Neumann criterion.

Curves 5a–5h in Fig. 2 represent the solutions of Eq. (9): intensities  $J_1$  of incident shocks at  $\gamma_3 = \gamma = 1.4$  and different values of pulsed energy release ( $\bar{\varphi} > 0$ , curves 5a–5f) or energy absorption ( $\bar{\varphi} < 0$ , curves 5g and 5h). Their comparison with curve 2, which corresponds to the flow of non-reactive gas, confirms that the von Neumann criterion shifts significantly toward weaker and stronger

incident shocks in the case of positive and negative energy release, respectively.

Curves 6a–6f in Fig. 2 illustrate the solutions of Eq. (9) with zero pulsed energy release ( $\bar{\varphi} = 0$ ), but with the adiabatic index of the gas mixture decreasing (curves 6a–6c) or increasing (curves 6d–6f) relative to the initial value ( $\gamma = 1.4$ ). Their comparison with „classical“ curve 2 reveals that a decrease in adiabatic index at the main shock at moderate and high Mach numbers leads to an increase in „von Neumann“ values of  $J_1$ ; as  $\gamma_3$  increases, the mentioned values become smaller. However, the shift of curves 6a–6f away from curve 2 is significantly smaller than the similar shift of curves 5a–5h; therefore, the pulsed energy release is the prevalent factor influencing the generalized von Neumann criterion.

According to the data presented in Fig. 4, angle of shock incidence  $\sigma_1$ , which is related to the incident shock intensity as

$$J_1 = (1 + \varepsilon) M^2 \sin^2 \sigma_1 - \varepsilon,$$

decreases by as much as 6–8° due to specific energy input. A change in adiabatic index of the gas mixture shifts angle  $\sigma_1$  by as much as 2–3°. At high Mach numbers, this angle of incidence is characterized by an asymptote that does not depend on  $\bar{\varphi}$ :

$$\sigma = \arcsin \sqrt{C/(1 + \varepsilon)},$$

$$\begin{aligned} &(1 - \varepsilon)^3 C^3 - (1 - \varepsilon)(2 + \varepsilon + \varepsilon \varepsilon_3 - 3\varepsilon^2 + \varepsilon^2 \varepsilon_3) C^2 \\ &+ (1 - \varepsilon^2)(1 + \varepsilon - 4\varepsilon^2 + 2\varepsilon^2 \varepsilon_3 + \varepsilon^3 - \varepsilon^3 \varepsilon_3) C \\ &- \varepsilon(1 - \varepsilon_3)(1 + \varepsilon)^2(1 - 2\varepsilon + \varepsilon \varepsilon_3) = 0, \end{aligned}$$

which becomes known from [3] at  $\gamma_3 = \gamma$  for gas without transformations:

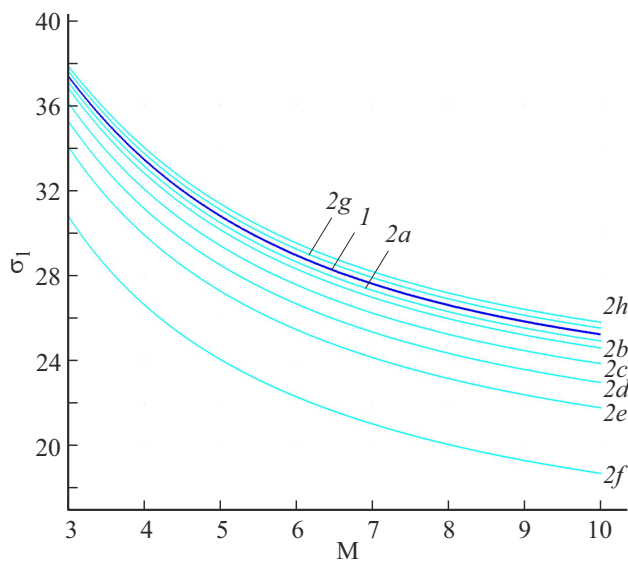
$$\sigma = \arcsin \sqrt{\frac{1 + \varepsilon - \varepsilon^2 + \varepsilon^3 + \sqrt{1 - 2\varepsilon + 3\varepsilon^2 + 4\varepsilon^3 - \varepsilon^4 - 2\varepsilon^5 + \varepsilon^6}}{2(1 - \varepsilon^2)}}.$$

Angle  $\beta_1$  of flow deflection at the incident shock in the transition configuration, which is specified by relation (3) at  $J = J_1$ ,  $\bar{\varphi} = 0$ , and  $\gamma_3 = \gamma$ , undergoes similar changes.

The conclusion regarding the weakening of incident shocks corresponding to the von Neumann criterion due to the release of energy and an increase in adiabatic index of gas at the main shock may be illustrated vividly on the plane of shock polars (e. g., by shifting the corresponding point on polar I from position  $N$  in the „classical“ case to position 1S at the maximum possible energy release; see Fig. 1, b).

The detailed analysis of solutions of (9) and their possible instability and infeasibility (especially at low and moderate Mach numbers) is a subject of separate study.

Thus, we obtained analytical relations specifying the generalized von Neumann criterion: the conditions of existence of a triple configuration with a normal main shock if a pulsed release or absorption of energy, as well as a change in chemical composition of the high-temperature gas mixture or high-temperature effects of real gas, which



**Figure 4.** Angles of incidence of shocks corresponding to the „classical“ (curve 1) and generalized (curves 2a–2h) von Neumann criteria at  $\gamma = \gamma_3 = 1.4$ . Curves 2a–2f — incident shock at an energy release constituting 0.1, 0.2, 0.4, 0.6, 0.8, and 1.0 of the critical value, respectively; curves 2g and 2h — incident shock at an energy absorption constituting 0.1 and 0.2 of the critical value.

lead to a change in adiabatic index, are observed at this shock. It was found that the energy release and an increase in adiabatic index shift significantly the von Neumann criterion toward weaker (relative to the „classical“ case) incident shocks, while inverse phenomena shift it toward stronger shocks. The obtained relations characterize qualitatively the shift of the von Neumann criterion within the Chapman–Jouguet stationary detonation model, but may be subject to numerical analysis and refinement with the application of more advanced gas detonation models.

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## Conflict of interest

The authors declare that they have no conflict of interest.

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