Evolution of waves in a porous medium when passing the boundary between bubbly and „pure“ liquids saturating the porous medium

© I.K. Gimaltdinov[1], I.G. Khusainov[1,2]

[1] State Petroleum Technical University, Ufa, Bashkortostan, Russia
[2] Ufa University of Science and Technology, Sterlitamak Branch, Sterlitamak, Russia
E-mail: iljas_g@mail.ru

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The reflection and transmission of acoustic waves in a porous medium across the boundary between bubbly and „clean“ liquids was studied. Dispersion relations for the reflection and transmission coefficients were obtained, accounting for the initiation of secondary („fast“ and „slow“ waves). The dynamics of finite-duration pulses was studied using the fast Fourier transform method. Keywords: porous medium, bubbly liquid, harmonic waves, reflection and transmission, pressure pulse, Fourier transform..

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The interest in propagation of acoustic waves in a porous medium saturated with liquid containing gas bubbles is associated with the need for acoustic monitoring of the effectiveness of water-gas treatment [1] or acid treatment of formations [2] aimed at oil recovery enhancement, with issues surrounding the formation and decomposition of gas hydrates in porous media [3], etc. Experimental studies of the impact of a shock wave on a dense layer of a granular medium were discussed in [4]. It was demonstrated that the signal of a pressure sensor located within the fill material may increase after the impact of the second and subsequent pressure pulses. Experimental studies into the propagation of pressure waves in a porous medium saturated with a gas-liquid mixture were carried out in [5]. The dependence of the wave propagation velocity on gas composition was examined. It was found that a „slow“ wave in a porous medium saturated with liquid containing carbon dioxide bubbles is attenuated to a lesser extent than in the case where the mentioned liquid contains air bubbles. The results of theoretical and experimental studies of the evolution of a pressure wave in a porous medium saturated with gas and liquid as a function of volumetric gas content were presented in [6]. It was demonstrated that even a relatively small amount of gas has a strong influence on the pressure wave evolution. The influence of interphase heat transfer on the propagation of acoustic waves in a porous medium saturated with bubbly liquid was analyzed (at different bubble size distributions) in [7]. It was found that the interphase heat transfer depends strongly on the bubble radius and weakly on the volume fraction of the gas phase in liquid. A numerical study of wave processes in a porous medium saturated with bubbly liquid was carried out in [8]. The nonlinearity of bubble oscillations was taken into account in this work.

The present study is the first to examine the dynamics of a wave propagating through a porous medium and passing the boundary between bubbly and „pure“ liquids with which the porous medium is saturated.

Let us assume that the porous medium consist of two layers saturated with bubbly $(0 \leqslant x \leqslant x_b)$ and „pure“ $(x_b < x \leqslant x_w)$ liquids. Pressure pulse $p_{in}(t)$ is induced at the boundary $(x = 0)$ of the first layer. The aim of the study is to analyze the features of passage of this pulse across the boundary between layers. The following system consisting of linearized equations of mass balance, number of bubbles, and momentum for a porous medium saturated with a gas-liquid mixture [9] was used for this purpose:

$$\frac{\partial \rho_{sk}}{\partial t} + \rho_{sk0}\frac{\partial v_{sk}}{\partial x} = 0, \quad \frac{\partial \rho_{l+g}}{\partial t} + \rho_{l+g0}\frac{\partial v_l}{\partial x} = 0,$$

$$\frac{\partial n_b}{\partial t} + n_{b0}\frac{\partial v_l}{\partial x} = 0, \quad \alpha_g = \frac{4}{3}\pi b^3 n_b,$$

$$\rho_{l+g0}\frac{\partial v_l}{\partial t} = -\alpha_{l0}\frac{\partial p_l}{\partial x} - F,$$

$$\rho_{l+g0}\frac{\partial v_l}{\partial t} + \rho_{sk0}\frac{\partial v_{sk}}{\partial t} = \frac{\partial \sigma^*_{sk}}{\partial x} - \frac{\partial p_l}{\partial x},$$

$$\rho_{sk} = \alpha_{sk}\rho^0_{sk}, \quad \rho_{l+g} = \alpha_l\rho^0_l + \alpha_g\rho^0_g, \quad \rho_g = \alpha_g\rho^0_g,$$

$$\rho_{l+g0} = \alpha_{l0}\rho^0_{l0} + \alpha_{g0}\rho^0_{g0}, \quad \alpha_g + \alpha_l + \alpha_{sk} = 1, \qquad (1)$$

where ρ_j v_j, and α_j are the reduced density, velocity, and volumetric content of phase j, respectively. Subscripts $j = sk$, l, g, and $l+g$ correspond to the porous medium skeleton, liquid, gas, and the mixture of liquid and bubbles, respectively. Additional subscript 0 denotes the parameters corresponding to the unperturbed initial state of the system; F is the interphase force; σ^*_{sk} and p_l are the reduced stress in the skeleton and the pressure in liquid, respectively; and n and b are the number of bubbles in unit volume and their radius. The skeleton of the porous medium is assumed to be viscoelastic:

$$\alpha_{sk0}\frac{\partial\varepsilon}{\partial t}=\frac{1}{E_{sk}}\frac{\partial\sigma^*_{sk}}{\partial t}+\frac{\sigma^*_{sk}}{\mu_{sk}},\quad \frac{\partial\varepsilon}{\partial t}=\frac{\partial v_{sk}}{\partial x},\qquad(2)$$

where E_{sk} and μ_{sk} are the effective modulus of elasticity and viscosity coefficient of the porous skeleton. Equations of state in the acoustic approximation are adopted for the solid and liquid phases; gas in the bubbles is considered to be calorically perfect:

$$\rho^0_{sk0}/\rho_{sk0}=1+\beta_{sk}\big(p_{sk}-p_{sk0}\big),\qquad p_{sk}=p_l-\sigma^*_{sk}/\alpha_{sk0},$$
$$\rho^0_{l0}/\rho_{l0}=1+\beta_l\big(p_l-p_{l0}\big),\qquad p_g=\rho^0_g RT_g.\qquad(3)$$

Here, β_j is the compressibility of phases, T_g and p_g are the gas temperature and pressure, and R is the gas constant. Characterizing the rate of change of bubble radius w [9,10], we assume that

$$db/dt=w=w_A+w_R,\qquad(4)$$

where w_R is determined based on the Rayleigh–Lamb equation for a porous medium and w_A is derived from the solution of the problem of spherical unloading on a sphere with radius b in carrier liquid in the acoustic approximation

$$\rho^0_{l0}b_0\frac{\partial w_R}{\partial t}=p_g-p_l-4\mu_l\frac{w_R}{b_0},\quad w_a=\frac{p_g-p_l}{\rho^0_{l0}C_l\varphi^{1/3}_{g0}}.\qquad(5)$$

where C_l is the speed of sound in „pure“ liquid and φ_{g0} is the volume fraction of the gas phase in bubbly liquid. The equation for pressure in the gas phase is written as [10]

$$\frac{\partial p_g}{\partial t}=-\frac{3\gamma p_g}{b_0}w-\frac{3(\gamma-1)}{b_0}q,\qquad(6)$$

where q is the intensity of gas–liquid heat transfer and γ is the polytropic index.

To determine the intensity of gas–liquid heat transfer q, we write the heat conductivity equation and boundary conditions in the linear approximation [10]:

$$\rho^0_{g0}c_g\frac{\partial T_g}{\partial t}=r^{-2}\left(\lambda_g r^2\frac{\partial T_g}{\partial r}\right)+\frac{\partial p_g}{\partial t},\quad r<b_0,\qquad(7)$$

$$T_g=T_l,\ q=-\lambda_g\left(\frac{\partial T_g}{\partial r}\right),\ r=b_0,\ \frac{\partial T_g}{\partial r}=0,\ r=0.\ (8)$$

The solution of system (1)–(8) is sought in the form of harmonic waves

$$\alpha_j,\rho^0_j,v_j,p_j,\sigma^*_s\cong\exp\big[i(Kx-\omega t)\big],$$
$$T_g=T^*_g(r)\exp\big[i(Kx-\omega t)\big],$$

where ω is the angular frequency, K is the complex wave number ($K=k+i\delta$), δ is the linear attenuation ratio, and r is the microcoordinate measured from the bubble center.

Two types of longitudinal waves propagate in the porous medium: „fast“ and „slow“ ones [11]. Having solved system (1)–(8), we obtained the wave numbers for the first and second layers: $K^{(1)}_f$, $K^{(1)}_s$ and $K^{(2)}_f$, $K^{(2)}_s$. Superscripts $i=1$, 2 correspond to the parameters of a wave of the first and second layers, respectively. Subscripts f and s correspond to the parameters of „fast“ and „slow“ waves. Performing certain transformations, we obtain the following dispersion equation for a porous medium layer saturated with bubbly liquid:

$$-\alpha_{l+g0}\chi_\mu(\xi/\phi)\left(\frac{K^{(1)}}{\omega}\right)^4+\left\{\xi\left(1-\alpha_{s0}\frac{\rho_{l+g0}}{\phi}\right)\right.$$
$$\left.+\varepsilon\chi_\mu\left(1-\frac{\rho_{l+g0}}{\phi}\right)\right\}\left(\frac{K^{(1)}}{\omega}\right)^2+c=0,\qquad(9)$$

where

$$\phi=\chi_V/i\omega,\quad \chi_\mu=-\alpha_{s0}E_s/(1+iE_s/\omega\mu_s),$$
$$c=\varepsilon\big[\rho_{l+g0}+\rho_{s0}(1-\rho_{l+g0}/\phi)\big],$$
$$\xi=1-(1-1/\gamma)i\omega b_0\rho^0_{l0}Q/P_{g0},$$
$$Q=\lambda_g T_{g0}(1-1/\gamma)\frac{\chi_\mu}{P_{g0}b_0}\big[\mathrm{cth}(y_g)-1\big],\ y_g=\sqrt{-\frac{i\omega b_0^2}{\kappa_g}},$$
$$\kappa_g=\lambda_g/\rho^0_{g0}c_g,$$
$$\chi_V=-i\eta_m/2+\eta_\mu\nu_l a_0^{-2}/\omega+\eta_B(1-i)a_0^{-1}\sqrt{2\nu_l/\omega},$$
$$\nu_l=\mu_l/\rho^0_{l0}.$$

With a „fast“ or „slow“ wave incident on the interface between layers, reflected „fast“ and „slow“ waves form in the first layer and transmitted „fast“ and „slow“ waves form in the second one. The resulting disturbances of effective stress in the skeleton and pressure in the liquid are determined for these waves in the following way:

$$\sigma^{*(i)}_{sk}=\big[A^{(i)}_{\sigma,f}\exp(iK^{(i)}_f x)+A^{(i)}_{\sigma,s}\exp(iK^{(i)}_s x)\big]\exp(-i\omega t),$$
$$p^{(i)}_l=\big[A^{(i)}_{p,f}\exp(iK^{(i)}_f x)+A^{(i)}_{p,s}\exp(iK^{(i)}_s x)\big]\exp(-i\omega t).\qquad(10)$$

Here, $A^{(i)}_{\sigma,f}$, $A^{(i)}_{p,f}$ and $A^{(i)}_{\sigma,s}$, $A^{(i)}_{p,s}$ are the amplitudes of effective stress and pressure of „fast“ and „slow“ waves, respectively. By analogy with (10), one may write expressions for the velocities of the skeleton of the porous medium and fluid with amplitudes $A^{(i)}_{vsk,f}$, $A^{(i)}_{vl,f}$ and $A^{(i)}_{vsk,s}$, $A^{(i)}_{vl,s}$.

We use the following boundary conditions at $x=x_b$ to obtain the reflection and transmission coefficients of harmonic waves: equality of the velocities of skeleton and fluid particles, equality of pressures in the fluid, and continuity of total stresses

$$v^{(1)}_{sk}=v^{(2)}_{sk},\quad \nu^{(1)}_l=\nu^{(2)}_l,\quad p^{(1)}_l=p^{(2)}_l,$$
$$-\alpha_{sk0}\sigma^{*(1)}_{sk}+(\alpha_{l0}+\alpha_{g0})p^{(1)}_l=-\alpha_{sk0}\sigma^{*(2)}_{sk}+\alpha_{l0}p^{(2)}_l.\qquad(11)$$

Inserting formulae (10) for the effective stress in the skeleton and the pressure in liquid and similar formulae

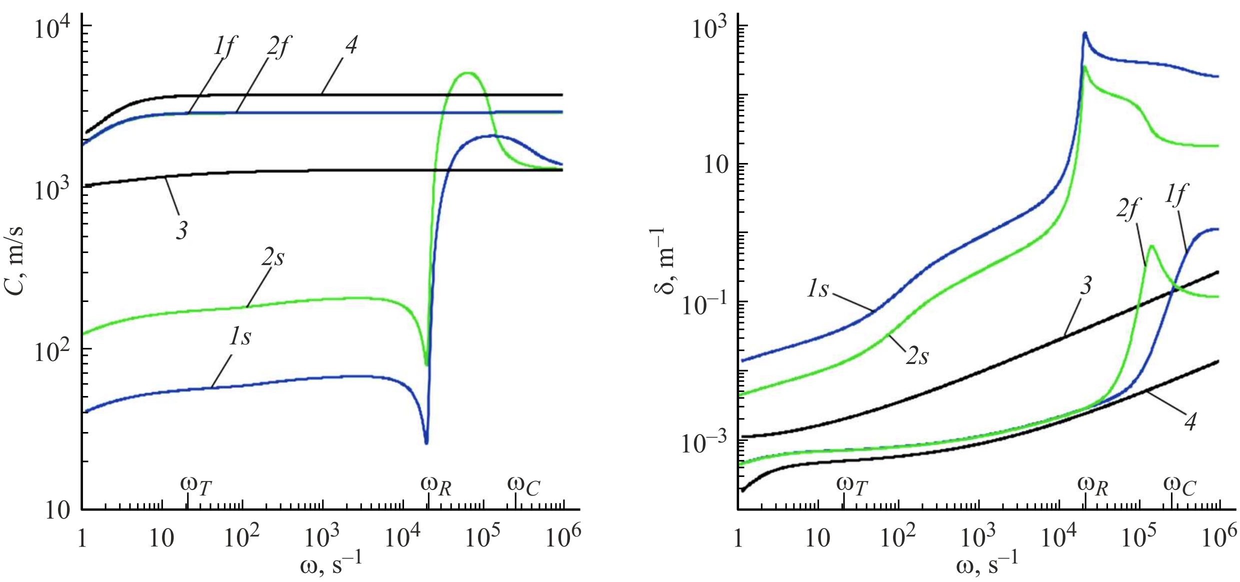

Figure 1. Phase velocities and linear attenuation ratio of „fast" (*1f*, *2f*) and „slow" (*1s*, *2s*) waves in the porous medium saturated with bubbly liquid. Curves *1f*, *1s* correspond to $\alpha_{g0} = 0.01$; curves *2f*, *2s*, to $\alpha_{g0} = 0.001$. Curves *3* and *4* represent „slow" and „fast" waves with the porous medium saturated with „pure" liquid.

for the velocity of the skeleton of the porous medium and fluid with amplitudes $A_{j,f}^{(i)}$, $A_{j,s}^{(i)}$ $(j = \sigma,\ p,\ vsk,\ vl)$ into system (1)–(6), we find the relations between amplitudes. Using these expressions, we then obtain the reflection and transmission coefficients from boundary conditions (11). These coefficients are determined using the ratio of pressure amplitudes in liquid. Let us denote the reflection coefficients as N_{yz} (y and z correspond to the type of incident and reflected waves, respectively). For example, reflection coefficients N_{ff} and N_{fs} for „fast" and „slow" waves, respectively, are obtained when an incident „fast" wave is reflected from boundary $x = x_b$. The transmission coefficient is denoted as M_{yz}.

$$M_{ys} = 2\delta_1, M_{yf} = \frac{2}{a_{4y}}(1 - \delta_1 a_{5y}), N_{yf} = \frac{2b_{4y}}{a_{4y}} - 2\delta_1\delta_2,$$

$$N_{ys} = 2\frac{1 - b_{4y}}{a_{4y}} - 2\frac{a_{5y} - a_{4y}}{a_{4y}}\delta_1 + 2\delta_1\delta_2 - 1, \qquad (12)$$

where

$$\delta_1 = \frac{a_{6y} - a_{4y}}{a_{5y}a_{6y} - a_{4y}a_{7y}}, \qquad \delta_2 = \frac{b_{4y}a_{5y} - b_{5y}a_{4y}}{a_{4y}},$$

$$A_y^{(i)} = \chi_\mu C_y^{(i)} K_y^{(i)}/\omega, \quad \psi = 1 - \phi/\rho^{(i)}, \quad \rho^{(1)} = \rho_{l+g0},$$

$$\rho^{(2)} = \rho_{l0}, \quad B_y^{(i)} = K_y^{(i)}/\omega\rho^{(i)} - \phi_y^{(i)} C_y^{(i)}/\rho^{(i)},$$

$$\phi_y^{(i)} = \rho_{s0} + \phi(K_y^{(i)}/\omega)^2, \quad C_y^{(i)} = \frac{\alpha^{(i)} K_y^{(i)}/\omega}{\phi - \psi\phi_y^{(i)}},$$

$$\alpha^{(1)} = \alpha_{l+g0},\ \alpha^{(2)} = \alpha_{l0},\ C_y^{(i)r} = -C_y^{(i)},\ B_y^{(i)r} = -B_y^{(i)},$$

$$A_y^{(i)r} = A_y^{(i)}, \quad a_{1s} = A_f^{(1)r}/A_s^{(1)}, \quad a_{1f} = A_s^{(1)r}/A_f^{(1)},$$

$$a_{2y} = A_f^{(2)}/A_y^{(1)}, \quad a_{3y} = A_s^{(2)}/A_y^{(1)}, \quad c_{1s} = C_f^{(1)r}/C_s^{(1)},$$

$$c_{1f} = C_s^{(1)r}/C_f^{(1)}, \quad c_{2y} = C_f^{(2)}/C_y^{(1)}, \quad c_{3y} = C_s^{(2)}/C_y^{(1)},$$

$$b_{1s} = B_f^{(1)r}/B_s^{(1)}, \quad b_{1f} = B_s^{(1)r}/B_f^{(1)},$$

$$b_{2y} = B_f^{(2)}/B_y^{(1)}, \quad b_{3y} = B_s^{(2)}/B_y^{(1)},$$

$$b_{4y} = \frac{b_{2y} - c_{2y}}{b_{1y} - c_{1y}}, \quad b_{5y} = \frac{b_{3y} - c_{3y}}{b_{1y} - c_{1y}},$$

$$a_{4y} = 1 + c_{2y} - b_{4y}(1 + c_{1y}),$$

$$a_{5y} = 1 + c_{3y} - b_{5y}(1 + c_{1y}),$$

$$a_{6y} = a_{2y} + c_{2y} - b_{4y}(a_{1y} + c_{1y}),$$

$$a_{7y} = a_{3y} + c_{3y} - b_{5y}(a_{1y} + c_{1y}).$$

In our calculations, the gas was air, the liquid was water, and the porous medium was sandstone with parameters

$$\rho_{g0}^0 = 1.29\,\mathrm{kg/m^3}, \quad c_g = 1027\,\mathrm{J/(kg\cdot K)},$$

$$\lambda_g = 0.027\,\mathrm{J/(m\cdot s\cdot K)}, \quad \gamma = 1.4, \quad \rho_{s0}^0 = 2560\,\mathrm{kg/m^3},$$

$$\mu_s = 10^9\,\mathrm{Pa\cdot s}, \quad E_s = 3.7\cdot 10^{10}\,\mathrm{Pa}, \quad C_l = 1500\,\mathrm{m/s},$$

$$\rho_{l0}^0 = 1000\,\mathrm{kg/m^3}, \quad \alpha_{g0} = 0.01, \quad \mu_l = 10^{-3}\,\mathrm{Pa\cdot s},$$

$$\alpha_{l0} = 0.4, \quad b_0 = 1\,\mathrm{mm}.$$

The phase velocities and linear attenuation ratios in the porous medium saturated with bubbly liquid, which correspond to different values of the volumetric gas content, are presented in Fig. 1. The following characteristic frequencies are indicated: ω_T is the frequency separating isothermal $(\omega < \omega_T)$ and adiabatic $(\omega > \omega_T)$ bubble behaviors, ω_R is the natural frequency of radial vibration of bubbles, and

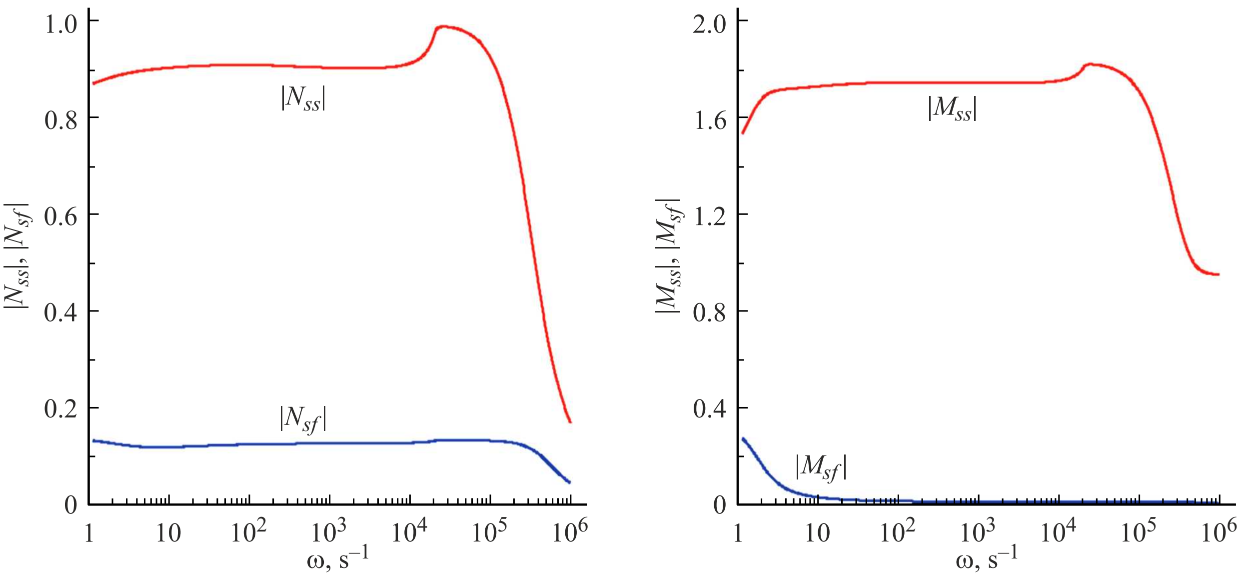

Figure 2. Dependences of the moduli of reflection coefficients of „slow" and „fast" waves initiated in the first layer and transmission coefficients of „slow" and „fast" waves initiated in the second layer on angular frequency in the case when a „slow" wave is incident on the boundary between the layers ($b_0 = 0.001$, $\alpha_{g0} = 0.01$).

ω_C is the frequency that specifies the right boundary of the opacity band (in Fig. 1, $\omega_R = 20\,500\,\mathrm{s}^{-1}$, $\omega_C = 85\,000\,\mathrm{s}^{-1}$, and $\omega_T = 20\,\mathrm{s}^{-1}$ at $\alpha_{g0} = 0.001$). The expressions for these frequencies are [10]

$$\omega_R = \sqrt{3\gamma p_0/\rho_{l0}^0}\Big/b_0, \quad \omega_C = \omega_R\sqrt{1+\rho_{l0}^0\alpha_{g0}C_l^2/\gamma p_0},$$

$$\omega_T = \kappa_g/b_0^2, \quad \kappa_g = \lambda_g/\rho_{g0}^0 c_g.$$

The minimum phase velocity and the maximum attenuation ratio of a „slow" wave correspond to natural frequency of bubble vibrations ω_R. Within the $0 < \omega < \omega_R$ frequency range, the velocity of a „slow" wave in the porous medium saturated with bubbly liquid is lower (curves *1s*, *2s*), while the attenuation is stronger than in the porous medium saturated with „pure" liquid (curve *3*). As the frequency increases, the velocity reaches the same levels as in the porous medium with „pure" liquid. The influence of volumetric content on the „slow" wave velocity is also strong within this range ($0 < \omega < \omega_R$): an order-of-magnitude reduction in volumetric content of gas leads to an approximately twofold increase in phase velocity (curves *1s*, *2s* in Fig. 1). The variation of volumetric content of gas in bubbly liquid does not affect the phase velocity of a „fast" wave (curves *1f*, *2f* in Fig. 1). When the porous medium is saturated with „pure" liquid, the „fast" wave velocity remains approximately 800 m/s higher within the entire frequency range (curve *4*). As the frequency grows to ω_R, the attenuation coefficient of a „slow" wave increases strongly (by approximately four orders of magnitude; curves *1s*, *2s*) at both values of the volumetric gas content. The attenuation coefficient of a „slow" wave in the second layer is lower (curve *3*) than in the first one (curves *1s*, *2s*) within the entire frequency range. The attenuation coefficient of a „fast" wave at frequencies $\omega < \omega_R$ does not depend on the type of liquid (with or without bubbles) that saturates the porous medium (curves *1f*, *2f*, *4*). Note that the frequency dependences of the phase velocity and the linear attenuation ratio for a „slow" wave are the same as those for bubbly liquid [12].

Figure 2 presents the dependences of moduli of reflection $|N_{ss}|$, $|N_{sf}|$ and transmission $|M_{ss}|$, $|M_{sf}|$ coefficients on angular frequency ω. At $0 < \omega < \omega_R$, we have $|N_{ss}| \approx 0.9$, $|M_{ss}| \approx 1.7$; in the $\omega_R \leqslant \omega \leqslant \omega_C$ opacity band, $|N_{ss}| \approx 1.0$, $|M_{ss}| \approx 1.9$. Thus, the reflection of an incident „slow" wave from boundary $x = x_b$ is similar to the reflection from a solid wall within the $0 < \omega < 10^6\,\mathrm{s}^{-1}$ frequency range; i. e., the modulus of transmission coefficient is greater than unity. With a „slow" wave incident on the boundary between layers, a reflected „fast" wave of a moderate amplitude ($|N_{sf}| \approx 0.13$) is initiated in the first layer, while a transmitted „fast" wave is virtually nonexistent at frequencies $\omega > \omega_T$ in the second layer: $|M_{sf}| \approx 0$.

With a „fast" wave incident on the boundary, a reflected „fast" wave is initiated in the first layer with the modulus of reflection coefficient approximately equal to 0.06 within the $0 < \omega < \omega_C$ frequency range; as the frequency increases, the modulus of reflection coefficient increases to 0.15 (Fig. 3). A transmitted „fast" wave is initiated in the second layer. At frequencies $1 < \omega < \omega_T$, the transmission coefficient decreases from $|M_{ff}| \approx 1.7$ to $|M_{ff}| \approx 1.1$; at $\omega_T < \omega < \omega_R - |M_{ff}| \approx 1.1$. With a further increase in frequency, the transmission coefficient decreases to 0.8; i. e., a „fast" wave is barely reflected from the boundary. At frequencies $\omega < \omega_R$, a reflected „slow" wave with

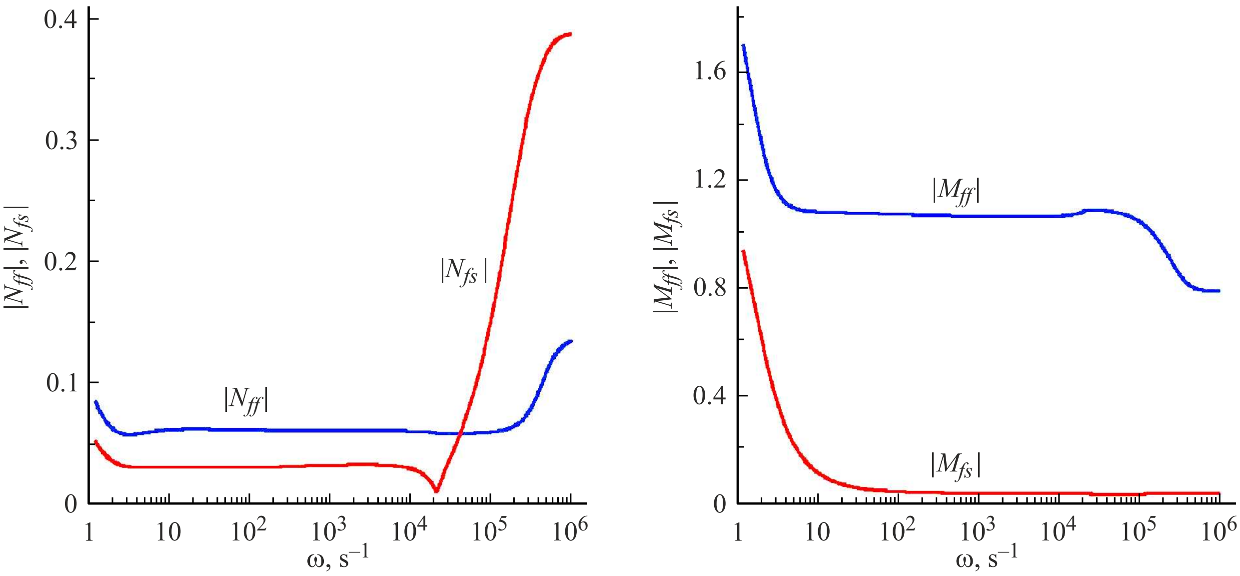

Figure 3. The same as in Fig. 2, but for a „fast" wave incident on the $x = x_b$ boundary. The calculation parameters are the same as in Fig. 2.

coefficient $|N_{fs}| \approx 0.06$ is initiated in the first layer. This coefficient increases gradually to 0.4 at $\omega_R < \omega$. The modulus of transmission coefficient $|M_{fs}|$ of a „slow" wave initiated in the second layer decreases from 1.0 to 0 as the frequency grows from 1 to ω_T; i.e., when a „fast" wave is reflected from boundary $x = x_b$, „slow" waves are not initiated in the second layer at frequencies $\omega > \omega_T$.

Based on the obtained wave numbers and reflection and transmission coefficients, the dynamics of a finite-duration wave incident on boundary $x = x_b$ was studied using the fast Fourier transform method. Let us assume that a „slow" wave in the form of a bell-shaped pulse with length $t_* = 10$ ms and a unit amplitude is induced at the left boundary of the first layer ($x = 0$; Fig. 4). The fundamental pulse frequency is $\sim 6000\,\mathrm{s}^{-1}$. The pulse amplitude decreases from 1 to 0.7 within a propagation distance of 5 m due to energy dissipation in bubbly liquid and interfacial forces (curve *2*). It should be noted that the wave profile does not feature oscillations characteristic of bubbly liquid, since the characteristic duration of the pulse is much greater than the period of natural vibrations of bubbles ($t_* \gg t_R$, $t_R = 2\pi/\omega_R$). The amplitude of the pulse reflected from the $x = x_b$ boundary decreases to 0.55 (curve *3*), while the transmitted wave amplitude increases to 1.2 (curve *4*). The amplitude of the reflected pulse at sensor D_1 is close to 0.4 (curve *5*). Figure 4, *b* presents the plots corresponding to the case when a „fast" wave in the form of a pulse is induced at the left boundary of the first layer ($x = 0$). The pulse travels a distance of 40 m (to the $x = x_b$ boundary) with almost no attenuation (curve *2*). Its amplitude is ~ 0.95. The pulse reflected from the $x = x_b$ boundary (curve *3*) has an amplitude of just ~ 0.05; i.e., a „fast" wave is weakly reflected. A „fast" wave in the form of a pulse passes through the $x = x_b$ boundary with an amplitude of ~ 1 (curve *4*). The reflected pulse „returns" to sensor D_1 with an amplitude of ~ 0.05 (curve *5*).

Only one wave is seen in Fig. 4, since reflected and transmitted „fast" and „slow" waves (curves *3* and *4*) are superimposed on each other. The reflected pulse reaching the $x = 0$ boundary (curve *5*) also has a single-wave structure, since the initiated „slow" wave (Fig. 4, *b*) has enough time to decay and the initiated „fast" wave (Fig. 4, *a*) has a very small amplitude.

We note in conclusion that the following facts were established:

— frequency dependences of the phase velocity and the linear attenuation ratio for a „slow" wave in a porous medium saturated with bubbly liquid are the same as those for a wave in bubbly liquid;

— reflection of a „slow" wave incident from the side of bubbly liquid (at frequencies $0 \leqslant \omega \leqslant 10^5\,\mathrm{s}^{-1}$) from the bubbly liquid–pure liquid boundary is similar to the reflection from a solid wall ($|N_{ss}| \approx 0.9$, $|M_{ss}| \approx 1.8$), and a „fast" wave within the same frequency range is barely reflected from this boundary ($|N_{ff}| \approx 0.05$, $|M_{ff}| \approx 1.0$).

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Conflict of interest

The authors declare that they have no conflict of interest.

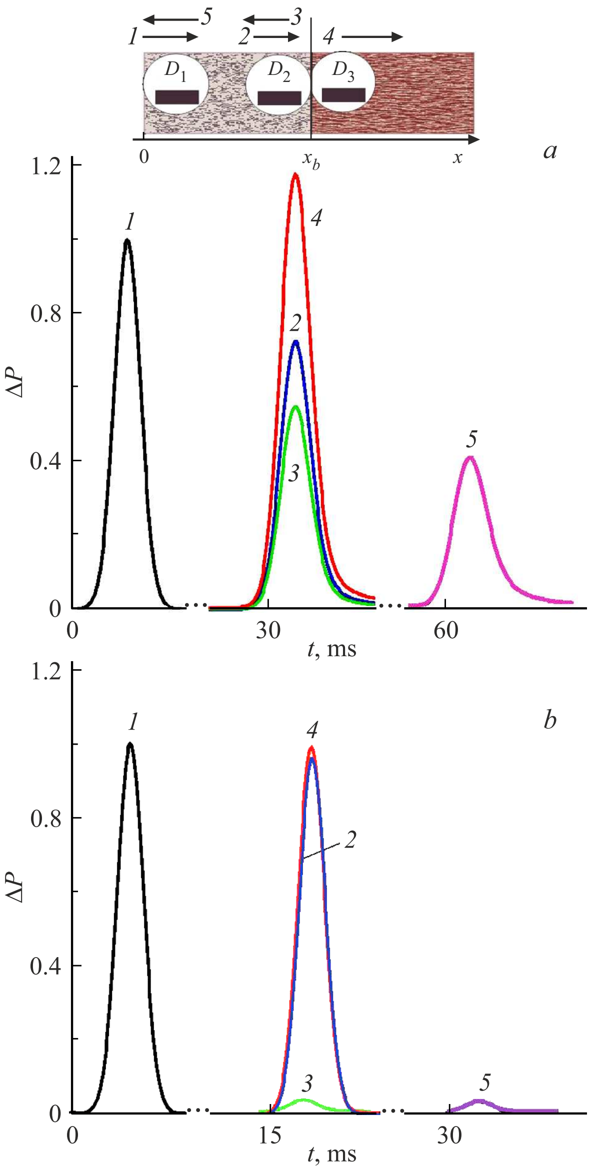

Figure 4. „Slow“ (a) and „fast“ (b) wave pulses reflected from and passing through the boundary between layers ($\alpha_{g0} = 0.01$). $x_b = 5$ (a) and 40 m (b). The schematic diagram of the problem is shown in the inset above: sensors D_1, D_2, and D_3 are positioned at boundary $x = 0$ of the calculation domain and to the left and right of boundary $x = x_b$, respectively. Numbers correspond to the calculated oscillograms: *1* — initial pulse, *2* — pulse propagating from left to right at the $x = x_b$ boundary, *3* — reflected pulse, *4* — transmitted pulse, and *5* — reflected pulse reaching the $x = 0$ boundary.

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