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Kinetic theory of the cathode layer in low-voltage beam discharges

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The work is devoted to the construction of the kinetic theory of the cathode layer of a low-voltage beam discharge in inert gases at Knudsen numbers of the order of 1. Under these conditions, this plasma system is promising for the creation of plasma electronic devices. It was found that the structure of the cathode layer of such a discharge differs fundamentally from that of a self-sustaining glow discharge. Thus, the spatial dependence of the electric field may be non-monotonic and contain a maximum. In this case, the values of the electric field near the cathode are significantly smaller, the spatial dependences of the electron and ion densities may have a minimum, and the lengths of the quasi-neutral pre-layer and the cathode part of the disturbed layer, where quasi-neutrality is significantly violated, increase significantly.

Keywords: cathode sheath, presheath, electron beam, elastic scattering of electrons.

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So-called beam discharges, where a beam of electrons moving in plasma carries current and provides the ionization rate needed for a discharge, are known [1–4]. The low-voltage beam discharge (LVBD) in inert gases belongs to this group. Studies into the dynamics of electron beams in plasma are important in the context of development of next-generation plasma electronic devices that utilize beam plasma [5,6].

The kinetic theory of wall layers (including cathode ones) for arbitrary conditions in gas-discharge plasma was developed in [7]. However, the authors examined a glow discharge, where plasma electrons ensure current transfer and ionization.

According to known concepts, the disturbed wall layer (DWL) has the following structure. It is divided into the so-called quasi-neutral pre-layer and the wall layer, where quasi-neutrality is violated to a significant degree. In the pre-layer, the electric field increases relatively weakly with distance from its boundary with undisturbed plasma, and when ions reach a certain average velocity, charge separation is initiated due to a strong reduction of the electron concentration compared to the ion one. This is accompanied by a marked intensification of the electric field. However, owing to the presence of a beam of fast electrons, the LVBD is characterized by a number of specific features of formation of the cathode layer, which make the kinetic theory developed in the mentioned studies inapplicable in this case [5,6].

It should also be noted that, under certain conditions, beam discharges are unstable with respect to weak perturbations of their parameters [8]. This hinders the application of LVBDs. It is known that the instability increments of harmonic perturbations of LVBD parameters have a maximum at the plasma frequency. Beam noise plays a leading part in the buildup of oscillations within the range of LVBD parameters that are promising in the context of practical applications [9]. Thus, it is important to study the beam noise spectrum, which is shaped, among other things, by the cathode layer structure.

Let us use the LVBD model developed in [9]. Under the conditions of application of LVBDs in plasma electronics with the interelectrode distance being shorter than 10–12 mm and the inert gas pressure amounting to several tenths of Torr, the following relation is normally satisfied:

$$d_c \ll d \sim \lambda_{ea}, \qquad (1)$$

where d_c, d, and λ_{ea} are the DWL width, the interelectrode distance, and the mean free path with regard to elastic collisions with atoms of the plasma-forming gas, respectively. The interelectrode potential difference in the DWL is close to the ionization potential of the plasma-forming gas. The mean free path of electrons with regard to inelastic collisions (λ_{eai}) is more than an order of magnitude greater than λ_{ea} [4]; therefore, they are taken into account by introducing plasma formed due to inelastic collisions into analysis. Since relations (1) are fulfilled, width d_c may be neglected when one characterizes the processes outside the DWL. The cathode is the source of a beam of electrons that carry the discharge current.

As beam electrons move, they undergo elastic collisions with atoms of the plasma-forming gas without losing their energy. Each elastic collision contributes to a significant isotropization of the directional distribution of beam electrons. It follows that the LVBD electron cloud within the energy range of the initial beam (after acceleration in the cathode layer) outside the DWL is a virtually monokinetic electron gas. This gas consists of two groups of electrons: (1) unscattered ones; (2) electrons that underwent elastic collisions with atoms of an inert gas.

Thus, under the considered conditions, we have three groups of electrons in the LVBD with significantly different distribution functions (EDFs):

— virtually unidirectional beam electrons that have not undergone any collisions;

— electrons with an energy close to the beam energy, which have undergone at least one elastic collision with atoms and are characterized by a directional distribution with a small average cosine between the velocity and the normal to the cathode plane;

— plasma electrons with a weakly anisotropic velocity distribution and average energy

$$E_e = 1.5kT_e \ll E_0, \qquad (2)$$

where $T_e \sim 1\,\mathrm{eV}$ is the EDF temperature in plasma and E_0 is the energy of beam electrons.

The above model provides the following description of the processes of formation of the cathode layer structure. The cathode potential is negative relative to plasma and the cathode layer. Let us introduce a Cartesian coordinate system XYZ with plane XY coinciding with the boundary of undisturbed plasma and axis Z being directed along the electric field of plasma toward the cathode. Electrons escape from the cathode heated to temperature $T_c \sim 1000\,\mathrm{K}$ and start to move in the direction opposite to the Z axis. Moving toward plasma, they accelerate to energies corresponding to the voltage drop across the cathode layer and form a virtually unidirectional beam, since $E_0 \gg 1.5kT_c$.

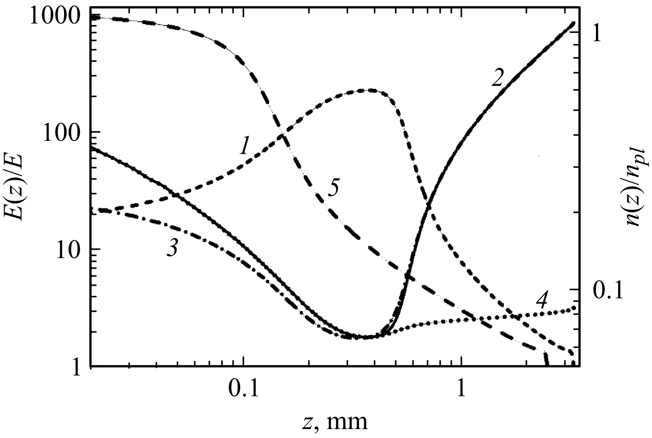
Thus, a group of elastically scattered electrons with the beam energy and plasma electrons with average energy E_e move in the cathode layer from plasma toward the cathode along the Z axis, slowing down in the layer field. Ions of the plasma-forming gas produce a positive charge between plasma and the cathode. One significant difference between the structure of the cathode layer in an LVBD and the one in a glow discharge is that the negative charge in this layer is produced not only by plasma electrons, but also by a unidirectional beam moving (from the cathode side) toward an isotropic flow of electrons with energy E_0 (from the plasma side), which satisfies inequality (2).

It was assumed in the quantitative description of the spatial structure of the cathode layer that resonant charge exchange is the key process shaping the ion distribution function (IDF) both in plasma and in the DWL. The approach developed in [7] was applied to this problem. A system of equations consisting of Boltzmann kinetic equations for the IDF, the distribution function of electrons emitted by the cathode and fast elastically scattered electrons, and the Poisson equation was formulated and solved. The EDF was considered to be Maxwellian both in plasma and in the DWL.

The developed theory was verified by comparing the calculation results with experimental data taken from other studies [4,10,11]. It was found that they matched closely.

Let us examine the results of calculation of the spatial distributions of DWL parameters under LVBD conditions. The figure shows the spatial dependences of concentrations of various groups of charged particles under typical LVBD conditions in He and the relative electric field in the cathode layer, including that determined without account for fast electrons. As can be seen, the electric field increases monotonically in the latter case, eventually becoming approximately 10^3 times stronger than the plasma one. When groups of fast electrons in the cathode region are taken into account, the growth of the field slows down. At a sufficiently high concentration of these electrons, the electric field is non-monotonic in accord with the non-monotonic dependences of the ion concentration and the total concentration of electrons, which consists of three different groups. At the same time, the calculations revealed that under LVBD conditions, the electric field in the part of the cathode layer adjacent to plasma is significantly stronger than it would be if fast electrons were not taken into account. In addition, when fast electrons are taken into account, the concentration of ions in a discharge with the same concentration of plasma electrons is higher, since the concentration of ions in plasma is equal to the total concentration of electrons (fast ones included). The cathode layer structure in the LVBD has a significant distinguishing feature in that the negative charge in the DWL part adjacent to plasma (small values of variable z in Fig. 1) is produced not by plasma electrons with an exponentially decreasing density, but by fast electrons with a relatively slowly decreasing density.

One of the key results of this study is that the first kinetic theory characterizing the structure of the cathode layer under LVBD conditions suitable for application of a beam discharge of this type in the design of plasma electronic devices has been formulated. The conditions for formation



Spatial dependences of relative electric field strength $E(z)/E$ (*1* — with account for fast electrons; *5* — with fast electrons excluded) and relative concentrations of charged particles $n(z)/n_{pl}$ (*2* — total electron concentration; *3* — ion concentration; *4* — fast electron concentration) in the cathode layer of the LVBD. $P_{\mathrm{He}} = 2\,\mathrm{Torr}$, $j = 1\,\mathrm{A/cm^2}$, $kT_e = 1.54\,\mathrm{eV}$, $kT_k = 0.06\,\mathrm{eV}$, and $d = 12\,\mathrm{mm}$ (P_{He}, j, T_e, and T_k are the plasma-forming gas pressure, the current density, and the temperatures of electrons in plasma and the cathode surface, respectively).

of the cathode LVBD layer differ significantly from those in a conventional glow discharge in that this layer supports not only a flow of ions from plasma to the cathode and plasma electrons, but also counter flows (from the cathode to plasma and vice versa) of fast electrons with coordinate-dependent energy. Their maximum energy is much greater than the average energy of electrons in plasma.

The obtained results provide an opportunity to investigate the spectrum of beam noise in the LVBD. This is crucial for the examination of LVBD stability, since, as was noted earlier, if the spectrum of beam noise contains the plasma frequency, this noise is exactly the one that governs the dynamics of perturbations in the system.

The reported results are of great importance not only for the LVBD, but also for other types of discharges with a heated cathode, where electrons enter the discharge from the cathode due to thermionic emission and are accelerated in the cathode layer to energies exceeding significantly the average energy of plasma electrons.

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Conflict of interest

The authors declare that they have no conflict of interest.

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