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Role of interparticle collisions in the transport properties of two-component two-dimensional electron systems with a parabolic spectrum

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The contribution of the Coulomb interaction to the conductivity of a two-dimensional electron system with two different effective masses of electrons is theoretically studied. It is shown that, unlike impurity scattering, in which the difference in effective masses is weakly manifested, the correction to conductivity from electron–electron scattering is extremely sensitive to the nanostructure parameters. For a significant difference in the electron effective masses, the low-temperature asymptotic behavior of the contribution of the electron–electron interaction to conductivity is $\propto T^2 \ln T$, with the sign of this contribution determined by the position of the Fermi level and the characteristics of disorder in the system.

Keywords:: electron-electron scattering, two-dimensional electron system, conductivity.

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1. Introduction

In physics of semiconductors and solid-state nanostructures, the systems where conductivity is provided by several types of charge carriers are common. When additional scattering channels appear between the particles of different types it radically changes the nature of internal interactions in the system during the experiment. For example, in structures with a parabolic electron spectrum, the Coulomb interaction between electrons does not contribute to conductivity due to conservation of total momentum of the electron gas and, as a result, its drift velocity. However, in multi-zone structures, the invariance of the system with respect to Galilean transformations is violated, as a result of which the role of the electron-electron interaction in the system's response to external disturbances becomes significant. Various aspects of transport phenomena in systems containing different types of electric charge carriers have been studied for many years. In particular, a number of studies highlighted the role of electron-hole scattering in semi-metals [1–6], and in [7] the scattering of the spin-split subband holes in GaAs-based heterostructures was theoretically and experimentally analyzed. The difference in the effective masses of electrons starts to manifest itself especially vividly under the influence of a high-frequency electromagnetic field of high intensity. When exposed to illumination by a circularly polarized field, electrons with different effective masses form bound states [8]. As a result, an exotic mixture of gases obeying Bose-Einstein and Fermi-Dirac statistics with unique properties is formed in the system [9].

The Galilean invariance is violated in any system in which the relationship of kinetic energy of particles with momentum is not quadratic, which is often typical for the quasiparticles in crystal structures. For example, the influence of a specific geometry of Fermi surfaces on conductivity was studied in [10,11], and recently the interaction of particles in systems with the Dirac type of the electron energy spectrum [12,13] was investigated. In addition, there is a condition under which the electron-electron interaction contributes to conductivity even in single-component electronic systems with a parabolic spectrum, namely, the dependence of the electron relaxation time on crystal lattice defects and impurity centers on momentum (or energy) [14]. As a rule, when describing two-component systems, they are limited to using constant relaxation times of electrons on impurity disorder, however, more precise approaches are also being undertaken, including consideration of shielded Coulomb impurities [15]. An interesting issue arises which is investigated in this paper: what are the properties of a system that satisfies two conditions for a nonzero contribution of inter-electron scattering to conductivity: the presence of two types of electrons with a parabolic energy spectrum and an energy-dependent relaxation time. At the same time, to get the maximum possible effect from the combination of these conditions, we'll expect the shielding potential of Coulomb impurities and electron-electron interaction to be completely suppressed. A discussion of the aftermath of taking the shielding into account is given at the end of the article.

2. Model

Let us consider a two-dimensional degenerate gas of electrons with different effective masses and a parabolic energy spectrum (Figure 1):

$$\varepsilon_{\mathbf{p}}^{i} = \frac{p^2}{2m_i} \tag{1}$$

where m_i — effective mass of i-th electron and $i = (a, b)$ index differentiates the electrons with different effective masses. Let's assume for certainty $m_a < m_b$. In nature, this type of energy spectrum is realized, for example, in the valence bands of graphane [16] and gallium arsenide [15].

In this paper, we are interested in the static conductivity σ of a two-component system, defined as the coefficient of proportionality between the electric current density and the electric field strength in Ohm's law:

$$\mathbf{j} = \sigma\mathbf{E}, \tag{2}$$

where $\mathbf{j} = \mathbf{j}_a + \mathbf{j}_b$ — density of total electric field, $\mathbf{E}$ — strength of the sweeping electric field. From a microscopic point of view, the electric current density in a two-dimensional system is, by definition, written as:

$$\mathbf{j} = 2e\int \frac{d\mathbf{p}}{(2\pi)^2}\sum_i \mathbf{v}_{\mathbf{p}}^{i} n_{\mathbf{p}}^{i}, \tag{3}$$

where $n_{\mathbf{p}}^{i}$ — nonequilibrium electron distribution function, $\mathbf{v}_{\mathbf{p}}^{i}$ — electron velocity, e — electron charge (here and further in the text we assume $\hbar = k_B = 1$, $\hbar$ and k_B — Planck and Boltzmann constants, respectively). By comparing the expressions (2) and (3) the desired expression for conductivity will be obtained.

As can be seen from expression (3), the problem narrows down to finding nonequilibrium distribution functions $n_{\mathbf{p}}^{i}$ in each zone. Within quasi-classical approximation, these functions satisfy the system of kinetic Boltzmann equations:

$$e\mathbf{E}\frac{\partial n_{\mathbf{p}}^{i}}{\partial \mathbf{p}} = \sum_{j=(a,b)}\left(I_{ij}^{\delta}\{n_{\mathbf{p}}^{i}\} + I_{ij}^{c}\{n_{\mathbf{p}}^{i}\} + Q_{ij}\{n_{\mathbf{p}}^{i}\}\right), \tag{4}$$

where the symbols I_{ij}^{δ} and I_{ij}^{c} denote the collision integrals for electrons having short-range defects and Coulomb impurities, respectively, and Q_{ij} characterize the scattering of electrons on each other. The matrix structure of collision integrals is related to the need to take into account both intraband and interband electron scattering. We'll search how to solve the system of equations (4) by using the method of successive approximations. Within the framework of this approach, it is assumed that nonequilibrium corrections slightly change the Fermi distribution of electrons over states in both zones:

$$n_{\mathbf{p}}^{i} \approx n_{\mathrm{F}}(\xi_{\mathbf{p}}) + f_{\mathbf{p}}^{i} + \delta f_{\mathbf{p}}^{i}, \tag{5}$$

$$\delta f_{\mathbf{p}}^{i} \ll f_{\mathbf{p}}^{i} \ll n_{\mathrm{F}}(\xi_{\mathbf{p}}), \tag{6}$$

where $n_{\mathrm{F}}(\xi_{\mathbf{p}}) = (e^{\xi_{\mathbf{p}}/T} + 1)^{-1}$ — Fermi-Dirac distribution, $\xi_{\mathbf{p}} = \varepsilon_{\mathbf{p}} - \mu$ — electron energy calculated from the Fermi level μ. In this paper, we will limit ourselves to the range of sufficiently low temperatures, in which the processes of electron scattering on impurities and defects in the crystal lattice make the main contribution to the resistivity of the system. Accordingly, in (5) the term $f_{\mathbf{p}}^{i}$ is due to impurity electron scattering, and $\delta f_{\mathbf{p}}^{i} \ll f_{\mathbf{p}}^{i}$ stands for the electron-electron interaction. According to assumption (6), the relations $I\{f\} \gg Q\{f\}$ and $I\{\delta f\} \sim Q\{f\}$ are valid between the collision integrals, which allow solving the system of equations (4) in two stages. At the first stage, the functions $f_{\mathbf{p}}^{i}$ are determined from the equations

$$\frac{e\mathbf{E}\mathbf{p}}{m_i}\frac{\partial n_{\mathrm{F}}(\xi_{\mathbf{p}}^{i})}{\partial \xi_{\mathbf{p}}^{i}} = \sum_{j=(a,b)}\left(I_{ij}^{\delta}\{f_{\mathbf{p}}\} + I_{ij}^{c}\{f_{\mathbf{p}}\}\right). \tag{7}$$

Then the already known nonequilibrium corrections $f_{\mathbf{p}}^{i}$ are substituted into the system of equations:

$$\sum_{j=(a,b)}\left(I_{ij}^{\delta}\{\delta f_{\mathbf{p}}\} + I_{ij}^{c}\{\delta f_{\mathbf{p}}\} + Q_{ij}\{f_{\mathbf{p}}\}\right) = 0, \tag{8}$$

which is solved by functions $\delta f_{\mathbf{p}}^{i}$.

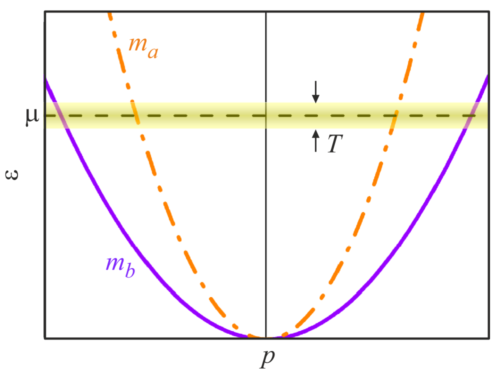

Figure 1. The energy spectrum of a two-dimensional system containing electrons with differing masses m_a (dashed curve) and m_b (solid curve). Horizontal dashed line denotes the Fermi level, μ.

2.1. Impurities contribution to conductivity

In equations (7), the integrals of collisions of electrons with impurities have the standard form:

$$I_{ij}^{\delta,c}\{f_{\mathbf{p}}\} = \frac{N_{\delta,c}}{2\pi}\int d\mathbf{q}|V_{\mathbf{q}}^{\delta,c}|^2\delta(\varepsilon_{\mathbf{p}}^{i} - \varepsilon_{\mathbf{q}+\mathbf{p}}^{j})(f_{\mathbf{q}+\mathbf{p}}^{j} - f_{\mathbf{p}}^{i}), \tag{9}$$

where N_δ and N_c — concentrations of the short-time defects and Coulomb impurities, $V_{\mathbf{q}}^{\delta} = V^{\delta} = \mathrm{const}$ and

$V_{\mathbf{q}}^c = 2\pi e^2/\epsilon q$ — Fourier images of the corresponding potentials, ϵ — permittivity of environment.

We'll search for a solution to equation (7) as follows:

$$f_{\mathbf{p}}^i = -\frac{e\mathbf{E}\mathbf{p}}{m_i}\frac{\partial n_{\mathrm{F}}(\xi_{\mathbf{p}}^i)}{\partial \xi_{\mathbf{p}}^i}\phi^i(p), \tag{10}$$

where the unknown functions $\phi^i(p)$ are assumed to be independent of the orientation of the momentum vector $\mathbf{p}$. Substituting (10) into (9) and performing momentum integration in (9) $\mathbf{q}$, we obtain

$$\sum_j I_{ij}^\delta\{f_{\mathbf{p}}\} = \frac{e\mathbf{E}\mathbf{p}}{m_i}\frac{\partial n_{\mathrm{F}}}{\partial \xi_{\mathbf{p}}^i}\frac{\phi^i(p)}{\tau}, \tag{11}$$

$$I_{ii}^c\{f_{\mathbf{p}}\} = \frac{e\mathbf{E}\mathbf{p}}{m_i}\frac{\partial n_{\mathrm{F}}}{\partial \xi_{\mathbf{p}}^i}\frac{\phi^i(p)}{\tau_{\varepsilon_{\mathbf{p}}^i}}, \tag{12}$$

$$I_{ab}^c\{f_{\mathbf{p}}\} = \frac{e\mathbf{E}\mathbf{p}}{m_a}\frac{\partial n_{\mathrm{F}}}{\partial \xi_{\mathbf{p}}^a}\frac{2\tilde{m}}{\tau_{\varepsilon_{\mathbf{p}}^a}}\left[\frac{\phi^a(p)}{m_a} - \frac{\phi^b\left(\sqrt{m_b/m_a}\,p\right)}{m_b}\right], \tag{13}$$

$$I_{ba}^c\{f_{\mathbf{p}}\} = \frac{e\mathbf{E}\mathbf{p}}{m_b}\frac{\partial n_{\mathrm{F}}}{\partial \xi_{\mathbf{p}}^b}\frac{2\tilde{m}}{m_b\tau_{\varepsilon_{\mathbf{p}}^a}}\left[\phi^b(p) - \phi^a\left(\sqrt{m_a/m_b}\,p\right)\right], \tag{14}$$

where $1/\tilde{m} = 1/m_a - 1/m_b$, $1/\tau = N_\delta V_\delta^2(m_a + m_b)$ — time of electrons relaxation on the short-time defects, $1/\tau_{\varepsilon_{\mathbf{p}}^a} = (\pi e^2/\epsilon)^2 N_c/\varepsilon_{\mathbf{p}}^a$ — electrons relaxation time on the Coulomb impurities. Using further in the equations (7) explicit expressions for impurity collision integrals (11)−(14), we arrive at an algebraic system of equations for the function ϕ:

$$\hat{M}\begin{pmatrix}\phi^a(p)\\ \phi^b(p')\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}, \tag{15}$$

$$\hat{M} = \begin{pmatrix} \dfrac{1}{\tau} + \dfrac{1}{\tau_{\varepsilon_{\mathbf{p}}^a}}\dfrac{3m_b - m_a}{m_b - m_a} & -\dfrac{1}{\tau_{\varepsilon_{\mathbf{p}}^a}}\dfrac{2m_a}{m_b - m_a} \\ -\dfrac{1}{\tau_{\varepsilon_{\mathbf{p}}^a}}\dfrac{2m_a}{m_b - m_a} & \dfrac{1}{\tau} + \dfrac{1}{\tau_{\varepsilon_{\mathbf{p}}^a}}\dfrac{m_b + m_a}{m_b - m_a} \end{pmatrix}, \tag{16}$$

where $p' = \sqrt{m_b/m_a}\,p$. Under low temperature conditions, $T \ll \mu$, the electron gas is degenerate, which makes it possible, neglecting the terms $\sim T/\mu \ll 1$, to consider the relaxation time on Coulomb impurities as a function of the Fermi energy, $\tau_{\varepsilon_{\mathbf{p}}^a} \approx \tau_\mu$. Let us introduce dimensionless variables for brevity: $\tilde{\phi} = \phi/\tau$, $x = m_a/m_b$ and $y = \tau_\mu/\tau$. The system of equations (15) is then written as follows:

$$\tilde{\phi}^{a,b}(x,y) = \frac{y(1-x)[(y \mp 1)(1-x) + 2(1+x)]}{[(y+1)(1-x)+2][(y-1)(1-x)+2] - 4x^2}, \tag{17}$$

where signs „–“ and „+“ in the numerator relate to a and b, respectively. Putting together expressions (3), (10), and (17), we write down the answer for the impurity contribution to the conductivity of a two-component system:

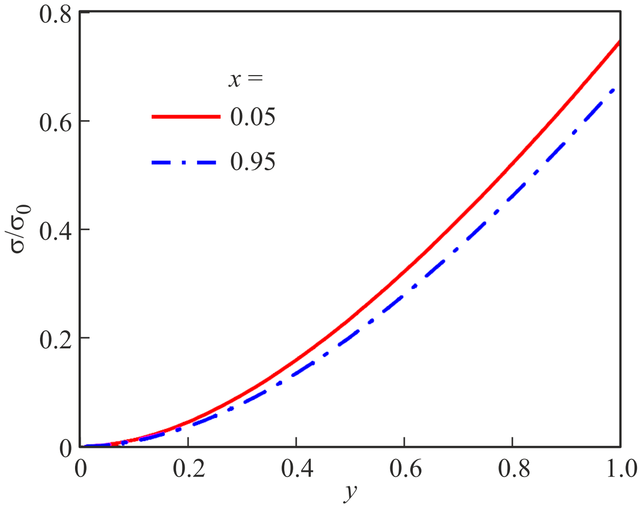

Figure 2. The dependence of the impurity contribution to the conductivity of a two-component system (18) on the Fermi energy, $y = \mu/\mu_0$, for two different ratios of effective electron masses, $x = m_a/m_b$.

$$\sigma = \sigma_0\frac{\mu}{\mu_0}\left[\tilde{\phi}^a\left(\frac{m_a}{m_b}, \frac{\mu}{\mu_0}\right) + \tilde{\phi}^b\left(\frac{m_a}{m_b}, \frac{\mu}{\mu_0}\right)\right], \tag{18}$$

where $\sigma_0 = e^2\tau\mu_0/\pi$, and μ_0 — is the Fermi energy level at which the times of electrons scattering on the two types of impurity centers, τ and τ_μ, coincide, i.e. $\mu = y\mu_0$.

In expression (18), we are primarily interested in the dependence of conductivity on the Fermi energy, since it is this parameter that can be controlled experimentally (Figure 2). From the analysis of expression (18) in extreme cases, it follows that from the standpoint of response to an electric field, a two-component two-dimensional electron gas practically does not differ from the systems with a 2D electron gas of the same mass. Indeed, the conductivities of both one- and two-component electron gases (18) behave in a similar way depending on the position of the Fermi level: in the region of $\mu \ll \mu_0$, Coulomb impurities and $\sigma \propto \mu^2$ play the main role; in the opposite limiting case, $\mu \gg \mu_0$, the short-time disorder prevails and $\sigma \propto \mu$. As can be seen from Figure 2, the ratio of the effective masses of electrons in the two zones has a weak effect on the amount of impurity contribution to the conductivity of the system. At a fixed value of $m_a + m_b$, the maximum difference is achieved when scattering processes at Coulomb centers, $1/\tau_\mu \gg 1/\tau$, dominate, and does not exceed one-third: $\sigma(m_a \ll m_b)/\sigma(m_a = m_b) < 4/3$. In the opposite extreme case, $1/\tau_\mu \ll 1/\tau$, the difference in the effective masses of the electrons does not affect the magnitude of the conductivity at all: $\sigma(m_a \ll m_b) = \sigma(m_a = m_b)$. However, we emphasize that by omitting the zone index for the matrix elements $V_{\mathbf{q}}^{\delta,c}$, we limit all the differences between the two types of electrons only to the envelope wave functions describing free motion in the plane. In general case, of course, the other components of the electron wave functions do not have to match, and taking these additional differences into account would lead to different relaxation rates of the two types of electrons, even in short-range disorder. For example, in paper [17], a system is studied that includes two separated parabolic subbands for electrons with the same effective mass. In this paper, it was shown that electron-electron scattering indirectly manifests itself by redistributing electrons between zones, but only when the rates of electron relaxation in each subband are different. In this paper, we are primarily interested in the role of differences in the effective masses of electrons, and the introduction of additional relaxation times on impurities would significantly complicate the study of the properties of electron relaxation due to the processes of their collisions with each other.

2.2. Contribution of the intra-electron scattering into conductivity

Let's calculate the contribution of the electron-electron interaction into conductivity of the two-component system. Using expressions for impurity collision integrals (11)−(14) and short symbols $x = m_a/m_b$ and $y = \tau_\mu/\tau$, we write a formal solution of the system of equations (8):

$$\begin{pmatrix} \delta f^a_{\mathbf{p}} \\ \delta f^b_{\mathbf{p}'} \end{pmatrix} = \tau y(1-x)\breve{M}^{-1} \sum_j \begin{pmatrix} Q_{aj}\{f_{\mathbf{p}}\} \\ Q_{bj}\{f_{\mathbf{p}'}\} \end{pmatrix} \tag{19}$$

$$\breve{M} = \begin{pmatrix} y(1-x)+3-x & -2x^{1/2} \\ -2x^{3/2} & y(1-x)+1+x \end{pmatrix} \tag{20}$$

In the right-hand side of expression (19), the integral of the electron-electron collisions, linearized over the nonequilibrium addition to the electron distribution function, is expressed as follows:

$$\begin{aligned} Q_{ij}\{f_{\mathbf{p}}\} = &-\frac{2}{(2\pi)^3 T}\int d\mathbf{k}d\mathbf{q}|V^c_{\mathbf{k}}|^2 \int\limits_{-\infty}^{\infty} d\omega\, \delta(\varepsilon^i_{\mathbf{p}} - \varepsilon^i_{\mathbf{p-k}} - \omega) \\ &\times \delta(\varepsilon^j_{\mathbf{q}} - \varepsilon^j_{\mathbf{q+k}} + \omega) n_{\mathrm{F}}(\xi^i_{\mathbf{p}})[1 - n_{\mathrm{F}}(\xi^i_{\mathbf{p-k}})] n_{\mathrm{F}}(\xi^j_{\mathbf{q}}) \\ &\times [1 - n_{\mathrm{F}}(\xi^j_{\mathbf{q+k}})] e\mathbf{E}\left\{ \frac{\mathbf{p}}{m_i}\phi_i(p) + \frac{\mathbf{q}}{m_j}\phi_j(q) \right. \\ &\left. - \frac{\mathbf{p-k}}{m_i}\phi_i(|\mathbf{p-k}|) - \frac{\mathbf{q+k}}{m_j}\phi_j(|\mathbf{q+k}|) \right\}. \end{aligned} \tag{21}$$

The Fourier image of the potential energy of the Coulomb repulsion of electrons, $V^c_{\mathbf{k}} = 2\pi e^2/\epsilon k$, is used as the matrix element of the interaction of electrons with each other. Using the filtering property of the delta functions, we integrate over the angular variables $\varphi_{\mathbf{k}}$ and $\varphi_{\mathbf{q}}$:

$$\begin{aligned} Q_{aa}\{f_{\mathbf{p}}\} = &-\frac{eEp\cos\varphi}{\pi^3 T m_a}\int\limits_0^\infty kdkqdq|V^c_k|^2 \\ &\times \int\limits_{-\infty}^{\infty} d\omega\, n_{\mathrm{F}}(\xi^a_{\mathbf{p}})[1 - n_{\mathrm{F}}(\xi^a_{\mathbf{p}} - \omega)] n_{\mathrm{F}}(\xi^a_{\mathbf{q}})[1 - n_{\mathrm{F}}(\xi^a_{\mathbf{q}} + \omega)] \\ &\times \left\{ \phi_a(\varepsilon^a_{\mathbf{p}}) - \phi_a(\varepsilon^a_{\mathbf{p}} - \omega) + \frac{\varepsilon^{a,2}_{\mathbf{k}} - \omega^2}{4\varepsilon^a_{\mathbf{p}}\varepsilon^a_{\mathbf{k}}}[\phi_a(\varepsilon^a_{\mathbf{q}}) - \phi_a(\varepsilon^a_{\mathbf{q}} + \omega)] \right. \\ &\left. + \frac{\varepsilon^a_{\mathbf{k}} + \omega}{2\varepsilon^a_{\mathbf{p}}}[\phi_a(\varepsilon^a_{\mathbf{p}} - \omega) - \phi_a(\varepsilon^a_{\mathbf{q}} + \omega)] \right\} \\ &\times \frac{\Theta[4\varepsilon^a_{\mathbf{p}}\varepsilon^a_{\mathbf{k}} - (\varepsilon^a_{\mathbf{k}} + \omega)^2]\Theta[4\varepsilon^a_{\mathbf{q}}\varepsilon^a_{\mathbf{k}} - (\varepsilon^a_{\mathbf{k}} - \omega)^2]}{\sqrt{4\varepsilon^a_{\mathbf{p}}\varepsilon^a_{\mathbf{k}} - (\varepsilon^a_{\mathbf{k}} + \omega)^2}\sqrt{4\varepsilon^a_{\mathbf{q}}\varepsilon^a_{\mathbf{k}} - (\varepsilon^a_{\mathbf{k}} - \omega)^2}}, \end{aligned} \tag{22}$$

$$\begin{aligned} Q_{ab}\{f_{\mathbf{p}}\} = &-\frac{eEp\cos\varphi}{\pi^3 T}\int\limits_0^\infty kdkqdq|V^c_k|^2 \\ &\times \int\limits_{-\infty}^{\infty} d\omega n_{\mathrm{F}}(\xi^a_{\mathbf{p}})[1 - n_{\mathrm{F}}(\xi^a_{\mathbf{p}} - \omega)] n_{\mathrm{F}}(\xi^b_{\mathbf{q}})[1 - n_{\mathrm{F}}(\xi^b_{\mathbf{q}} + \omega)] \\ &\times \left\{ \frac{\phi_a(\varepsilon^a_{\mathbf{p}}) - \phi_a(\varepsilon^a_{\mathbf{p}} - \omega)}{m_a} + \frac{(\varepsilon^a_{\mathbf{k}} + \omega)(\varepsilon^b_{\mathbf{k}} - \omega)}{4m_b\varepsilon^a_{\mathbf{p}}\varepsilon^b_{\mathbf{k}}} \right. \\ &\times [\phi_b(\varepsilon^b_{\mathbf{q}}) - \phi_b(\varepsilon^b_{\mathbf{q}} + \omega)] \\ &\left. + \frac{\varepsilon^a_{\mathbf{k}} + \omega}{2\varepsilon^a_{\mathbf{p}}}\left[\frac{\phi_a(\varepsilon^a_{\mathbf{p}} - \omega)}{m_a} - \frac{\phi_b(\varepsilon^b_{\mathbf{q}} + \omega)}{m_b} \right] \right\} \\ &\times \frac{\Theta[4\varepsilon^a_{\mathbf{p}}\varepsilon^a_{\mathbf{k}} - (\varepsilon^a_{\mathbf{k}} + \omega)^2]\Theta[4\varepsilon^b_{\mathbf{q}}\varepsilon^b_{\mathbf{k}} - (\varepsilon^b_{\mathbf{k}} - \omega)^2]}{\sqrt{4\varepsilon^a_{\mathbf{p}}\varepsilon^a_{\mathbf{k}} - (\varepsilon^a_{\mathbf{k}} + \omega)^2}\sqrt{4\varepsilon^b_{\mathbf{q}}\varepsilon^b_{\mathbf{k}} - (\varepsilon^b_{\mathbf{k}} - \omega)^2}}, \end{aligned} \tag{23}$$

where the angle φ sets the direction of the vector $\mathbf{p}$, the electric field strength vector is assumed to be directed along the axis x, $\mathbf{E} = (E, 0, 0)$. The expressions for the collision integrals Q_{ba} and Q_{bb} have the form similar to formulas (22) and (23).

An exact analytical calculation of the remaining integrals is not possible, so it is necessary to make a number of simplifications in the sub-integral expressions. First of all, we note that the exponential behavior of the Fermi distribution functions ensures convergence of the integral with respect to the integration variable ω at the upper and lower limits. In this case, the main part of this integral is typed in the range of values of the order of temperature, $\omega \sim T \ll \mu$. This allows us to decompose the functions enclosed in curly brackets by the small ratio $\omega/\mu \ll 1$. In order to avoid exceeding the accuracy and to take into account all contributions of the same order of magnitude, the decomposition of the functions ϕ shall be performed up to the third term:

$$\phi(\mu \pm \omega) \approx \phi(\mu) \pm \omega\phi'(\mu) + \frac{\omega^2}{2}\phi''(\mu), \tag{24}$$

where we assume the initial energies of the electrons in the subbands to be equal to the Fermi energy, $\varepsilon^a_{\mathbf{p}}, \varepsilon^b_{\mathbf{q}} \approx \mu$. The radicals in the denominators of expressions (22) and (23) should also be simplified. Given that the zeros of the denominator, as functions of $\varepsilon^a_{\mathbf{k}}$, are separated in both integrals by a gap of the order of 4μ, we obtain (23) for the intra-subband electron scattering:

$$\frac{1}{\sqrt{(4\mu\varepsilon^a_{\mathbf{k}} - (\varepsilon^a_{\mathbf{k}} + \omega)^2)(4\mu\varepsilon^b_{\mathbf{k}} - (\varepsilon^b_{\mathbf{k}} - \omega)^2)}} \approx \frac{1}{4\mu\sqrt{x}}\left\{\frac{1}{\sqrt{x[4\mu - \varepsilon^a_{\mathbf{k}}][4\mu/x - \varepsilon^a_{\mathbf{k}}]}} + \frac{1}{\sqrt{[\varepsilon^a_{\mathbf{k}} - \omega^2/(4\mu)][\varepsilon^a_{\mathbf{k}} - \omega^2/(4\mu x)]}}\right\}. \tag{25}$$

For the intra-subband collision integral Q_{aa}, similar actions give

$$\frac{1}{\sqrt{(4\mu\varepsilon^a_{\mathbf{k}} - (\varepsilon^a_{\mathbf{k}} + \omega)^2)(4\mu\varepsilon^a_{\mathbf{k}} - (\varepsilon^a_{\mathbf{k}} - \omega)^2)}} \approx \frac{1}{4\mu}\left\{\frac{1}{\sqrt{[4\mu a_- - \varepsilon^a_{\mathbf{k}}][4\mu a_+ - \varepsilon^a_{\mathbf{k}}]}} + \frac{1}{\sqrt{[\varepsilon^a_{\mathbf{k}} - \omega^2 a_+/(4\mu)][\varepsilon^a_{\mathbf{k}} - \omega^2 a_-/(4\mu)]}}\right\}, \tag{26}$$

where $a_\pm = 1 \pm \omega/(2\mu)$.

The accepted assumptions are sufficient for the analytical taking of integrals over variables k and q. The result of integration, however, boils down to rather cumbersome expressions, so we present the answer in the following form:

$$Q_{ij}(\mathbf{p}) = \frac{eEp\cos\varphi}{2\pi^3 T}\frac{K_{ij}}{N_c\mu} \times \int_{-\infty}^{\infty} d\omega n_{\mathrm{F}}(\xi^i_{\mathbf{p}})[1 - n_{\mathrm{F}}(\xi^i_{\mathbf{p}} - \omega)]\omega e^{\omega/T} n_{\mathrm{B}}(\omega), \tag{27}$$

where $n_{\mathrm{B}}(\omega) = (e^{\omega/T} - 1)^{-1}$ — Bose-Einstein distribution, and K_{ij} functions are given in (A1), (A2), (A3). Note that in expression (27), for brevity, we omitted contributions with even powers ω before the exponent, since in the subsequent calculation of the conductivity of the system, these contributions will turn out to be odd functions of energy ω and, thus, will accumulate during integration.

After substituting the nonequilibrium additions (19) and the integrals of the electron-electron collisions (27) into the expression of current density (3) and subsequent pulse integration $\mathbf{p}$, we arrive at the expression to correct for conductivity of the system due to the electron-electron interaction:

$$\delta\sigma = \frac{\sigma_0\pi e^4}{3\epsilon^2 N_\delta V_\delta^2}\left(\frac{T}{\mu_0}\right)^2 \frac{m_a}{m_a + m_b} F\left(\frac{m_a}{m_b}, \frac{\mu}{\mu_0}, \frac{T}{\mu_0}\right), \tag{28}$$

where the dimensionless function F is given in Appendix (see expression (A4)). Despite the fact that the integration was carried out completely in the accepted approximations, the final expressions, (28) and (A4), are very difficult to perceive, and we will provide a qualitative description of the results obtained.

3. Discussion of the results and conclusion

So, in contrast to the impurity contribution to the conductivity, the correction to the conductivity from the electron-electron interaction turned out to be extremely sensitive to the ratio of the effective masses of electrons in the two bands (Figure 3). In addition to the non-monotonic dependence on the chemical potential, this contribution demonstrates the possibility of taking both positive and negative values. To formulate a physical interpretation of this behavior of conduction (28), let us return to the definition of the collision integral (21). In this expression, the terms in the curly bracket have length dimension, in particular, $\mathbf{l}^i_{\mathbf{p}} = \mathbf{p}\phi_i/m_i$ — mean free path of ith electron with momentum $\mathbf{p}$, and the entire bracket

$$\Delta\mathbf{l}^i_{\mathbf{p}} = \mathbf{v}^i_{\mathbf{p}}\phi_i(p) + \mathbf{v}^j_{\mathbf{q}}\phi_j(q) - \mathbf{v}^i_{\mathbf{p}-\mathbf{k}}\phi_i(|\mathbf{p}-\mathbf{k}|) - \mathbf{v}^j_{\mathbf{q}+\mathbf{k}}\phi_j(|\mathbf{q}+\mathbf{k}|) \tag{29}$$

may be interpreted as a free path variation due to the electron-electron collisions. The vector $\Delta\mathbf{l}^i_{\mathbf{p}}$ can be directed both along and against the momentum vector $\mathbf{p}$, and it is this mutual orientation of vectors $\mathbf{p}$ and $\Delta\mathbf{l}^i_{\mathbf{p}}$ that determines the sign of the corresponding nonequilibrium correction to the distribution function.

Let's consider the limiting cases. Let's assume that Coulomb impurities ($\mu \ll \mu_0$) are dominant in the system and there's only one type of electrons, then $\phi(\mu) = \tau_\mu \propto \mu$ and in the curly bracket of expression (22) each difference of functions ϕ will provide a multiplier $\propto \omega$. As further analysis shows, when calculating the conductivity of all the terms in the curly bracket, we should leave only the even ones in the variable ω. Only the last term meets this condition, which, however, changes the general sign of the collision integral (22) from $(-)$ to $(+)$, and the corresponding nonequilibrium addition to the distribution function $\delta f_{\mathbf{p}}$ increases the conductivity of the system. In this case, the positive correction to conductivity should be interpreted as a consequence of the general increase in the velocity of electrons in inelastic collisions with each other. This is due to the fact that the relaxation of electrons on Coulomb impurities in states with higher kinetic energy

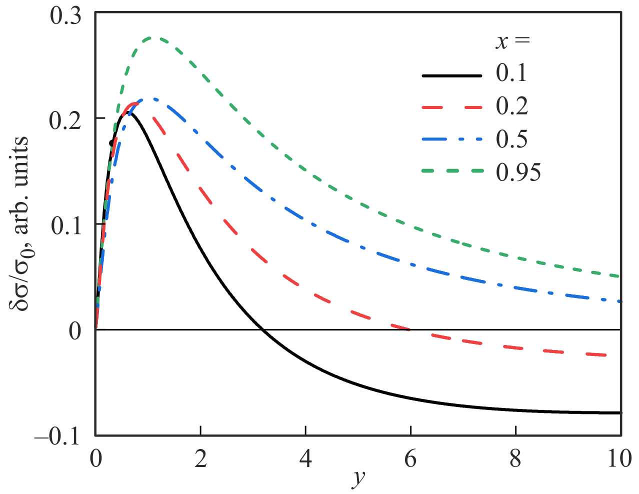

Figure 3. Contribution of the electron-electron collisions into conductivity of the two-component system (28) versus Fermi energy, $y = \mu/\mu_0$, at a temperature of $T/\mu_0 = 1/100$ and various ratios of the effective electron masses $x = m_a/m_b$.

takes longer. If there are two types of electrons in the system, these arguments do not lose their validity, since in the absence of short-range defects, the dependence of the functions $\phi_i(\mu) \propto \tau_\mu \propto \mu$ on energy remains linear.

In the opposite limiting situation, when there is only short-range disorder in the system ($\mu \gg \mu_0$), the relaxation time of electrons does not depend on their energy, $\phi_i(\mu) \propto \tau = \text{const}$, and the integral of intraband electron-electron collisions (22) identically vanishes. In the language of classical physics, the scattering of electrons on each other, being a manifestation of internal forces, does not change the total momentum of the electron gas, which for systems with parabolic energy dispersion of electrons means keeping the drift velocity. Thus, the conductivity is not affected by the electron-electron interaction. However, these considerations are not applicable to interband electron scattering. As can be seen from (29), a pair of electrons from different zones receive different velocity increments after momentum exchange $\mathbf{k}$, and, consequently, $\triangle \mathbf{l}^i_{\mathbf{p}} \propto \mathbf{v}^i_{\mathbf{k}} - \mathbf{v}^j_{\mathbf{k}} \neq 0$ even at $\phi_i = \phi_j = \text{const}$. The calculation showed that this type of relaxation has a retarding effect on the mixture of two electron gases and the corresponding correction to conductivity is negative. A necessary condition for the implementation of the interband relaxation is the difference in effective electron masses, therefore, in the absence of Coulomb impurities ($y \gg 1$) and in the limit $m_a \to m_b$, this contribution disappears, which is confirmed by the corresponding limiting expression for the function (B1):

$$F \propto \ln(1-x)\,\frac{(1-x)^2}{x^{3/2}} \xrightarrow[x\to 1]{} 0, \qquad (30)$$

where, it should be mentioned, $x = m_a/m_b$.

The cases described above qualitatively explain the course of the curves in Figure 3. If the difference in effective electron masses is small (Figure 3, green curve, short strokes), then the negative correction to conductivity from interband scattering is suppressed and the electron-electron interaction generally contributes to higher conductivity. At the same time, with the growth of y, that is, with the transition to the prevailing short-range disorder, the correction gradually tends to zero. It should be stressed that tendency of $\delta\sigma$ to zero in the limit $y = \tau_\mu/\tau \to 0$ is explained by the absence of the charge carriers at $\mu = 0$. However, it should be borne in mind that the field of applicability of the developed theory limits the permissible values of y from below: $y \gg T/\mu_0$.

The large difference in the effective masses of electrons enhances the role of interband electron scattering. Since in the considered conditions, intra- and interband collisions of electrons work in the opposite way, there is such a value of the Fermi energy μ_c, in which both mechanisms completely compensate each other. For example, as follows from Figure 3 (black solid curve), with a tenfold difference in the masses of electrons, $m_a/m_b = 1/10$, compensation occurs with an approximately three-fold difference in relaxation times on short-range disorder and Coulomb impurities, $\tau_\mu \approx 3\tau$.

Let's consider further the description of the temperature dependence of the effect. Systems with significant differences in the effective masses of electrons are of the greatest interest, therefore, let us take $m_a/m_b = 1/10$ as an example (Figure 4). As can be seen in the graph, the temperature change affects only the vertical scale of the curves, while the characteristic zero point of the contribution of electron-electron collisions y_c remains stationary on y axis. This is associated with the fact that in $x \ll 1$ limit in equation $F(x, y_c, T/\mu_0) = C_1(x, y_c) + C_2(x, y_c)\ln T = 0$ defining the value μ_c, the term with the temperature logarithm prevails. Thus, the asymptotics of correction for conductivity $\delta\sigma \propto T^2 \ln T$ and all curves in Figure 4 cross the x-axis approximately in one point y_c.

Experimental detection of the contribution of electron-electron interaction to conductivity can be carried out by measuring the temperature dependence of the effect. Figure 5 demonstrates that at different positions of the Fermi level, heating of the system can both increase and decrease the conductivity of a two-component electron gas. It should be remembered that the developed theory is valid only in the low temperature range, therefore, regardless of the position of the Fermi level, starting from a certain temperature value, the resistivity of the electron gas will grow. We emphasize that the described nontrivial properties of a mixture of two electron gases do not have any exotic grounds. In particular, the studied system was not assumed to be strongly correlated or to have features in the energy spectrum.

In conclusion, we will discuss the role of the shielding of the Coulomb potential by an electron gas in the considered system. As mentioned at the beginning of the article, the new theory uses the naked Coulomb potential of the electron-electron interaction to unambiguously account for

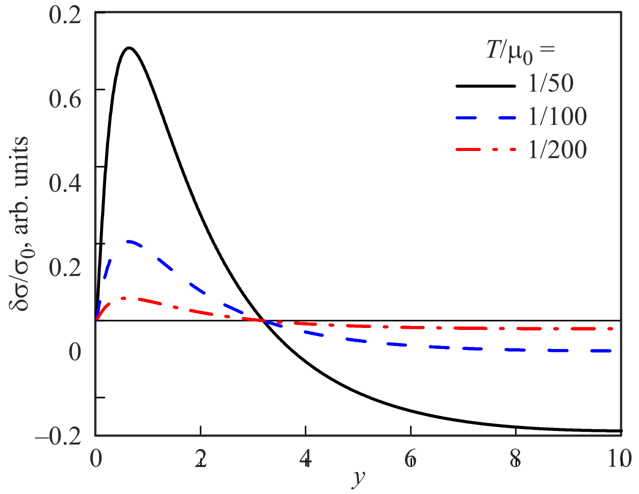

Figure 4. Contribution of electron-electron collisions to conductivity of the two-component system (28) versus Fermi energy, $y = \mu/\mu_0$, at the ratio of effective electron masses $x = m_a/m_b = 1/10$ and various temperatures.

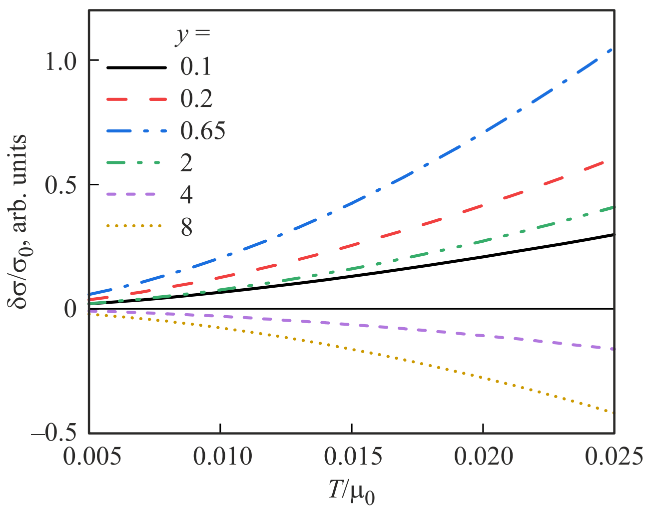

Figure 5. Contribution of electron-electron collisions to conductivity of the two-component system (28) versus temperature at the ratio of effective electron masses $x = m_a/m_b = 1/10$ and various positions of the Fermi level, $y = \mu/\mu_0$.

the presence of Coulomb impurity centers in the system and thus make the relaxation time of electrons dependent on their energy. The shielding of the Coulomb potential by an electron gas will, on the one hand, effectively weaken the dependence of the impurity electron relaxation time on energy by reducing the effective range of the Coulomb centers (which is equivalent to a shift towards large y in the graphs Figure 3 and Figure 4), and on the other hand, it will change the temperature dependence of the effect. Let us consider as an example the limiting case of strong shielding, when all defects of the crystal lattice and impurity centers are somewhat short-time acting. In this situation, the potential of electron-electron interaction in the framework of the random phase assumption (RPA) in a two-component system is described by the permittivity as follows:

$$\epsilon^{ee}_{\mathbf{k}} = (1 - V^{aa}_{\mathbf{k}}\Pi^a_{\mathbf{k}})(1 - V^{bb}_{\mathbf{k}}\Pi^b_{\mathbf{k}}) - (V^{ab}_{\mathbf{k}})^2\Pi^a_{\mathbf{k}}\Pi^b_{\mathbf{k}}, \qquad (31)$$

where the functions $\Pi^{a,b}_{\mathbf{k}}$ characterize the polarizability of electron gases in the corresponding energy bands and have a simple form in the quasi-ballistic mode of electron transport: $\Pi^{a,b}_{\mathbf{k}} \approx -m_{a,b}/\pi$. Both types of electrons have the same electric charge and lie in the same layer, so the naked Coulomb potentials are equal, $V^{aa}_{\mathbf{k}} = V^{bb}_{\mathbf{k}} = V^{ab}_{\mathbf{k}} = 2\pi e^2/\epsilon k$, and in the long-wavelength limit, the electrons begin interact with each other in a contact way:

$$\frac{V^c_{\mathbf{k}}}{\epsilon^{ee}_{\mathbf{k}}} \to \frac{\pi}{m_a + m_b}, \qquad (32)$$

When using in the interband electron-electron collision integrals the solution of equations (7) with only point impurity centers, $\phi_a = \phi_b = \tau_i$, and shielded Coulomb potential (32), we'll get the expression:

$$\delta\sigma = -\sigma_D \frac{\pi\tau_i T^2}{3\mu}\left(\frac{1-x}{1+x}\right)^2 \times \left[\frac{1-x}{\sqrt{x}} + \frac{1+x}{2x}\ln\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)\right], \qquad (33)$$

where $\sigma_D = e^2\tau_i\mu/\pi$ — Drude conductivity. The correction to conductivity (33), as expected, tends to zero when the Galilean invariance of the system is restored, that is, at $x \to 1$. In addition, the expression (33) is strictly negative, which means only an increase in the resistivity of the system with the rising temperature under conditions of strong shielding of the Coulomb interaction. Formal root divergence of correction (33) in the limit $m_a \ll m_b$ has no any physical sense and implies going beyond the applicability of the obtained expressions: $m_a/m_b \gg T/\mu$.

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Conflict of interest

The authors declare that they have no conflict of interest.

Appendix I. Functions K_{ij}

$$K_{ii} = 4\ln\left(\frac{2\mu}{T}\right)\frac{\partial\tilde{\phi}_i}{\partial y} + 2(1-\ln 2)y\,\frac{\partial^2\tilde{\phi}_i}{\partial y^2}, \tag{A1}$$

$$K_{ab} = \frac{1}{x}\left\{\operatorname{arcth}\sqrt{x}\left[4\frac{\partial}{\partial y}\left(y\frac{\partial\tilde{\phi}_a}{\partial y}\right) + 2(3x-1)\frac{\partial\tilde{\phi}_b}{\partial y} - \frac{\tilde{\phi}_a - x\tilde{\phi}_b}{y} - 2y(1+x)\frac{\partial^2\tilde{\phi}_b}{\partial y^2}\right]\right.$$
$$\left. - \frac{\tilde{\phi}_a - x\tilde{\phi}_b}{2y\sqrt{x}}\ln\left[\left(\frac{4\mu}{T}\right)^2\frac{4x}{1-x}\right] + 2\sqrt{x}y\,\frac{\partial^2\tilde{\phi}_b}{\partial y^2}\right\}, \tag{A2}$$

$$K_{ba} = x\left\{\operatorname{arcth}\sqrt{x}\left[4\frac{\partial}{\partial y}\left(y\frac{\partial\tilde{\phi}_b}{\partial y}\right) + 2\left(\frac{3}{x}-1\right)\frac{\partial\tilde{\phi}_a}{\partial y} - \frac{\tilde{\phi}_b - \tilde{\phi}_a/x}{y} - 2y\left(1+\frac{1}{x}\right)\frac{\partial^2\tilde{\phi}_b}{\partial y^2}\right]\right.$$
$$\left. - \sqrt{x}\,\frac{\tilde{\phi}_b - \tilde{\phi}_a/x}{2y}\ln\left[\left(\frac{4\mu}{T}\right)^2\frac{4x}{1-x}\right] + \frac{2y}{\sqrt{x}}\frac{\partial^2\tilde{\phi}_a}{\partial y^2}\right\}. \tag{A3}$$

Appendix II. Function F

$$F(x,y,T) = 4\ln\left(\frac{2\mu}{T}\right)\left(\tilde{\phi}_a\frac{\partial\tilde{\phi}_a}{\partial y} + \frac{1}{x}\tilde{\phi}_b\frac{\partial\tilde{\phi}_b}{\partial y}\right) + 2(1-\ln 2)y\left(\tilde{\phi}_a\frac{\partial^2\tilde{\phi}_a}{\partial y^2} + \frac{1}{x}\tilde{\phi}_b\frac{\partial^2\tilde{\phi}_b}{\partial y^2}\right)$$
$$+ \frac{1}{x}\operatorname{arcth}\sqrt{x}\left\{4\tilde{\phi}_a\frac{\partial}{\partial y}\left(y\frac{\partial\tilde{\phi}_a}{\partial y}\right) + 4x\tilde{\phi}_b\frac{\partial}{\partial y}\left(y\frac{\partial\tilde{\phi}_b}{\partial y}\right) + 2(3x-1)\tilde{\phi}_a\frac{\partial\tilde{\phi}_b}{\partial y} + 2(3-x)\tilde{\phi}_b\frac{\partial\tilde{\phi}_a}{\partial y}\right.$$
$$\left. - \frac{(\tilde{\phi}_a - x\tilde{\phi}_b)(\tilde{\phi}_a - \tilde{\phi}_b)}{y} - 2y(1+x)\left(\frac{\tilde{\phi}_a\partial^2\tilde{\phi}_b}{\partial y^2} + \frac{\tilde{\phi}_b\partial^2\tilde{\phi}_a}{\partial y^2}\right)\right\}$$
$$- \ln\left[\left(\frac{4\mu}{T}\right)^2\frac{4x}{1-x}\right]\frac{(\tilde{\phi}_a - x\tilde{\phi}_b)^2}{2yx^{3/2}} + \frac{2y}{\sqrt{x}}\left(\tilde{\phi}_a\frac{\partial^2\tilde{\phi}_b}{\partial y^2} + \tilde{\phi}_b\frac{\partial^2\tilde{\phi}_a}{\partial y^2}\right) \tag{A4}$$

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