

Charge neutrality in semiconductor lasers

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A short theoretical review of the problem of charge neutrality in semiconductor lasers is presented. It is shown that the global charge neutrality holds in laser structures at any pump currents both at steady-state and non-stationary conditions. In the context of injection lasers with a low-dimensional active region, the charge neutrality condition reads as equality of the total electron concentration in a bulk waveguide region and a low-dimensional active region to the total hole concentration in those regions. It is shown that, due to the fact that at high injection currents each of the electron and hole concentrations in the waveguide region is significantly higher than each of these concentrations in the active region, the global charge neutrality condition in the laser structure at such currents effectively reduces to the charge neutrality condition in the bulk waveguide region, and the local charge neutrality in the low-dimensional active region does not have to hold.

Keywords: injection lasers, semiconductor heterostructures, low-dimensional active region, quantum well lasers, quantum dot lasers, waveguide region, charge neutrality.

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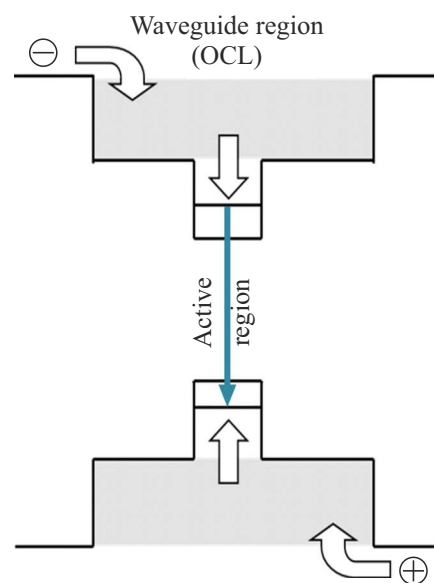
1. Introduction

The idea of using semiconductor lasers based on double heterostructures was put forward more than sixty years ago [1–4]. The advancement of new technological methods for creating semiconductor laser heterostructures began in the 70–80s of the 20th century: liquid phase epitaxy (LPE) [5], metalorganic chemical vapor deposition (MOCVD) [6] and molecular beam epitaxy (MBE) [7] have been developed. The use of these methods made it possible to create lasers on double heterostructures with a separate confinement, and then lasers with a low-dimensional active region [8–10]. The band energy diagram of a laser with a low-dimensional active region is schematically shown in Figure [11]. The idea of using a low-dimensional active region in semiconductor lasers was proposed in Ref. [12]. Currently, one or more quantum wells (QW) or one or more layers with quantum dots are used as a low-dimensional active region [8–11,13–26].

Lasers with a low-dimensional active region demonstrate improved performance compared to lasers with a bulk active region [8,9]. In particular, the threshold current is noticeably reduced in lasers with a low-dimensional active region and the optical output power is increased.

This paper provides a brief theoretical overview of the issue of charge neutrality in semiconductor lasers with a two-dimensional active region (quantum well lasers). The significance of this work is due, in particular, to the fact that, in articles (see, for example, [27,28]), as well as in monographs [29,30] devoted to the theoretical consideration of semiconductor lasers with nanoscale active

regions, it is often assumed that local charge neutrality holds in the active region, and the dependence of internal optical loss on the concentrations of charge carriers in the waveguide and active regions is also not taken into account. As discussed in Ref. [31] [see the text in that paper immediately after formula (13)], the use of these two assumptions in the equation representing the laser generation condition (the condition that optical gain equals



Band energy diagram of a semiconductor laser with a low-dimensional active region [11].

total optical loss) immediately leads to the conclusion that the concentration of charge carriers in the active region is stabilized (pinned) above the generation threshold at its threshold value, which, as discussed below, is not confirmed experimentally.

In laser structures with a low-dimensional active region, a broadening of the generation spectrum and an increase in the intensity of spontaneous radiation in the waveguide region at high pump currents beyond the generation threshold were experimentally observed [32–35]. The broadening of the generation spectrum with an increase in the pump current was explained by an increase in the concentrations of charge carriers in the active and waveguide regions. Thus, it was experimentally found in Refs. [32–35] that the concentrations of charge carriers continue to increase with the pump current beyond the generation threshold in both low-dimensional active and bulk waveguide regions, and hence these concentrations depend on the pump current. This means that, in the generation mode in lasers with a low-dimensional active region, the concentration of charge carriers does not stabilize (pin) at its threshold value (see also Ref. [36]). An increase in the concentration of charge carriers beyond the generation threshold is an important feature of lasers with a low-dimensional active region, which significantly determines their output characteristics. The theory explaining this effect and its impact on the light-current characteristic and the internal differential quantum efficiency of the laser was first developed in Refs. [31,37].

Further studies in Refs. [38–43] showed that the concentrations of electrons and holes in a low-dimensional active region can differ greatly from each other both at the generation threshold [38,39] and above the generation threshold [40–43], i. e., the local charge neutrality does not hold in a low-dimensional active region.

2. Condition of global charge neutrality

The condition of global charge neutrality is proved here strictly mathematically and, thus, is a theorem. It is derived from the rate equations for electrons and holes in the bulk waveguide and low-dimensional active regions of the laser. In this paper, we consider a two-dimensional active region consisting of undoped quantum wells placed in an undoped waveguide region (optical confinement layer (OCL)).

The rate equations for a quantum well laser are presented below [43]. The equation for free electrons in a bulk waveguide region:

$$b \frac{\partial n^{\text{OCL}}}{\partial t} = \frac{j}{e} + N_{\text{QW}} \frac{n^{\text{QW}}}{\tau_{n,\text{esc}}} - N_{\text{QW}} v_{p,\text{capt},0} (1 - f_n) n^{\text{OCL}} - b B_{3\text{D}} n^{\text{OCL}} p^{\text{OCL}}; \quad (1)$$

the equation for free holes in a bulk waveguide region:

$$b \frac{\partial p^{\text{OCL}}}{\partial t} = \frac{j}{e} + N_{\text{QW}} \frac{p^{\text{QW}}}{\tau_{p,\text{esc}}} - N_{\text{QW}} v_{p,\text{capt},0} (1 - f_p) p^{\text{OCL}} - b B_{3\text{D}} n^{\text{OCL}} p^{\text{OCL}}; \quad (2)$$

the equation for electrons localized in the QW:

$$\frac{\partial n^{\text{QW}}}{\partial t} = v_{n,\text{capt},0} (1 - f_n) n^{\text{OCL}} - \frac{n^{\text{QW}}}{\tau_{n,\text{esc}}} - B_{2\text{D}} n^{\text{QW}} p^{\text{QW}} - c_g g^{\text{max}} (f_n + f_p - 1) n_{\text{ph}}; \quad (3)$$

the equation for holes localized in the QW:

$$\frac{\partial p^{\text{QW}}}{\partial t} = v_{p,\text{capt},0} (1 - f_p) p^{\text{OCL}} - \frac{p^{\text{QW}}}{\tau_{p,\text{esc}}} - B_{2\text{D}} n^{\text{QW}} p^{\text{QW}} - c_g g^{\text{max}} (f_n + f_p - 1) n_{\text{ph}}. \quad (4)$$

Equations (1)–(4) are nonlinear and thus describe a nonlinear electron-photon system. The unknown quantities in them are: n^{OCL} and p^{OCL} — concentrations of free electrons and holes in the bulk waveguide region, n^{QW} and p^{QW} — two-dimensional concentrations of electrons and holes localized in the QW, n_{ph} — two-dimensional concentration of stimulated radiation photons, f_n and f_p — occupancies of states corresponding to the lower edge of the electron subband and the upper edge of the hole subband of the size quantization in QW. It should be noted that all of the above concentrations, as well as f_n and f_p , depend on the density of the injection current j .

The equations (1)–(4) also include the following parameters: b — thickness of the waveguide region, e — electron charge, N_{QW} — number of identical (having the same width and composition) quantum wells, $v_{n,\text{capt},0}$ and $v_{p,\text{capt},0}$ — capture velocities of electrons and holes into an empty quantum well, $\tau_{n,\text{esc}}$ and $\tau_{p,\text{esc}}$ — thermal escape times of electrons and holes from the QW into the waveguide region, $B_{3\text{D}}$ and $B_{2\text{D}}$ — coefficients of spontaneous radiative recombination in the bulk (waveguide) region and in the two-dimensional (active) region [15,44], c_g — group speed of light, g^{max} — maximum optical gain coefficient in each QW.

The occupancies f_n and f_p are related to the two-dimensional concentrations of electrons and holes in the QW n^{QW} and p^{QW} as follows [45–47]:

$$f_n = 1 - \exp\left(-\frac{n^{\text{QW}}}{N_{c,v}^{2\text{D}}}\right), \quad f_p = 1 - \exp\left(-\frac{p^{\text{QW}}}{N_v^{2\text{D}}}\right), \quad (5)$$

where $N_{c,v}^{2\text{D}} = m_{e,hh}^{\text{QW}} T / (\pi \hbar^2)$ are two-dimensional effective densities of states in the conduction band and in the valence band in the QW; $m_{e,hh}^{\text{QW}}$ are effective masses of electrons and holes in the QW; T is the temperature in energy units.

The thermal escape times of electrons and holes from the QW into the waveguide region are given according to Refs. [43,48]:

$$\begin{aligned}\tau_{n,\text{esc}} &= \frac{1}{v_{n,\text{capt},0}(1-f_n)} \frac{N_c^{2D}}{n_1}, \\ \tau_{p,\text{esc}} &= \frac{1}{v_{p,\text{capt},0}(1-f_p)} \frac{N_v^{2D}}{p_1},\end{aligned}\quad (6)$$

where

$$\begin{aligned}n_1 &= N_c^{3D} \exp\left(-\frac{\Delta E_c - \varepsilon_n^{\text{QW}}}{T}\right), \\ p_1 &= N_v^{3D} \exp\left(-\frac{\Delta E_v - \varepsilon_p^{\text{QW}}}{T}\right),\end{aligned}\quad (7)$$

where $N_{c,v}^{3D} = 2[m_{c,v}^{\text{OCL}}T/(2\pi\hbar^2)]^{3/2}$ are three-dimensional effective densities of states in the conduction band and in the valence band in the waveguide region, $m_{c,v}^{\text{OCL}}$ are effective masses of electrons and holes in the waveguide region, $\Delta E_{c,v}$ are conduction band and valence band offsets at the hetero-interfaces of the QW and waveguide region, $\varepsilon_n^{\text{QW}}$ and $\varepsilon_p^{\text{QW}}$ are the energies of the lower and upper edges of the electron and hole subbands of the size quantization in QW.

The capture velocities of electrons $v_{n,\text{capt},0}$ and holes $v_{p,\text{capt},0}$ from the waveguide region to the empty QW are determined by the compositions and sizes of the QW and the waveguide region [11] and do not depend on the injection current. The finite (i. e., not infinitely high) values of these velocities themselves reflect the fact that charge carriers are not captured instantaneously in the active region of the laser. The total capture velocities (capture velocities taking into account the occupancy of the QW with charge carriers [11,31,37,43,49]), determined by the expressions

$$\begin{aligned}v_{n,\text{capt}}(j) &= v_{n,\text{capt},0} \exp\left[-\frac{n^{\text{QW}}(j)}{N_c^{2D}}\right], \\ v_{p,\text{capt}}(j) &= v_{p,\text{capt},0} \exp\left[-\frac{p^{\text{QW}}(j)}{N_v^{2D}}\right],\end{aligned}\quad (8)$$

depend on the pump current [40] and, as can be seen from (8), have lower values than the capture velocities into an empty QW. Noninstantaneous capture of electrons and holes into the active region, combined with spontaneous electron-hole recombination in the waveguide region, leads to sublinearity of the laser light-current characteristic [31,37,43].

It should be noted that capture velocities can also be introduced for quantum dots. However, unlike QW, these velocities are not characteristics of a single quantum dot, but of an array of quantum dots (quantum dot ensemble). These velocities can be expressed in terms of capture cross sections, which are the intrinsic characteristics of a single quantum dot [15,31,50].

We would also like to note here that the internal optical loss, which depends on the concentrations of charge carriers and consequently increases with the pump current, leads to

an additional sublinearity of the laser light-current characteristic, namely, to rollover of light-current characteristic at high pump currents [51,52]. This loss is thus one of the main factors limiting the optical power output in lasers with a low-dimensional active region [51,52].

Adding up for electrons equation (1) and multiplied by N_{QW} (number of QW) equation (3), as well as adding up for holes equation (2) and multiplied by N_{QW} equation (4), and subtracting the resulting equation for electrons from the resulting equation for holes, we obtain the following equation:

$$\begin{aligned}\frac{\partial}{\partial t} \{ [bp^{\text{OCL}}(j) + N_{\text{QW}}p^{\text{QW}}(j)] \\ - [bn^{\text{OCL}}(j) + N_{\text{QW}}n^{\text{QW}}(j)] \} = 0.\end{aligned}\quad (9)$$

Equation (9) is a condition for total charge conservation in the laser structure:

$$\begin{aligned}e[bp^{\text{OCL}}(j) + N_{\text{QW}}p^{\text{QW}}(j)] \\ - e[bn^{\text{OCL}}(j) + N_{\text{QW}}n^{\text{QW}}(j)] = \text{const}(t).\end{aligned}\quad (10)$$

Since the laser structure is initially uncharged ($\text{const} = 0$), equation (10) yields

$$bp^{\text{OCL}}(j) + N_{\text{QW}}p^{\text{QW}}(j) = bn^{\text{OCL}}(j) + N_{\text{QW}}n^{\text{QW}}(j).\quad (11)$$

Equation (11) holds for any pump current under both stationary and non-stationary conditions and is a condition of global charge neutrality in a semiconductor laser structure, according to which the total concentration of electrons in the waveguide and active regions is equal to the total concentration of holes in these regions. Each of the four concentrations included in equation (11) changes with the change in the pump current density j . However, changes in these concentrations occur in such a way that the condition (11) is fulfilled.

It should be noted that, in Ref. [53], the global charge neutrality condition was derived for lasers of a new design — quantum dot lasers with asymmetric barrier layers. Due to their improved characteristics, lasers of this design are more promising than conventional quantum dot lasers (see, for example, [54,55]).

3. Violation of local charge neutrality in quantum wells

Despite a straightforward derivation of the global charge neutrality condition from rate equations, properly taking this condition into account is very important for understanding and correctly describing physical processes in laser structures with a low-dimensional active region.

As discussed in Ref. [40], in quantum well lasers, the concentrations of electrons and holes in the bulk waveguide region significantly exceed the concentrations of electrons and holes in the quantum wells themselves at high pump

currents. The following conclusions can be drawn from this fact.

1) As can be seen from the global charge neutrality condition (11), the concentrations of electrons and holes in the waveguide region should be close to each other at such pump currents, and, thus, the global charge neutrality condition practically reduces to the charge neutrality condition in the waveguide region.

2) The concentrations of electrons and holes in quantum wells can differ greatly from each other, i.e., there may be a violation of local charge neutrality in the two-dimensional active region of the laser (see also [56] for quantum dot lasers).

4. Conclusion

A brief overview of the issue of charge neutrality in semiconductor lasers is given. Due to the widespread use of injection lasers, a proper account of this issue is required for an adequate theoretical description of the performance characteristics of such lasers.

The paper mathematically deduces the condition of global charge neutrality in quantum well lasers. It is shown that at any pump currents, both in stationary and dynamic generation modes, the total concentration of electrons in the bulk waveguide region and the two-dimensional active region (quantum wells) is equal to the total concentration of holes in these regions.

In the case of a spatially nonuniform distribution of charge carriers in the waveguide region and a nonuniform occupancy of quantum wells by carriers, the charge neutrality condition will include coordinate-averaged carrier concentrations in the waveguide region and quantum well-averaged carrier concentrations in the wells.

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Conflict of interest

The authors declare that they have no conflict of interest.

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