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# Study of the transformation of singular Laguerre–Gauss beams in reflection from a rough surface

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Using a numerical model based on scalar diffraction of singular beams, we qualitatively show the peculiarities of the evolution of phase singularities upon reflection from a surface possessing a microrelief of a given height and shape. The cases of degeneration of the vortex „core", formation of topological dipoles, and doubling of singularities under the influence of phase inhomogeneities on the initial beam are demonstrated. The obtained results demonstrate the applicability of singular beams in the problems of analyzing the topography of reflecting surfaces whose longitudinal scale of inhomogeneities is of the order of quarter wave length.

**Keywords:** optical vortex, Laguerre–Gauss beam, diffraction.

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Laser beams with wavefront singularities (optical vortices) represent a promising research subject for optical measurement applications. It is known that the wavefront geometry of a scalar optical vortex forms a helicoid with a singular point on the axis. The wave phase at this point is undefined and experiences a jump multiple of $2\pi$ when encircling the point along a closed contour [1]. This wave structure of singular beams exhibits certain stability during diffraction on amplitude and phase inhomogeneities and features a characteristic intensity minimum serving as a vortex position marker [2]. This enabled the application of vortex beams in optical manipulators as well as for measuring physical quantities such as refractive index, thicknesses, surface relief height, and roughness [3–5]. However, one barrier to using singular beams is the complexity of accurately analyzing their phase portraits. This stems from structural distortions of beam intensity and phase during diffraction on surface inhomogeneities of measured objects.

The aim of this work is to determine the applicability limits of singular beams for metrological surface studies where surface roughness distorts the optical vortex structure. Since surfaces with stepped relief structures are commonly encountered in practice, this work focuses on a numerical model of singular beam diffraction on reflecting surface featuring repeating rectangular protrusions and depressions [6]. This model is schematically shown in Fig. 1.

Using the known expression for a Laguerre–Gaussian beam with zero radial index and switching to Cartesian coordinates for modeling convenience, the resulting complex amplitude field distribution in the initial plane $z = 0$ is written, up to a normalizing factor [7]:

$$E(x, y, z = 0) \propto \exp\left(-\frac{x^2 + y^2}{2\omega_0^2}\right) \exp\big(il \arctan(y/x)\big), \quad (1)$$

where $\omega_0$ — Gaussian beam waist radius, $l$ — vortex topological charge. The roughness profile with strips of varying width and height is defined by the transmission function $\exp(ik\Delta)$, where $k$ — wavenumber. This function accounts for the geometric optical path difference of waves reflected from surface sections differing in height $h$ (Fig. 1, $b$).

This work focuses on analyzing the two-dimensional distribution of the electric field vector amplitude modulus and phase of singular beams primarily in the far diffraction zone to approximate experimentally realizable conditions. The spatial evolution of the beam field $E(X, Y, z)$propagating in free space at distance $z$ from the surface (Fig. 1, $a$) is obtained using the Fresnel–Kirchhoff evolution integral [8]:

$$\begin{aligned} E(X, Y, z) = \frac{k}{2\pi iz} \int\limits_{-\infty}^{a_1} \int\limits_{-\infty}^{\infty} E(x, y, 0) \\ \times \exp\left\{\frac{ik}{2z}\left[(X - x)^2 + (Y - y)^2\right]\right\} dxdy \\ + \frac{k \exp(ik\Delta)}{2\pi iz} \int\limits_{a_1}^{a_2} \int\limits_{-\infty}^{\infty} E(x, y, 0) \\ \times \exp\left\{\frac{ik}{2z}\left[(X - x)^2 + (Y - y)^2\right]\right\} dxdy \\ + \frac{k}{2\pi iz} \int\limits_{a_2}^{\infty} \int\limits_{-\infty}^{\infty} E(x, y, 0) \\ \times \exp\left\{\frac{ik}{2z}\left[(X - x)^2 + (Y - y)^2\right]\right\} dxdy. \end{aligned} \quad (2)$$

The resulting amplitude distribution calculation was performed as a sum over all inhomogeneous relief sections with

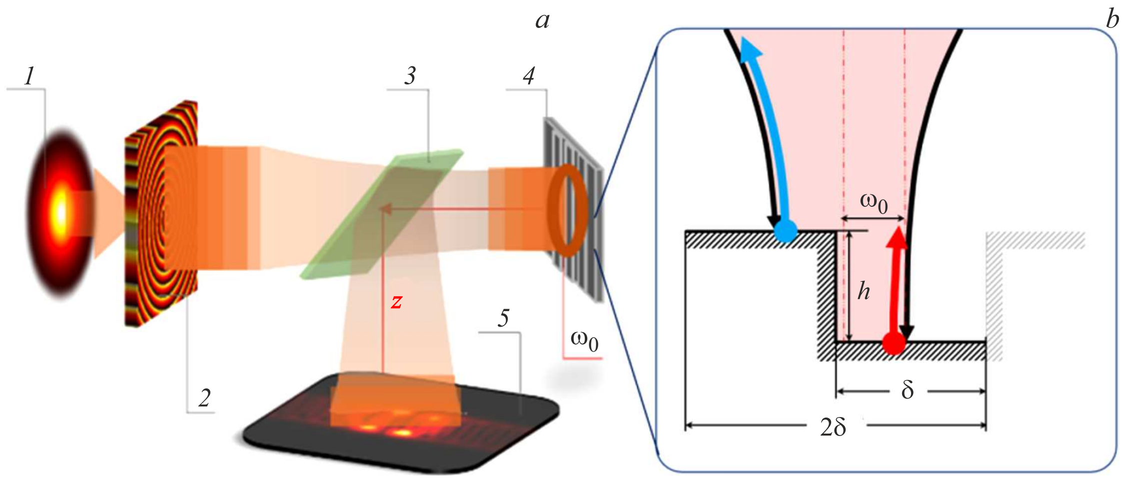


**Figure 1.** (*a*) Schematic of singular beam formation and diffraction: passage of Gaussian beam (*1*) through phase transparency (*2*), reflection from surface (*4*), and propagation of diffracted beam (*3*) over distance $z$ to observation plane (*5*); (*b*) parameters of stepped reflecting surface.

integration limits divided by $dx$ according to the number of these sections, with the minimum number being three but without upper limit.

The model input parameters include the Gaussian beam waist radius $\omega_0$ in the initial plane $z = 0$, linear scale of inhomogeneities ($\delta = a_2 - a_1$), distance to observation plane $z$ and geometric optical path difference $\Delta$. We trace the amplitude evolution of the beam and spatial dynamics of wavefront singularities after reflection from such a surface.

Fig. 2 shows the numerical modeling results of vortex beam diffraction with topological charges $|l| = 1$ and $|l| = 2$ on a stepped quasi-periodic three-dimensional protrusion-depression structure with relief height $h = \Delta/2$. The structure period was $2\delta = 2.4\,\mu$m, distance from source plane to observation plane 0.75 m in Rayleigh length units ($z_R = k\omega_0^2/2$). Initial beam waist radius is $\omega_0 = 37.0\,\mu$m. The phase portrait reveals a „chain" of phase singularities appearing as „forks" — splits in equal-phase lines. The initial axial vortex retains its spiral phase structure (Fig. 2, *a*), though its amplitude distribution is deformed by adjacent diffraction orders. Doubling the topological charge increases the number of helical dislocations in the beam field. Meanwhile, the axial singularity, appearing as a double spiral on the phase portrait, undergoes degeneration (Fig. 2, *b*). Consequently, a pair of optical vortices with unit modulus topological charge localizes near the beam axis. This behavior aligns with previously obtained edge diffraction results of optical vortices on amplitude and phase screens by A.Ya. Bekshaev's team [9–11].

Developing this problem, we focus on the evolution of a singular beam propagating near an inhomogeneity edge. The geometric optical path difference parameter $\Delta$ was chosen as a multiple of $\lambda/2$ achieving maximum helicoidal wavefront perturbation [12]. Parameter $a$ representing the inhomogeneity edge displacement relative to the beam center, was calculated in waist radius units ranging from one (edge at beam periphery) to zero (edge crossing beam center).

With progressive edge displacement through the beam cross-section (right to left) along the abscissa axis, the Laguerre–Gaussian beam amplitude modulus and phase (Fig. 3, *a*) undergo substantial changes: the axial optical vortex shifts to the beam periphery, revealing a so-called topological dipole on the phase portrait (at $a = \pm 0.5\omega_0$). Meanwhile, the phase spiral rotates by $\pi$ radians relative to its initial position, enabling step height detection in $\lambda/4$. From the perspective of preserving the vortex beam ring shape, the transverse inhomogeneity size was chosen as $\delta = 0.9\omega_0$.

Most interesting is the moment when the inhomogeneity edge passes through the beam center, resulting in the far-field diffraction pattern showing a doubled initial vortex. The phase portrait localizes equal-phase line bifurcations indicating formation of a „copy" of the initial optical vortex, mirror-displaced relative to the center (Fig. 3, *b*). This figure clearly illustrates axial optical vortex displacement with increasing surface inhomogeneity height $h$ plotted along the abscissa in wavelength units. Maximum displacement corresponds to the height achieving geometric optical path difference $\Delta = \lambda/2$ between beam parts reflected from stepped inhomogeneity planes (Fig. 1, *b*).

This case is conveniently traced evolutionarily at various distances measured in Rayleigh length units from the source plane containing the rough surface. Fig. 3, *c* shows amplitude modulus and phase distribution transformation at distances $z$ ranging from several centimeters to one meter with fixed inhomogeneity height and displacement parameters. The phase profile reveals the moment of optical vortex pair formation starting at $0.5z_R$, while the amplitude modulus distribution acquires an N-shaped outline. In the far diffraction zone, it transforms into a twin vortex beam with a common center as an amplitude field maximum.

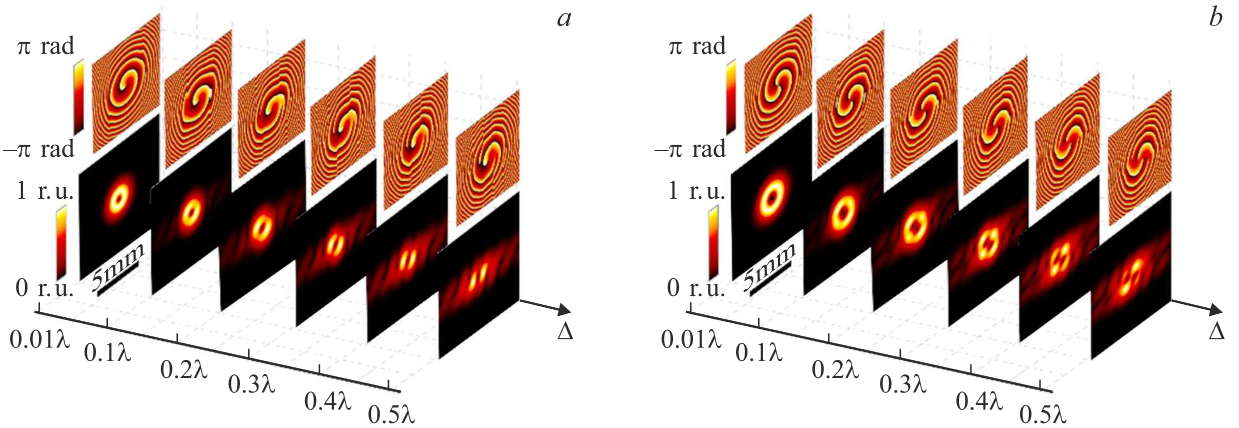


**Figure 2.** Numerical calculation of transverse distribution of amplitude modulus and phase of diffracted Laguerre–Gaussian beam with topological charges $l = 1$ $(a)$ and $l = 2$ $(b)$ on stepped quasi-periodic three-dimensional protrusion-depression structure with relief height $h = \Delta/2$. Structure periodicity is $2.4\,\mu$m. Distance from source plane to observation plane $\sim 40z_R$ (0.75 m). Initial beam waist radius is $\omega_0 = 37.0\,\mu$m.

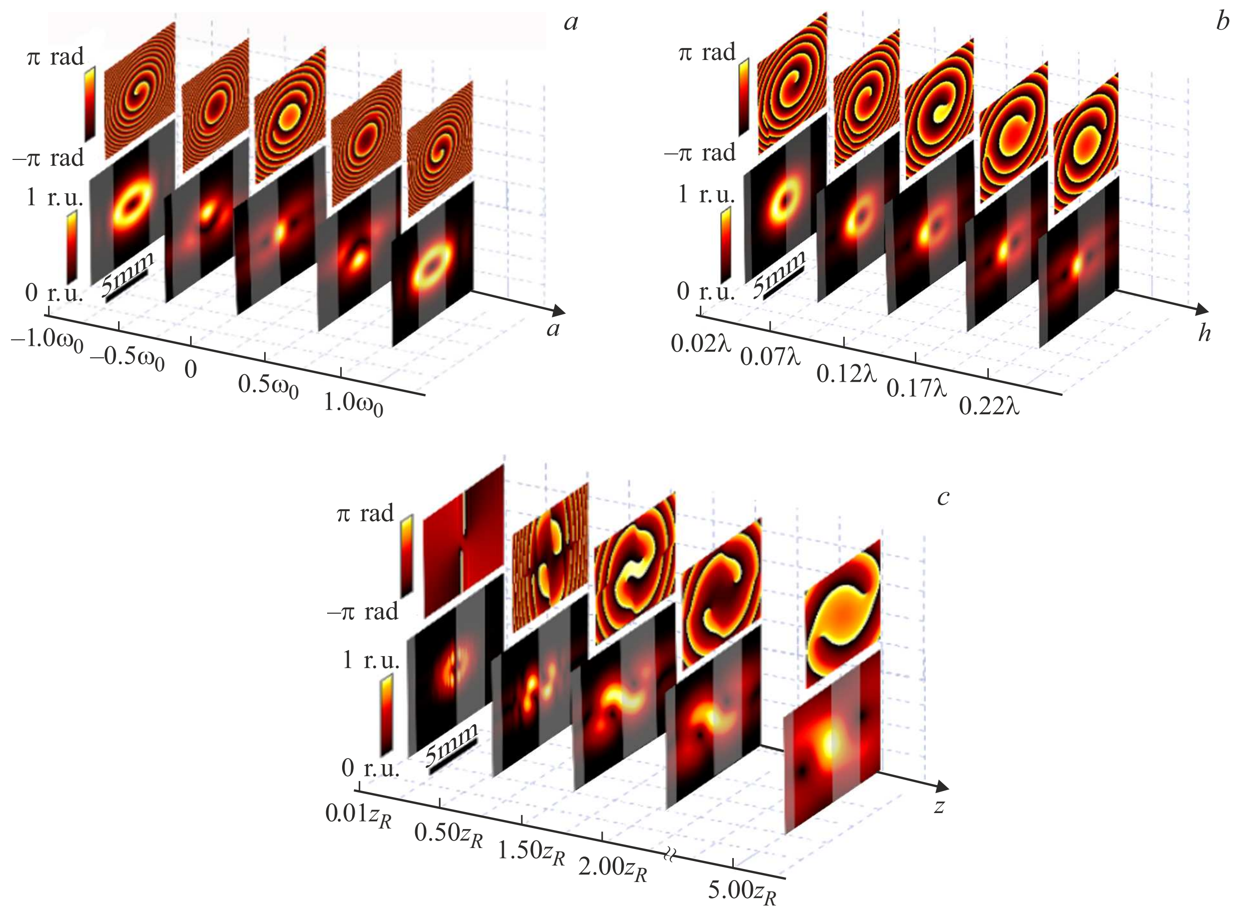


**Figure 3.** Numerical calculation of transverse distribution of amplitude modulus and phase of diffracted Laguerre–Gaussian beam with topological charge $l = 1$ depending $(a)$ on position of stepped inhomogeneity edge (shown as light stripe) with relief height $h = \lambda/4$ and width $\delta = 0.9\omega_0$, $(b)$ on relief height $h$ with edge positioned at beam center, $(c)$ $z$-diffraction of Laguerre–Gaussian beam on step with relief height $h = \lambda/4$. Beam waist radius taken as $\omega_0 = 225.0\,\mu$m.

Thus, the numerical diffraction model demonstrates qualitative features of amplitude and phase transformation of singular beams reflected from rough surfaces, promising for developing new nondestructive testing approaches. The obtained results demonstrate the applicability of singular beams for analyzing reflecting surface relief, with maximum longitudinal inhomogeneity scale on the order of $\lambda/4$. In this case, the capability to extract information about the surface characteristics from the phase portrait of the singular beam and directly from the characteristic amplitude field distribution is preserved.

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### Conflict of interest

The authors declare that they have no conflict of interest.

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