Development of a Method for Analyzing Lunar Laser Ranging Data

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A method for analyzing modern lunar laser ranging (LLR) observations has been developed. The main elements of the algorithm for the reduction of LLR data are considered. The features of the computational process are described, including the determination of individual residual differences and the relationship between the theoretical modeling of the Moon's rotation and laser ranging, aimed at obtaining new planetophysical data.

Keywords: physical libration of the Moon, reduction of lunar laser ranging observations.

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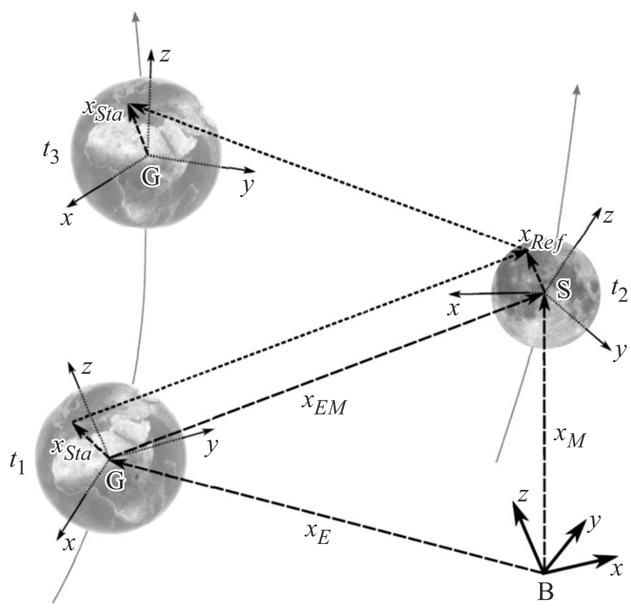
The time of radiation and signal registration by a laser rangefinder is the main observed value in case of the reduction of lunar laser ranging (LLR) [1]. Thus, the interrelation of observations with the theory of the Moon's rotation is carried out through the solution of the inverse problem. When processing LLR data, it is necessary to take into account the fact that during the propagation of the laser signal, the angular reflector located on the Moon and the laser rangefinder located on the Earth's surface change their positions in space due to the movement of the Earth and the Moon [2,3]. In addition, it is important to investigate many other factors affecting the movement of celestial bodies. It is necessary to know the exact ephemerides of celestial bodies (at least for the solid-state model) and the tidal force calculation model, to take into account the deflection of the light beam due to the curvature of space-time during its movement between the Earth and the Moon and the time delay [4] during the passage of the signal through the Earth's atmosphere. All this makes the task of creating a method for processing LLR data quite difficult even before the stages of solving the inverse problem and the process of modeling the effects of external effects.

Let there be three points in time: t_1 — time of signal emission, t_2 — time of signal reflection from the reflector, t_3 — time of signal arrival at the station (see figure). The Barycentric Celestial Reference System (BCRS) will act as the basic inertial reference system, and time will be measured in the Barycentric Dynamic Time (TDB). In Figure B is the barycenter, S is the position of the angular reflector on the surface of the Moon, G is the laser optical system on Earth. It should be said that the feature of BCRS is that the origin coincides with the barycenter of the Solar system, i.e. the center of mass of the entire Solar system.

The time interval between the emission of a laser signal and the reception of its reflected beam by an optical system can be written as



Coordinate systems.

$$t_2 - t_1 = \frac{|l_{\text{BCRS}}(t_2) - s_{\text{BCRS}}(t_1)|}{c} + \Delta_{grav}(t_1, t_2) + \Delta_{atm}(t_1, t_2),$$

$$t_3 - t_2 = \frac{|s_{\text{BCRS}}(t_3) - l_{\text{BCRS}}(t_2)|}{c} + \Delta_{grav}(t_3, t_2) + \Delta_{atm}(t_3, t_2), \qquad (1)$$

where Δ_{grav} is the gravitational delay, Δ_{atm} is the atmospheric delay along the signal path.

We will use the recommendation of the IAU (International Astronomical Union) under the IERS 2010 (International Earth Rotation and Reference Systems Service) convention for calculating the station and reflector vectors [2]:

$$s_{\rm BCRS} = r_E + s_{\rm GCRS}\left(1 - \frac{U_E}{c^2} - L_c\right) - \frac{1}{2}\left(\frac{\dot{r}_E \cdot s_{\rm GCRS}}{c^2}\right)\dot{r}_E, \qquad (2)$$

where r_E is the position of the Earth's center relative to the barycenter, and $s_{\rm GCRS}$ are the coordinates of the station in the inertial geocentric celestial reference system, corrected for the tidal and other delays, U_E is the gravitational potential in the center of the Earth, without taking into account the mass of the Earth, $L_c = 1.48082686741 \cdot 10^{-8}$ is the relativistic parameter. The coordinates of the stations can be found as follows:

$$S_{\rm GCRS} = R_{T2C}(\mathbf{S}_{\rm TRS} + \Delta_{pole} + \Delta_{solid} + \Delta_{ocean}), \qquad (3)$$

where R_{T2C} is the rotation matrix from the terrestrial Reference system TRS (Terrestrial Reference System), which rotates with the Earth, in GCRS (Geocentric Celestial Reference System), the vector $\mathbf{S}_{\rm TRS}$ consists of three components of the station position, Δ are corrections for station coordinates resulting from pole motion, solid-state and oceanic tides.

Similar equations can be written to obtain the coordinates of the reflectors:

$$l_{\rm BCRS} = \mathbf{r}_M + l_{\rm LCRS}\left(1 - \frac{U_M}{c^2}\right) - \frac{1}{2}\left(\frac{\dot{r}_M - l_{\rm LCRS}}{c^2}\right)\dot{r}_M,$$

$$\mathbf{l}_{\rm LCRS} = R_{L2C}\, l_{PA} + \Delta^{(E)}_{solid\ moon} + \Delta^{(S)}_{solid\ moon'}, \qquad (4)$$

where $\mathbf{l}_{\rm LCRS}$ is the coordinate vector of the reflectors in the inertial (non-rotating) selenocentric coordinate system (where LCRS is the Lunar Celestial Reference System), $\mathbf{r}_M$ is the barycentric vector of the center of the Moon, U_M is the gravitational potential without taking into account the mass of the Moon, $R_{\rm L_2C}$ is the matrix of transformation from a rotating reference frame to LCRS, $\mathbf{l}_{PA}$ is the vector of coordinates of stations in the reference frame associated with the main axes, Δ is the tidal addition from the Earth and the Sun. The formula for the tidal delay can be written, according to the IAU IESR2010 convention, as

$$\Delta_{solidmoon} = \frac{\mu_A R_M^4}{\mu_M r_{MA}^3}\left[\frac{h_2}{2}(3(\mathbf{r}_{MA}\cdot\mathbf{l})^2 - 1)\mathbf{r}_{MA} + 3l_2(\mathbf{r}_{MA}\cdot\mathbf{l})(\mathbf{r}_{MA} - (\mathbf{r}_{MA}\cdot\mathbf{l})\mathbf{l})\right], \qquad (5)$$

where h_2, l_2 are the tidal Love's (Shida) numbers, $\mathbf{l} = R_{L2C}\mathbf{l}_{PA}$ is a unit vector in the LCRS inertial coordinate system (the carriage symbol indicates the unit length of the vector), vector $\mathbf{r}_{MA} = \mathbf{r}_A - \mathbf{r}_M$ is the relative distance between the Moon and the body that creates the tide (A is the Earth or the Sun), R_{L2C} is the rotation matrix describing the physical libration of the Moon.

It should be noted that the time of sending the signal is given in the UTC coordinated Universal time scale, however, the TDB scale is used in our calculations. To convert the time, we will use a data set called C04 [4], which describes unmodeled effects at the celestial pole. This set includes the coordinates of the earth's pole, the correction to the celestial pole, and the differences UT1−UTC in one-day increments.

To obtain the coordinates of the celestial pole, we will use the model laid down by the IAU standard in the SOFA software package. It is necessary to download the average longitudes of the planets of the Solar system for the program to work. Tables of precession according to the IAU 2006 model and nutation according to the IAU 2000A model are attached in the program code. The time point is set using the following formula to increase accuracy:

$$T = ((\text{DATE1} - \text{DJ00}) + \text{DATE2})/\text{DJC}, \qquad (6)$$

where DJC is 36525. The program returns the two components of the celestial pole in radians.

If a correction from the IERS C04 solution [5] is added to the obtained components, then the true (instantaneous) values of the celestial pole can be obtained. To obtain the coordinates of the Earth's pole, it is necessary to calculate intra-day fluctuations caused by tides from the ocean, libration effects, and additional fluctuations of the UT1 scale.

After receiving all the corrections, the rotation matrix R_{T2C} iau_C2TXY can be calculated from the SOFA [6] software package. The input receives times in two different scales — in TT (the sum of two times TTA + TTB) and in UT1 (the sum of UTA + UTB), single components of the celestial and terrestrial poles are also fed to the input. A matrix of dimension 3×3 can be obtained at the output, which allows changing the geocentric for the Earth reference system. An inverse or transposed matrix is needed for our purpose.

When using the Earth's reference system, the station components must be adjusted for tides, since the system is built in such a way as not to take into account the instantaneous values of the displacement points. First correction — Earth's pole shift Δ_{pole}. It is also calculated based on IAU,IERS 2010, where the correction is specified in the geodetic coordinate system, which should be converted to the Cartesian coordinate system.

$$S_r = -33\sin(2\theta)(m_1\cos(\lambda) + m_2\sin(\lambda));$$

$$S_\theta = -9\cos(2\theta)(m_1\cos(\lambda) + m_2\sin(\lambda));$$

$$S_\lambda = 9\cos(\theta)(m_1\cos(\lambda) - m_2\sin(\lambda)), \qquad (7)$$

where the symbol m indicates the difference between the coordinate position of the Earth's pole, according to CO4, and the position set in IERS 2010.

The solid-state tides Δ_{tidal} to the stations are calculated based on the program developed in Ref. [7]. When using this program, the coordinates of the stations, the Sun, and the Moon in the geocentric system are entered, as well as the value of the observation time. The program returns the displacement vector of the station due to lunar and solar tides. Oceanic tides Δ_{ocean} are calculated based on a program using the FES2012 tidal ocean model. Next, it remains to take into account the last two corrections — gravitational Δ_{grav} and atmospheric Δ_{at}m. To account for the atmospheric delay, the program [8] is used, where the latitude and longitude of the observation site are entered at the input, and then, based on the temperature and the angle at which the laser signal was released (adjusted for compression), the compression function is calculated. The angle can be found by the formula:

$$position = \frac{position}{1 - f(2 - f)}, \quad (8)$$

$$\text{TEA} = \arcsin(\cos(position, direction)), \quad (9)$$

where *position* is the coordinate vector of the stations, *direction* is the direction to the reflector, f is the compression of the Earth. Based on this compression function, you can use the program [5], which inputs geodetic latitude, height above the ellipsoid, surface pressure, laser wavelength, and water vapor pressure to calculate the zenith delay (in meters), which must be divided by the velocity to obtain a correction in units of time.

The physical meaning of the gravitational delay is the curvature of space [9], which, in turn, leads to a time delay of the signal. It is necessary to pay attention to the fact that this delay should be calculated for two time segments: when the signal moves from the Earth to the Moon and when the signal moves from the Moon to the Earth. The distance from the disturbing body to the trajectory is chosen in the following formula as the minimum during calculations. If this factor is not taken into account, a negative value will appear in the logarithm:

$$c(t_2 - t_0) = R_{20} + \sum \frac{2GM_A}{c^2} \ln \frac{R_{2A} + R_{0A} + R_{20}}{R_{2A} + R_{0A} - R_{20}} + \frac{8G^2M_S^2}{c^4} \frac{R_{20}}{R_{20}^2 - (R_{2S} + R_{0S})^2}, \quad (10)$$

$$c(t_0 - t_1) = R_{01} + \sum \frac{2GM_A}{c^2} \ln \frac{R_{1A} + R_{0A} + R_{01}}{R_{1A} + R_{0A} - R_{01}} + \frac{8G^2M_s^2}{c^4} \frac{R_{01}}{R_{01}^2 - (R_{1S} + R_{0S})^2}. \quad (11)$$

This completes the process of describing the laser location processing algorithm.

In conclusion, we will give a brief description of the results obtained Ref.[1], the methodology of which became the basis of our research, and the capabilities of our proposed LLR analysis algorithm. As shown in Ref. [1], the combined use of DE430 and EPM with the IERS Conventions 2010 models reduced the LLR inconsistencies to 1–3 cm. The values 2.4–2.7 cm (YAG laser, 1987–2005) and 2.2–2.7 cm (MeO-laser, 2009–2013) were obtained based on the analysis of the residual differences O–C from the study in Ref. [1] for the CERGA station. Moreover, the O–C discrepancies were reduced by fixing some of the parameters (for example, gravity coefficients from GRAIL), which reduced the number of parameters to be adjusted, as well as by taking into account more accurate models of tides (IERS vs. DE) due to the introduction of empirical corrections (for example, additional terms in libration). Our method, which optimizes the calculation of R_{T2C} and $\Delta_{solidmoon}$ corrections, can further reduce systematic errors for modern stations (Apache Point, Matera), especially when processing short observation series where traditional methods are less effective.

The key difference between our method is the automated accounting of nonlinear effects in Δ_{grav} and Δ_{atm} (formulas (8)–(11)), which is especially important for stations with rapid changes in observation conditions (for example, Matera). A promising direction is to adapt our method for joint use with GRAIL data, which will allow us to refine the parameters of the physical libration of the Moon.

The proposed method can optimize the processing of LLR data by comprehensively accounting for various effects and using modern models. This opens up new possibilities for studying the physical libration of the Moon and other planetary physical phenomena.

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Conflict of interest

The authors declare that they have no conflict of interest.

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