Mathematical modeling of influence of plasma and gravitational inhomogeneities on refraction of cosmic radiation

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Apparatus of numerical-analytical modeling of refraction cosmic radiation in gravitational field of mass objects in presence random plasma inhomogeneities and gravitational noise is developed on based beam approximation and theory of perturbations. The propagation of radiation in the stochastic gravitational field was considered as process in Euclid's space with the effective refraction index of vacuum expressed by gravitational potential. The results of calculating of stochastic blurring of gravitational lensing effects are presented at depending on the position of radiation source and receiver for various types of inhomogeneities of cosmic space.

Keywords: electromagnetic radiation, geometrical optics, gravitational field, gravitational and plasma inhomogeneities.

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Introduction

It is known [1–5] that massive astrophysical objects make a significant contribution to the refraction of electromagnetic radiation during propagation in the space environment. A group of gravitational objects leads to the formation of complex electromagnetic field distributions. Analyzing such effects, we can proceed to solving the inverse problem of restoring environmental parameters, as well as evaluate the characteristics of objects that are invisible in the electromagnetic range, but manifest themselves through gravitational interaction. Meanwhile, when studying objects at cosmological distances, additional refractive effects associated with the influence of disturbances in the cosmic environment may occur in the structure of the received radiation. In particular, such perturbations include random plasma inhomogeneities [2,4]. For a detailed reconstruction of the disturbing gravitational potentials of hidden objects based on the characteristics of the received radiation, it is necessary to take into account not only the masking effect of cosmic plasma, but also the presence of areas of stochastic inhomogeneities of gravitational fields in the surrounding space, since the latter can also lead to partial blurring of gravitational effects

1. Numerical analysis simulation apparatus

Stochastic radial differential equations in the Lagrange–Euler form in the special spherical coordinate system [6], derived from Fermat's variational principle were used to calculate the effect of plasma and gravitational inhomogeneities on the refraction of cosmic radiation in the gravitational field:

$$\frac{dR}{d\varphi} = R\,\mathrm{ctg}\,\beta,$$

$$\frac{d\beta}{d\varphi} = (1+\sin^2\beta\,\mathrm{tg}^2\alpha)\left(\frac{1}{\tilde{n}}\left(\frac{\partial\tilde{n}}{\partial\varphi}\,\mathrm{ctg}\,\beta - R\,\frac{\partial\tilde{n}}{\partial R}\right) - 1\right),$$

$$\frac{d\delta}{d\varphi} = \mathrm{tg}\,\alpha,$$

$$\frac{d\alpha}{d\varphi} = \frac{1}{\tilde{n}}(1+\cos^2\alpha\,\mathrm{ctg}^2\beta)\left(\frac{\partial\tilde{n}}{\partial\delta} - \frac{\partial\tilde{n}}{\partial\varphi}\,\mathrm{tg}\,\alpha\right), \qquad (1)$$

where R, δ, φ are the radial and angular coordinates of the beam, respectively; α, β are the current refractive angles of the beam; $\tilde{n}$ is the effective refractive index, taking into account random inhomogeneities of the space environment and the additive contribution of gravitational fields objects in the general gravitational field [4,5]:

$$\tilde{n} = n_0 + \tilde{n}_1,\ \ n_0 = 1 + \frac{R_g}{R} + \sum_{i=1}^{N} A_i$$

$$\times \exp\left[-b_{\varphi i}(\varphi-\varphi_{Li})^2 - b_{\delta i}(\delta-\delta_{Li})^2 - b_{Ri}(R-R_{Li})^2\right], \qquad (2)$$

where n_0 is the effective refractive index of the regular gravitational field; $\tilde{n}_1$ describes random inhomogeneities of the space environment; R_g is the gravitational radius of the main object of gravity; N is the number of additional regular inhomogeneities of the refractive index; $A_i, \varphi_{Li}, \delta_{Li}, R_{Li}, b_{\varphi i}, b_{\delta i}, b_{Ri}$ is respectively, the intensity, coordinates of the localization center and scales of the i-th regular disturbance of the gravitational field. The geometry of the problem corresponds to Fig. 1 in Ref. [5]. As a result of solving the system (1) in the approximation of the perturbation method (at $\tilde{n}_1 \ll 1$), a generative system of equations was obtained for calculating the refraction of radiation in a regular gravitational field (system (1) at $\tilde{n}_1 = 0$), as well as a system of equations to calculate the variances of the lateral deflections of the rays in the observer's picture plane [4,5]:

$$\frac{d\sigma_\delta^2}{d\varphi} = \frac{\mu}{4}\sqrt{\frac{\pi}{Q}}\left(\frac{DP^2}{Q} + 16\left(D - \frac{K}{Q}\right)(\varphi J_1 - J_2)\right),$$

$$\frac{dJ_1}{d\varphi} = P^2, \ \frac{dJ_2}{d\varphi} = \varphi P^2, \tag{3}$$

where

$$P = \frac{1}{\cos^2\alpha_0} + \mathrm{ctg}^2\beta_0, \ Q = \frac{1}{\nu_\varphi^2} + \frac{1}{\nu_\delta^2}\,\mathrm{tg}^2\alpha_0 + \frac{R_0^2}{\nu_R^2}\,\mathrm{ctg}^2\beta_0,$$

$$K = \left(\frac{1}{\nu_\varphi^2} - \frac{1}{\nu_\delta^2}\right)^2 \mathrm{tg}^2\alpha_0;$$

$$\mu = \gamma\mu_0,$$

$$\gamma = \exp\lfloor -m_R(R - R_L')^2 - m_\varphi(\varphi - \varphi_L')^2 - m_\delta(\delta - \delta_L')^2 \rfloor, \tag{4}$$

$R_0, \delta_0, \alpha_0, \beta_0$ — refractive characteristics of the beam at $\tilde{n}_1 = 0$; $\mu_0, \nu_R, \nu_\varphi, \nu_\delta$ — intensity and spatial radii of correlation of inhomogeneities of the space environment; $R_L', \varphi_L', \delta_L', m_R, m_\varphi, m_\delta$ — the coordinates of the center and the size of the localization area of the inhomogeneities of the space environment. The equations (3) are obtained under the assumption that the turbulence of the cosmic medium is characterized by a quasi-uniform random field of refractive index inhomogeneities. For simplicity of estimates, the Gaussian form of the homogeneous part of the correlation function was used. It should be noted that in the general case, chaotic inhomogeneities of a multiscale space environment are described by a power spectrum. Meanwhile, in a number of cases [7], when calculating the lowest moments of fluctuations in the direction of propagation of transmission signals, an effective Gaussian spectrum can be used if the external scale of cosmic turbulence defined by a power spectrum is considered as the spatial scale of inhomogeneities. This is attributable to the fact that the high-frequency part of the spectrum of inhomogeneities has a greater effect on the amplitude of the signal than on its phase [7].

2. Calculation results and discussion

Fig. 1–3 shows the results of calculations of the refractive characteristics of radiation based on the systems of equations (1), (3) for the case of an offset point source relative to the line of sight along the angular coordinate δ at a fixed distance R. The initial conditions were $\varphi_n = 0$, $R_n = 50$ cul (cul is the conventional unit of length). Two source positions with angular coordinates $\delta_n = 0.3$ rad and $\delta_n = 0.6$ rad were considered to assess the effect of the displacement of the radiation source from the line of sight on the observed effect. The aiming angular parameter α_n varied in the range (-0.75)–$(+0.75)$ rad, and β_n varied in the ranges (-0.75)–(-0.03) and $(+0.03)$–$(+0.75)$ rad. The calculation was carried out up to the radial coordinate $R_k = 50$ cul, where the observer's picture plane was formed (hereinafter —„ray pattern“) with the final angular coordinates $(\varphi_k; \delta_k)$ of the ray marked on it. Fig. 1 shows the ray pattern in the observer's plane during the transport of cosmic radiation through the regular gravitational field of a single object. For clarity, the final angular values $(\varphi_k; \delta_k)$ are shown here in Cartesian coordinates: $x_k = R_k \cos\varphi_k \cos\delta_k$, $y_k = R_k \sin\varphi_k \cos\delta_k$.

It can be seen from the calculation results that the formation of the ray pattern depends on the location of the point source. In the upper part of the distribution of the points of arrival of the rays into the observer's picture plane, the peripheral area is condensed (arc formation) with a gradual decrease in size. In the lower part of the distribution, the lens region shifts to the central part of the distribution with a decrease in spatial scales. The transition of the lens effect from the upper peripheral part of the distribution to the lower region of the ray pattern should be also noted in Fig. 1, *b*. Due to the displacement of the radiation source in the image plane of the observer, arc-shaped lens effects begin to form. Also, in the distribution of points of arrival in the picture plane, a candle-like formation should be noted, expressed by a noticeable structuring of points in the central area above the plane of the visual beam (relative to $y_k = 0$). This effect corresponds to the appearance of the radiation source in the line of sight for the observer. The higher the source is above the plane of the visual beam, the more pronounced the degree of structuring is.

One of the problems leading to the loss of information about the radiation source is the random distribution of mass in the gravitational field of a massive object or on the line of sight (hereinafter —„gravitational noise“). The effect of gravitational noise on the radiation pattern was calculated using the following parameters: $\nu_R = 0.1$ cul, $\nu_\varphi = \nu_\delta = 0.1$, $R_L' = 0$ cul, $\varphi_L' = \delta_L' = 0$, $\mu_0 = 10^{-5}$, $m_R = 25\,\mathrm{cul}^{-2}$, $m_\varphi, m_\delta \to \infty$. Fig. 2 shows the simulation results, where the lateral red segments correspond to the root-mean-square lateral deviations of the rays in the observer's plane under the influence of the gravitational-noise „cloud“. The longer the length of the segment, the greater the deflection of the beam. The values of the lengths of the segments of the lateral deflections of the rays in the observer's picture plane were obtained using the ratios $\sigma_x = \sigma_\delta R_k \cos\varphi_k \sin\delta_k$, $\sigma_y = \sigma_\delta R_k \sin\varphi_k \sin\delta_k$. It follows from Fig. 2 that when the gravitational-noise „cloud“ is localized near a massive object, stochastic blurring of the central part of the ray pattern occurs. As the optical path of the beam increases, as it propagates in the gravitational field, its lateral deviations in the pictorial plane increase. In the considered case, the arc-shaped regions of gravitational focusing are not deformed under the influence of gravitational noise due to the finite size of the „cloud“. However, as the distances between the sources

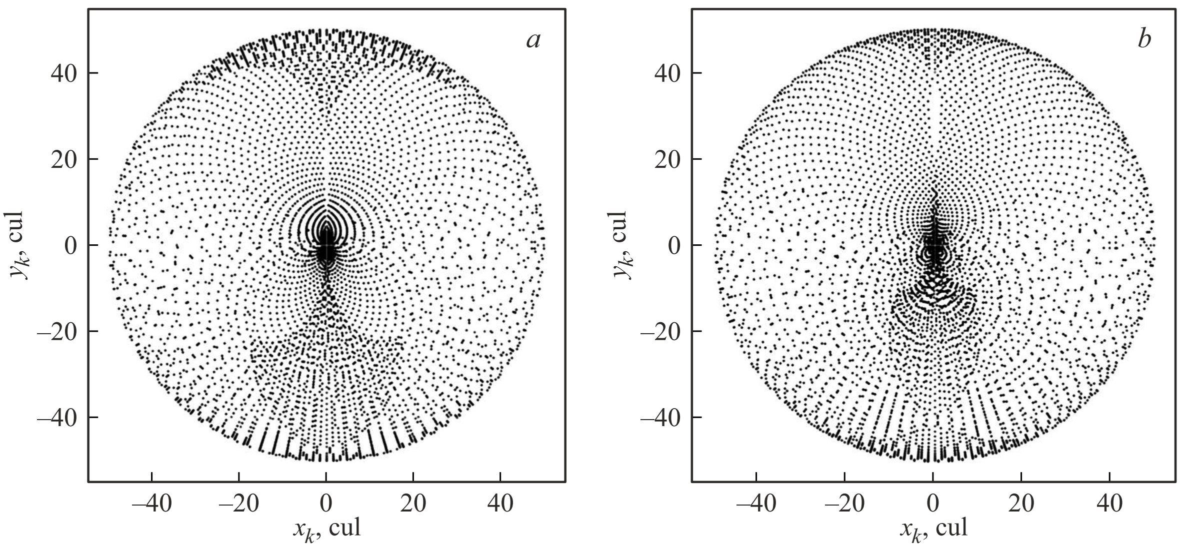

Figure 1. The ray pattern in the observer's plane during the transport of cosmic radiation near a single massive object with a different displacement of the point source at a fixed distance R along the angular coordinate δ from the line of sight: $\delta_n = 0.3$ rad (a) and 0.6 rad (b), in the absence of random irregularities. The marked points correspond to rays coming at a fixed distance R_k.

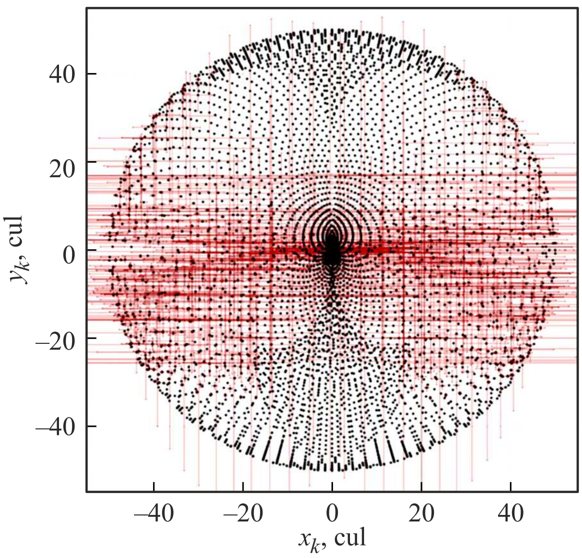

Figure 2. The ray pattern in the observer's plane during the transport of cosmic radiation near a single massive object immersed in gravitational noise. The radiation source is located at $\delta_n = 0.3$ rad. The red segments correspond to the RMS lateral deviations of the rays in the observer's picture plane under the influence of gravitational noise.

and gravitationally lensed objects increase, even a relatively small „cloud" can lead to blurring of the focus areas, which will result in the loss of information about the radiation source.

In order to obtain reliable information about the radiation source located at cosmological distances, it is also necessary to take into account the influence of random plasma inhomogeneities. The calculation of the blurring effect of gravitational lensing in cosmic plasma was carried out in the case of a uniform distribution of inhomogeneities throughout the medium, i.e. $m_R, m_\varphi, m_\delta \to \infty$. The intensity of the inhomogeneities was set as $\mu_0 = \mu_0'(f_{pl}/f)^2$, where f_{pl} is the plasma frequency, f is the frequency of cosmic radiation. The task parameters were: $\nu_R = 0.1$ cul, $\nu_\varphi = \nu_\delta = 0.1$, $R_L' = 0$ cul, $\varphi_L' = \delta_L' = 0$, $\mu_0' = 0.01$, $f_{pl} = 12$ MHz. Fig. 3 shows the calculation results for $f = 6$ GHz (Figure $3, a$) and $f = 300$ MHz (Figure $3, b$). It follows from Fig. 3 that, as in the case of Fig. 2, the greatest effect of stochastic blurring of the radial pattern is observed in the central part. It should be noted that the blurring effect is insignificant at a radiation frequency of 6 GHz (Fig. $3, a$), which leads to the preservation of the clarity of the arcuate lens effects. This is especially true for the central focus area. At a radiation frequency of 300 MHz (Fig. $3, b$), the radiation pattern undergoes significant blurring under the influence of plasma inhomogeneities. In this case, the lens effect in the central part of the ray pattern will be blurred, and the arc at the periphery will remain, but with a partial blurring of the outline.

Conclusion

The ray approximation and perturbation theory are used to calculate the effect of plasma and gravitational inhomogeneities on the refraction of cosmic radiation in the gravitational field of massive objects. Calculations of the refractive characteristics of radiation in gravitational fields of various configurations have been performed. It is noted that when the radiation source is shifted relative to the line of sight to a gravitationally lensed object, two arc-shaped focus areas are formed in the ray pattern. Under the influence of random gravitational and plasma inhomogeneities, these

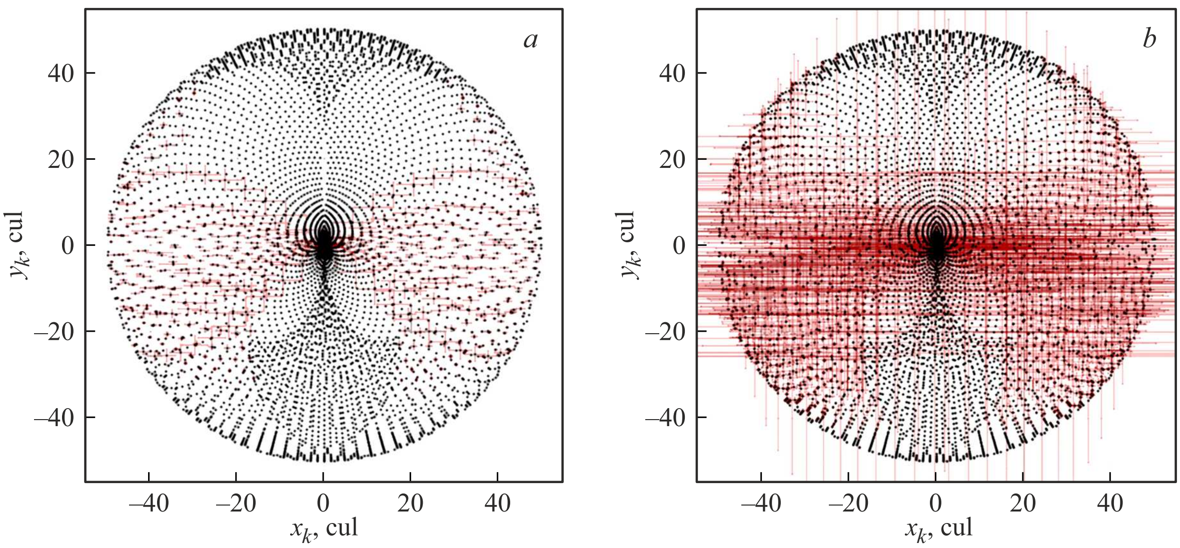

Figure 3. Blurring of gravitational lensing in cosmic plasma. Radiation frequencies — 6 GHz (*a*) and 300 MHz (*b*). Parameters: $\mu_0' = 0.01$, $f_{pl} = 12$ MHz.

effects can undergo stochastic washing out. For an arc localized on the periphery of the ray pattern, the effect will be negligible, unlike in the central focus area. At the same time, the degree of blurring of the radiation pattern also depends on the wavelength of the radiation, the intensity and spatial radii of the correlation of the inhomogeneities of the space environment. These features must be taken into account when interpreting observational data on complex gravitational-lens effects.

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Conflict of interest

The authors declare that they have no conflict of interest.

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