Peculiarities of relativistic rotation of solar system bodies, using some of Jupiter's satellites as an example

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The relativistic effects of geodetic rotation around their axes of 10 Jupiter satellites (J4, J6, J8, J11, J12, J16, J18, J46, J49, and J62) were studied. The difference in the angular velocity vectors of geodetic rotation was demonstrated depending on the choice of coordinate reference system. As a result, the most significant periodic terms of the geodetic rotation (geodetic nutation) of these celestial bodies were determined for the first time. The resulting analytical values of geodetic nutation for the celestial bodies can be used for numerical studies of their rotation in the relativistic approximation.

Keywords: Geodetic nutation, relativistic rotation, Jupiter, regular and irregular satellites.

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The geodetic rotation effect is the most significant effect in the relativistic rotation of celestial bodies around their axis and it consists of two effects: the systematic (or secular) effect — geodetic precession [1] and the periodic effect — geodetic nutation [2].

As is known, the formula of the angular velocity vector of geodetic rotation for any body in the Solar system has the following form [3]:

$$\boldsymbol{\sigma}_i = \frac{1}{c^2}\sum_{j\neq i}\frac{Gm_j}{|\mathbf{R}_i-\mathbf{R}_j|^3}(\mathbf{R}_i-\mathbf{R}_j)\wedge\left(\frac{3}{2}\dot{\mathbf{R}}_i-2\dot{\mathbf{R}}_j\right)$$
$$=\frac{1}{c^2}\sum_{j\neq i}\frac{Gm_j}{|\mathbf{r}_{ji}|^3}\mathbf{r}_{ji}\wedge\left(2\dot{\mathbf{r}}_{ji}-\frac{1}{2}\dot{\mathbf{R}}_i\right), \tag{1}$$

where c is the speed of light in vacuum; G is the gravitational constant; index i corresponds to the studied object of the Solar system, and index j are disturbing bodies (they are the Sun, Moon, Pluto and large planets in this paper); $\mathbf{R}_i$, $\dot{\mathbf{R}}_i$, $\mathbf{R}_j$, $\dot{\mathbf{R}}_j$ are barycentric vectors the positions and velocities of the studied body (Jupiter's moons) and the j-th disturbing body, respectively; m_j is the mass of the j-th body; the symbol $\wedge$ means the vector product; $\mathbf{r}_{ji}=\mathbf{R}_i-\mathbf{R}_j$; $\dot{\mathbf{r}}_{ji}=\dot{\mathbf{R}}_i-\dot{\mathbf{R}}_j$. All values used in formula (1) are taken from the Horizons On-Line Ephemeris System ephemeris [4]. The difference of the velocity vectors on the right side of the vector product of the formula (1) is not symmetric due to the different values of the coefficients in the difference of the barycentric velocity vectors of the investigated and perturbed bodies. Thus, the resulting value of the vector calculated using this formula will depend on the selected coordinate system. For clarity of the above, let us compare the two studied vectors of the angular velocity of geodetic rotation calculated in different coordinate systems — relative to the Solar System Barycenter (SSB) $\boldsymbol{\sigma}_{(\mathrm{SSB})_i}$ and relative to the Jovian System Barycenter (JSB) $\boldsymbol{\sigma}_{(\mathrm{JSB})_i}$. As a result, we get the expression of the difference of these vectors:

$$\boldsymbol{\sigma}_{(\mathrm{SSB})_i}-\boldsymbol{\sigma}_{(\mathrm{JSB})_i}=\frac{1}{2c^2}\sum_{j\neq i}\frac{Gm_j}{|\mathbf{r}_{ji}|^3}\mathbf{r}_{ji}\wedge\dot{\mathbf{r}}_{(\mathrm{SSB})(\mathrm{JSB})}, \tag{2}$$

where $\mathbf{R}_{(\mathrm{SSB})_i}$, $\dot{\mathbf{R}}_{(\mathrm{SSB})_i}$, $\mathbf{R}_{(\mathrm{SSB})_j}$, $\dot{\mathbf{R}}_{(\mathrm{SSB})_j}$, $\mathbf{R}_{(\mathrm{JSB})_i}$, $\dot{\mathbf{R}}_{(\mathrm{JSB})_i}$, $\mathbf{R}_{(\mathrm{JSB})_j}$, $\dot{\mathbf{R}}_{(\mathrm{JSB})_j}$ — barycentric vectors (relative to SSB and JSB) of the positions and velocities of the i-th and j-th body, respectively;

$$\mathbf{r}_{ji}=\mathbf{R}_{(\mathrm{SSB})_i}-\mathbf{R}_{(\mathrm{SSB})_j}=\mathbf{R}_{(\mathrm{JSB})_i}-\mathbf{R}_{(\mathrm{JSB})_j},$$

$$\dot{\mathbf{r}}_{ji}=\dot{\mathbf{R}}_{(\mathrm{SSB})_i}-\dot{\mathbf{R}}_{(\mathrm{SSB})_j}=\dot{\mathbf{R}}_{(\mathrm{JSB})_i}-\dot{\mathbf{R}}_{(\mathrm{JSB})_j},$$

$$\dot{\mathbf{r}}_{(\mathrm{SSB})(\mathrm{JSB})}=\dot{\mathbf{R}}_{(\mathrm{JSB})_i}-\dot{\mathbf{R}}_{(\mathrm{SSB})_i}$$

— velocity vector of the SSB and JSB relative to each other. Hence $\boldsymbol{\sigma}_{(\mathrm{SSB})_i}\neq\boldsymbol{\sigma}_{(\mathrm{JSB})_i}$. The difference formula (2) shows the dependence of the difference in the angular velocity vectors of geodetic rotation on the magnitude of the masses of the disturbing bodies, the distance between the selected coordinate systems and the speed of their motion relative to each other. As a result, for the same body under study in two different coordinate systems, two different vectors are obtained that do not transform into each other by parallel transfer or angular rotations (as happens for angular velocity vectors in Euclidean space), and, consequently, two different values of geodetic rotation are obtained. Thus, our study shows the dependence of the magnitude of the geodetic rotation effect on the choice of the coordinate system based on the type of formula itself. Thus, our study was conducted in relation to these two barycenters. The magnitude of this effect

$$\Delta\mathrm{SJ}=\int\left(|\boldsymbol{\sigma}_{(\mathrm{SSB})_i}|-|\boldsymbol{\sigma}_{(\mathrm{JSB})_i}|\right)dt=\mathrm{P}_{\mathrm{G\,SSB}}-\mathrm{P}_{\mathrm{G\,JSB}}$$

The main secular, periodic, and mixed terms of the geodetic rotation of Jupiter and its moons in the absolute magnitude of the angular rotation vector

	Argument	Harmonic period	Coefficient C at cos(argument), μas	Coefficient S at sin(argument), μas	Spectrum, Sp,$''^2$
Jupiter (B5): $a = 5.2038$ au, $P = 11.862$ yr, $e = 0.049$, $i = 1^\circ.3$, $P_{GSSB} = 0.3117815\,t$					
	λ_5	11.862 yr	$-21.2545-4.5696\,t$	$82.8332-0.1682\,t$	$7.3\cdot10^{-9}$
Metis (J16): $a = 128\,000$ km, $P = 0.295$ days, $e = 0.000$, $i = 2^\circ.2$, $\Delta SJ = 0.2730$					
$P_{GSSB} = 26536.4280292\,t$, $P_{GJSB} = 26536.1550121\,t$					
SSB	$\lambda_{516}-\lambda_5$	0.29480 days	$-15.0686-111.2248\,t$	$467.6472-3.6060\,t$	$2.2\cdot10^{-7}$
	J8	5.931 yr	$228.3097-21.2154\,t$	$269.5940-14.5597\,t$	$1.3\cdot10^{-7}$
	λ_5	11.862 yr	$-39.0834-9.2480\,t$	$149.3536-1.3534\,t$	$2.4\cdot10^{-8}$
JSB	λ_5	11.862 yr	$-28.2690-6.0697\,t$	$110.1806-0.2230\,t$	$1.3\cdot10^{-8}$
Callisto (J4): $a = 1882700$ km, $P = 16.690$ days, $e = 0.007$, $i = 1^\circ.8$, ΔSJ $= -0.0980$					
$P_{GSSB} = 32.179034\,t$, $P_{GJSB} = 32.2770074\,t$					
SSB	$\lambda_{54}-\lambda_5$	16.7536 days	$-0.6246-2.6452\,t$	$121.6603-0.1582\,t$	$1.5\cdot10^{-8}$
	λ_5	11.862 yr	$-21.3556-0.2802\,t$	$83.4659-3.1708\,t$	$7.4\cdot10^{-9}$
	Ω_{54}	559.876 yr	$56.1097-4.5738\,t$	$-63.7232-0.8979\,t$	$7.2\cdot10^{-9}$
JSB	λ_5	11.862 yr	$-28.2464-6.5188\,t$	$110.0464-0.5296\,t$	$1.3\cdot10^{-8}$
Themisto (J18): $a = 7398500$ km, $P = 130.028$ days, $e = 0.340$, $i = 43^\circ.8$, ΔSJ $= 0.0386$					
$P_{GSSB} = 1.5019\,t$, $P_{GJSB} = 1.4633\,t$					
SSB	Ω_{54}	559.876 yr	$3557.9407-31993.1286\,t$	$4734.0932-708.1949\,t$	$3.5\cdot10^{-5}$
	2Jn$_{518}$	210.099 yr	$-1150.8495-5341.0484\,t$	$-662.6680-1473.0951\,t$	$1.8\cdot10^{-6}$
	λ_5	11.862 yr	$2.6368-171.8260\,t$	$89.5481-345.2842\,t$	$8.0\cdot10^{-9}$
JSB	Ω_{54}	559.876 yr	$3849.9434-34759.5963\,t$	$5127.8191-630.0411\,t$	$4.1\cdot10^{-5}$
	2Jn$_{518}$	210.099 yr	$-1248.7294-5789.0359\,t$	$-718.7491-1601.0717\,t$	$2.1\cdot10^{-6}$
	λ_5	11.862 yr	$-2.3989-211.4619\,t$	$114.0998-410.2903\,t$	$1.3\cdot10^{-8}$
Himalia (J6): $a = 11440600$ km, $P = 250.562$ days, $e = 0.160$, $i = 28^\circ.1$, ΔSJ $= -0.0814$					
$P_{GSSB} = 0.669283\,t$, $P_{GJSB} = 0.750727\,t$					
SSB	λ_5	11.862 yr	$-20.5955-0.2714\,t$	$78.9670-11.7096\,t$	$6.7\cdot10^{-9}$
	$\lambda_{56}-\lambda_5$	265.942 days	$-38.5984-0.0464\,t$	$-14.6274-0.1644\,t$	$1.7\cdot10^{-9}$
JSB	λ_5	11.862 yr	$-27.5581-2.9352\,t$	$105.8590-6.6993\,t$	$1.2\cdot10^{-8}$
	2Ja$_{56}$	69.039 yr	$33.1548-23.2843\,t$	$-20.6042-33.4799\,t$	$1.5\cdot10^{-9}$
Carpo (J46): $a = 17042300$ km, $P = 1.2492$ yr, $e = 0.416$, $i = 53^\circ.2$, ΔSJ $= -0.0797$					
$P_{GSSB} = 0.46071\,t$, $P_{GJSB} = 0.54042\,t$					
SSB	2Jn$_{546}$	56.119 yr	$-23.4885-2223.8034\,t$	$-223.7604-275.4034\,t$	$5.1\cdot10^{-8}$
	λ_5	11.862 yr	$-21.3693-32.8235\,t$	$80.6539-5.6939\,t$	$7.0\cdot10^{-9}$
JSB	2Jn$_{546}$	56.119 yr	$-23.4967-2195.0117\,t$	$-220.3481-273.4600\,t$	$4.9\cdot10^{-8}$
	λ_5	11.862 yr	$-27.6233-21.6303\,t$	$109.4420-0.1806\,t$	$1.3\cdot10^{-8}$
Valetudo (J62): $a = 18694200$ km, $P = 1.4445$ yr, $e = 0.217$, $i = 34^\circ.5$, ΔSJ $= -0.0967$					
$P_{GSSB} = 0.41123\,t$, $P_{GJSB} = 0.50797\,t$					
SSB	λ_5	11.862 yr	$-22.2789-4.0809\,t$	$81.5033-4.7185\,t$	$7.1\cdot10^{-9}$
JSB	λ_5	11.862 yr	$-29.4513-5.2190\,t$	$109.3150-4.6221\,t$	$1.3\cdot10^{-8}$

Table (*continued*)

Ananke (J12): $a = 21034500$ km, $P = 1.7243$ yr, $e = 0.237$, $i = 147°.6$, $\Delta SJ = -0.0792$					
$P_{\mathrm{GSSB}} = 0.27421\,t$, $P_{\mathrm{GJSB}} = 0.35337\,t$					
SSB	Ω_{54}	559.876 yr	$-68.1193-407.1962\,t$	$-84.0840-189.6849\,t$	$1.2 \cdot 10^{-8}$
	λ_5	11.862 yr	$-11.4085-43.3038\,t$	$76.4858-271.2264\,t$	$6.0 \cdot 10^{-9}$
JSB	λ_5	11.862 yr	$-25.7977-29.0915\,t$	$109.0988-77.7144\,t$	$1.3 \cdot 10^{-8}$
Carme (J11): $a = 23144400$ km, $P = 2.0101$ yr, $e = 0.256$, $i = 164°.6$, $\Delta SJ = -0.0959$					
$P_{\mathrm{GSSB}} = 0.26117\,t$, $P_{\mathrm{GJSB}} = 0.35704\,t$					
SSB	Ω_{54}	559.876 yr	$-59.4380-229.7027\,t$	$-68.7652-272.3038\,t$	$8.3 \cdot 10^{-9}$
	λ_5	11.862 yr	$-42.9788-1.2298\,t$	$77.4253-173.9092\,t$	$7.8 \cdot 10^{-9}$
JSB	λ_5	11.862 yr	$-42.2075-4.5571\,t$	$110.5616-84.5666\,t$	$1.4 \cdot 10^{-8}$
Pasiphae (J8): $a = 23468200$ km, $P = 2.0359$ yr, $e = 0.412$, $i = 148°.4$, $\Delta SJ = -0.0748$					
$P_{\mathrm{GSSB}} = 0.29046\,t$, $P_{\mathrm{GJSB}} = 0.36529\,t$					
SSB	λ_5	11.862 yr	$-58.6039-1.4186\,t$	$18.0760-16.7520\,t$	$3.8 \cdot 10^{-9}$
JSB	λ_5	11.862 yr	$-44.1149-3.5013\,t$	$82.7846-2.0733\,t$	$8.8 \cdot 10^{-9}$
Kore (J49): $a = 24205200$ km, $P = 2.1267$ yr, $e = 0.328$, $i = 141°.5$, $\Delta SJ = -0.0838$					
$P_{\mathrm{GSSB}} = 0.29053\,t$, $P_{\mathrm{GJSB}} = 0.37433\,t$					
SSB	λ_5	11.862 yr	$-19.4835-230.1387\,t$	$80.0491-21.2041\,t$	$6.8 \cdot 10^{-9}$
JSB	λ_5	11.862 yr	$-28.4329-95.7619\,t$	$109.2469-12.3620\,t$	$1.3 \cdot 10^{-8}$

(see the table) for the studied satellites varies from 0.2730 arcsec/tjy (arcseconds in the Julian millennium) for satellite J16 to -0.0980 arcsec/tjy for satellite J4.

As a result, the most significant periodic terms of geodetic rotation (geodetic nutation) of the studied satellites of Jupiter in the absolute magnitude of the angular rotation vector were determined for the first time

$$|\mathbf{\Lambda}| = \left| \int \boldsymbol{\sigma}\, dt \right|$$

relative to SSB and JSB (see table). The contribution distribution of these harmonics of geodetic nutation is determined by the power spectrum Sp. In the table: t is dynamic barycentric time (TDB), which is measured in Julian millennia (tjy) (365250 jd (Julian days)) from epoch J2000.0; P is the sidereal period of rotation (Jupiter around SSB, moons around JSB); a is the semi-major axis of the orbit of the studied body; i is the inclination of the mean orbit of the studied body of epoch J2000.0 to the ecliptic; e is the eccentricity of the orbit of the studied body (Jupiter around SSB, satellites around JSB); P_{GSSB}, P_{GJSB} is the secular terms of geodetic rotation (geodetic precession) of the studied celestial bodies in the absolute magnitude of their angular rotation vector relative to SSB and JSB, respectively (their values are taken from our previous study [5]); $Sp = C^2 + S^2$ is the harmonic of the power spectrum (at $t = 0$); P_{5j} is the sidereal period j of the Jovian moon, Pa_{5j} and Pn_{5j} is the precession periods of the argument of the pericenter and longitude of the ascending node j of Jupiter's moon, respectively (their values for irregular satellites of Jupiter are taken from the ephemeris Horizons On-Line Ephemeris System [4]); λ_{5j} is the mean Jupiter-centered longitudes of Jupiter's moons;

$$\lambda_{5j} = (360 \cdot 365.25 \cdot 100/P_{5j})T,$$

$$\mathrm{Ja}_{5j} = (360 \cdot 100/\mathrm{Pa}_{5j})T,$$

$$\mathrm{Jn}_{5j} = (360 \cdot 100/\mathrm{Pn}_{5j})T$$

(hereafter T is the dynamic barycentric time (TDB) is measured in Julian centuries (cjy) (36525 jd) from the epoch J2000.0); Ω_{54} is the longitude of the ascending node of the orbit of Callisto on the Laplace plane (the mean orbit of this satellite of the epoch J2000.0); λ_5 is the mean longitude of Jupiter. The mean longitude of Jupiter is taken from Ref. [6], the mean longitudes, J8 and longitudes of the ascending nodes of the regular satellites of Jupiter are taken from the Ref. [7].

The mass of the Sun is dominant in the Solar system, so one of the parts of the angular velocity vector of geodetic rotation $\boldsymbol{\sigma}$ for the satellites of the planets is the result of their orbital motion around SSB. Jupiter and its moons are on mean at the same distance from the Sun and move relative to it at an mean speed. As a result, the values of their geodetic rotation caused by the influence of the Sun should be quite close to each other. Confirmation of this can be seen in the values of the harmonic contributions of the periodic terms of geodetic nutation with the argument λ_5 in the power spectrum Sp (see table), which are also quite close to each other. The harmonic with the argument of the mean longitude of Jupiter λ_5 is the dominant harmonic in terms of the magnitude which is the fundamental harmonic of the geodetic nutation of Jupiter (see table). It reflects its orbit around the SSB, since its moons (compared to Jupiter) have a low mass and have little effect on its geodetic rotation. Therefore, the influence from the Sun is the main influence for the geodetic rotation of Jupiter. The geodetic rotation of Jupiter's moons is determined not only by the Sun, but also by Jupiter. Therefore, the other part of the angular velocity vector of geodetic rotation relative to SSB $\boldsymbol{\sigma}$ for planets with satellites and their satellites is the result of their orbital motion around the barycenter of the satellite system of the planet. Thus, the values of the geodetic nutation of the satellites significantly exceed the similar value of the geodetic nutation of Jupiter (see table). This is due to the fact that, due to the close distance to them, Jupiter has a greater influence on their geodetic rotation than the Sun does on Jupiter. The rotation of Jupiter's moons around the two barycenters JSB and SSB is reflected in the appearance for each regular satellite of two dominant harmonics of geodetic nutation with the argument $\lambda_{5j}-\lambda_5$ and the next largest harmonic with the argument λ_5 (see table). Due to the proximity of the regular satellites to Jupiter, for the angular velocity vector of the geodetic rotation of the satellite under study relative to SSB, the contribution of the harmonics of geodetic nutation with the argument $\lambda_{5j}-\lambda_5$ is the largest (see table). For the angular velocity vector of the geodetic rotation of the satellite under study relative to JSB, the contribution of harmonics of periodic terms of geodetic nutation with the argument $\lambda_{5j}-\lambda_5$ in the power spectrum Sp is much less than the contributions of other harmonics (see table). The table shows that in the periodic terms of the geodetic rotation of the body under study relative to JSB, the disturbance from the Sun is predominant (in them the harmonic with the argument λ_5 dominates), and in the periodic terms of the geodetic rotation of the body under study relative to SSB for regular satellites, the disturbance from Jupiter comes first, and then from the Sun. It should be noted that the J18 satellite has the largest geodetic nutation (of the considered Jupiter moons) of the same order and is comparable to the magnitude of the geodetic nutation of Mercury, the planet closest to the Sun ($\mathrm{Sp} = 3.5 \cdot 10^{-5}$).

Using the example of Solar System bodies, it is shown that the magnitude of geodetic rotation can be significant not only for objects that revolve around supermassive relativistic central bodies, but also for bodies with a small distance to a less massive central body. In our Solar System, one of these objects is the regular moons of Jupiter (for which Jupiter is a less massive disturbing central body than the Sun). In particular, our previous study [5] showed that the magnitude of the geodetic precession of the closest of Jupiter's inner moons — J16 is -26536 arcsec/tjy, which is 90 000 times greater, than Jupiter (-0.3 arcsec/tjy), orbiting its more massive central body (the Sun), and 100 times larger than Mercury (-214 arcsec/tjy), which is closest to the Sun planets. The magnitude of the geodetic precession of the farthest of the Galilean moons of Jupiter — J4 is -32.2 arcsec/tjy, which is 100 times greater than that of Jupiter.

It should be noted that further study of the effects of geodetic rotation and other relativistic effects may be useful in studying the influence of internal physical processes of the studied bodies on their rotations, which may be comparable in magnitude to them. Namely, to separate them from each other.

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Conflict of interest

The authors declare that they have no conflict of interest.

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