12

Box-shaped lens with a low aberration level

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We have found parameters of the box-shaped lens with the low aberration level. A method of phase diagrams was used for analysis of lens properties and creation of an algorithm of iteration procedures. Linearity of transformation of lens trajectories is achieved by varying parameters of elliptic gaps between electrodes and values of electrode potentials. At a lens outlet, a linearly-formed beam has a width of 40 mm and a phase diagram shaped as a rectangle $40\,\mathrm{mm} \times 0.1\,\mathrm{mrad}$.

Keywords: ion optics, emittance, lenses, phase diagrams, aberrations

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Introduction

The present study describes a technology of designing the box-shaped lens with the low aberration level and its operating principle. The lens is designed for use in high-quality mass-spectrometers and mass-separators for isotope separation. A quality of the specified instruments is defined by a level of aberrations in an electrostatic focusing system and by aberrations of dipole magnets.

In a linear approximation, resolution of the magnetic mass-analyzers is determined by a ratio of the magnetic flux along particle trajectories to beam emittance. For this reason, at constant emittance, resolution increases with a width of the beam before the magnet. Therefore, good-quality installations can be created with linear optics for forming wide ion beams and compensated second-order aberrations in magnets. These quadratic aberrations are eliminated when using magnet poles with round boundaries, in which there are no aberrations.

Another method is to use electrostatic correctors described in [1]. This is the cheapest and most accurate method, since it is simpler to manufacture and adjust the corrector than cutting edges of pole tips of a sector magnet.

It is much more difficult to achieve linear focusing of the ion beams in electrostatic lenses. Aberrations are a fundamental limitation in creating any optics and in practice they limit capabilities of beam instruments, especially optics with wide strip particle beams. This case is far away from a paraxial approximation and according to the aberration theory aberrations increase with a deviation of the particle from the beam axis.

Despite great success in the charged-particle optics, currently, we do not know aberration-free electrostatic optical elements. Lenses are described, whose electrodes are various curved surfaces, devices with involvement of current conductors, charged meshes, etc. are tested, but it is still impossible to find a free-aberration configuration. Powerful methods are developed for analysis and calculation of ion-optical systems and their aberrations [2]. But, as a result, knowing aberration values just makes it possible to judge a focusing quality without responding to a question about how to decrease the aberration effect or to make the optical system fully linear.

With the known aberration equations, one can formulate an inverse problem: to find an electrostatic field's distribution that would provide linear focusing. This problem is not solved, but if this solution would exist, then there was a problem of realizing this field by means of electrodes.

1. Lens with the low aberration level

One can try compensate aberrations and many efforts have been made thereto [2]. But it is better to search for optical elements and systems which themselves would provide the required properties at minimum or zero aberrations. Usually, it is sought for by trials and errors when there are common sense, large computing powers and a set of system parameters to be varied in order to obtain an optimal result.

Only one lens type is known for implementing such a program. It is a lens with plane-parallel electrodes separated by curved gaps. This lens is described in [1]. A positive property of this lens is that there are several degrees of freedom when designing the lens. A normal to a curved boundary indicates a direction of the electrostatic field. If this field is directed towards the axis, then this field focuses the particles. This is the case in an accelerating field, when the gap between the electrodes is convexly shaped, i.e. a curvature center is behind the gap between the electrodes along the course of the particle. Or it can be in a decelerating field with a concavely-shaped gap. In this case, the curvature center is in front of the curvilinear gap. In the opposite cases, the particle is scattered. It takes place during acceleration with the concave boundary or deceleration with the convex curvature. The most important fact is that during designing a curvature value can be varied depending on a particle distance from the lens axis, thereby locally changing an optical force. The lens parameters include an electrode length, an accelerating or decelerating electrode potential, two ellipse radii (ellipse half-axes), if the gap between the electrodes is an ellipse arc, and a gap curvature sign. In order to produce the ribbon beam, the particles are first scattered in the lens and then focused into the parallel beam.

The lens described in [1] has a linearly-focusing electrostatic field. It is difficult to realize this field in practice, since technological details are required for assembling and fastening the electrodes. These details introduce distortions to the field distribution, then the charged electrodes create electric fields with vacuum chamber walls or other external subjects and these fields can affect the field inside the electrodes via side gaps in the electrodes.

2. Box-shaped lens and the phase diagrams

The present study considers a disturber free lens that is a lens from [1], which is has side walls. In this case, the lens becomes a box-shaped one. Its view with the ion source and the ribbon beam is shown in Fig. 1. The author does not know description of such lenses in the literature.

The lens parameters are found by a procedure of computer simulation described in [1,3]. The procedure means solving the Laplace equation on a three-dimensional grid. Then, after calculating the distribution of the electrostatic potential, the Runge-Kutta method is used to calculate parameters of the trajectory of the particle. A number of trajectory solution is determined by a desirable level of statistics. Then, the phase diagram is recorded in the selected cross-section of the beam. This diagram is a main tool for analyzing the beam properties and its appearance determines the subsequent iterative actions.

The phase diagrams and their areas called emittance were historically originated in the accelerator physics [4]. Since transverse momentums of particles are small in the accelerators, beams of these particles are well described by the paraxial (i.e., linear) approximation. Due to cyclicity of motion, impact of high-frequency fields on the particles and specific features of the ion sources, the phase diagrams of the beams are elliptic. The ellipses are described by three Twiss parameters. These are sizes of the half-axes and an angle of orientation of the ellipse in a phase space.

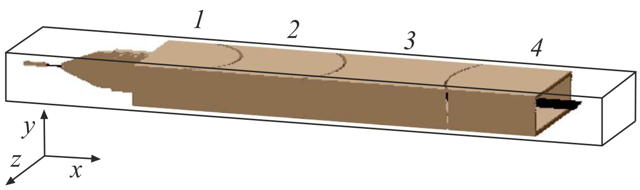

Figure 1. Ion source, the box-shaped lens and the strip ion beam.

But in the ion optics, it is more convenient to represent the phase diagram as a parallelogram [5], since inlet beams formed by means of slits and diaphragms always have the phase diagrams of this kind. Then, the phase diagram of the beam outgoing from a small hole of the ion source is usually shaped as a parallelogram or rhomb as well.

According to the Liouville's theorem, during motion in the electromagnetic fields emittance of the beam is preserved: $\varepsilon = \overline{x}_i \overline{x}'_i \sqrt{\varphi_i} = \text{const}$, here φ_i is a potential in the beam cross-section, while the area of the phase diagram is expressed via average values.

The second important property of the diagram is that phase points on a diagram contour in transformation of the trajectories are still on the contour irrespective of whether the transformation is linear or not.

In the ion optics, in the linear (paraxial) case, the particles emitted from an object point are collected in a point in a Gaussian plane — an image plane. But because of aberrations, the particles create a smeared spot in the image plane. It means that a straight-line portion in the phase space of the object, which reflects trajectories outgoing from the point, becomes a curved line in the phase space of the image.

If the contour of the inlet phase diagram is represented by straight-line portions, then with the linear transformations the outlet diagram also has linear boundaries. In the nonlinear case, the diagram has a curved shape. By measuring the phase diagram in various points of a beam path, one can determine a place of origin of aberrations as well as specify a distance of the trajectories from the axis, at which it is necessary to locally correct focusing, since aberrations manifest themselves as an excess or deficiency of the focusing force.

In [1], the linearly-focusing four-electrode lens has the phase diagram of the ion beam shaped as the rectangle with the sizes $40\,\text{mm} \times 0.1\,\text{mrad}$.

The beam in this work is obtained with the ion source under the potential of 30 kV and the charged ions are drawn by the electrode of the potential of 17.5 kV from a hole of the diameter of 1 mm.

Adding the walls in the lens [1] changes the distribution of the electrostatic field in the external space of the electrodes and the lens losses the linear properties. Its phase diagram is shown in Fig. 2.

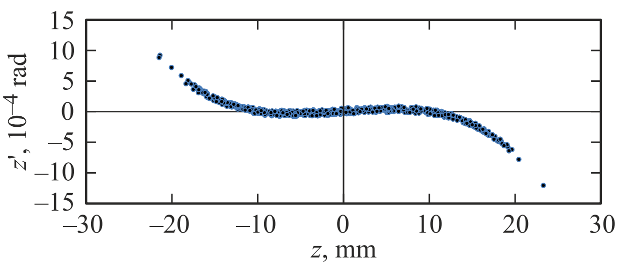

Figure 2. Phase diagram of the beam of the box-shaped lens obtained from the lens [1] by adding the side walls.

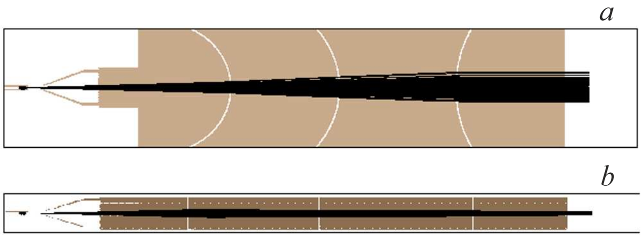

Figure 3. *a* — a horizontal projection of the beam, *b* — a vertical projection.

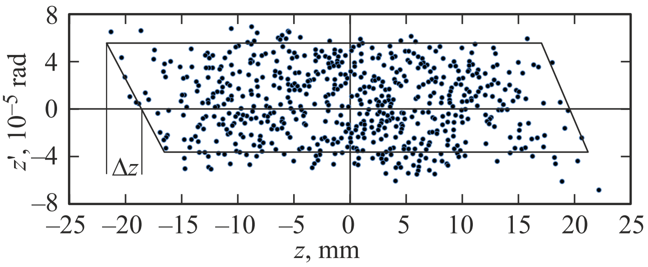

Figure 4. Phase diagram of the beam at the outlet of the box-shaped lens.

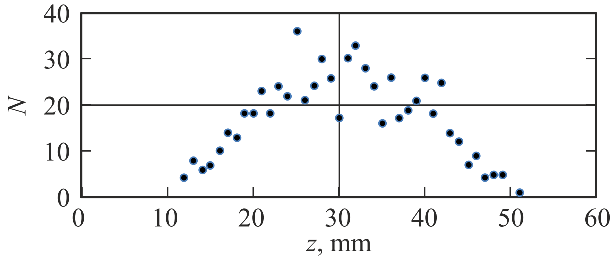

Figure 5. Beam profile.

According to this figure, the side walls sharply increase of focusing of the external trajectories of the particles, indicating that when it is away for more than 10 mm, the focusing field should be reduced. This attenuation is achieved with an increase of the ellipse half-axis r_z, which is transverse to the motion direction. By successive iterations with varying the curvature radii in the elliptic gaps as well as by selecting the potentials for each of the gaps, linearity of the phase diagram can be obtained at the lens outlet.

Horizontal and vertical projections of the beam trajectories are shown in Fig. 3, *a* and *b*, respectively.

The beam is disrupted in the cross-section, where the phase diagrams are measured.

The horizontal phase diagram shaped as the parallelogram is shown in Fig. 4, while the beam profile is shown in Fig. 5.

The beam profile is a result of convolution of data of Fig. 4 by the angle x' .

The number of the particles at the beam edges decreases at both the figures.

The parameters of the box-shaped lens that linearly transforms the beam trajectories are determined in Table.

3. Beam crossovers

Fig. 4, a longer diagonal of the parallelogram is oriented in the second and the fourth quadrant. This indicates that the beam converges and the crossover point is ahead along the beam path. With further propagation of the beam, the diagram will be changed. An upper side of the phase parallelogram will be shifted to the right, so is a lower one to the left so that in a distance $l = \Delta z/z'$ that is approximately 60 m, the parallelogram turns into the rectangle. It reflects the known fact that with linear transformation the crossover is a beam symmetry point.

According to Table, the electric fields in the second and third gaps (Fig. 1) of the lenses are focusing ones (they deviate the particles towards the beam axis).

There is also the second solution, when the first and second gaps scatter the particles and focusing takes place only at the third gap. In doing so, the beam is widened and all the trajectories that are away from the axis for more than 20 mm become nonlinear. Angles of the trajectories directed towards the beam axis quickly increase with an increase of the distance of the particle from the beam axis. This nonlinearity can not be compensated by selecting the ellipse half-axes. For the distances from the axis, which exceed ± 20 mm, sharper attenuation of the gap curvature is required and it even can be with a change of the curvature sign.

The design software programs of the Autocad or SolidWorks type make it possible to realize any type of a curvilinear form of the gap. But other curvilinear forms, except for the elliptic one, have not been considered in in the present work.

A ribbon property of the beam is demonstrated by Fig. 6 depicting the phase diagram in the vertical plane.

According to Fig. 3, *b* and Fig. 6, in the observation point the beam has the size of about 6 mm. The beam diverges (the phase diagram passes through the second and fourth quadrants). After a distance *L*, where the quantity $L = y/\overline{y}'$ represents the average value of the angles of the trajectories when they are deviated by the value y, the beam reaches the crossover position. It occurs in 1150 mm. In this case, the phase diagram is shown in the left lower insert in Fig. 6. The width of the linearly focused beam is 0.6 mm with the base size of 2 mm. At the same time, the base-narrowest beam of 0.9 mm is in front of the crossovers as demonstrated by the right upper insert in Fig. 6. We remind that for the aberration-free beams the narrowest location of the beam coincides with the crossover. For comparison, we indicate that the horizontal width of the beam is 40 mm.

Thus, it is shown in the present work that the box-shaped lenses can form the wide ribbon linearly-focused ion beams. Low angular divergence indicates that these beams can be transported for significant distances.

Aberrations are observed in cases when aberration angles significantly exceed the linear-trajectory slope angles and quickly increase with the deviation from the beam axis. According to Fig. 2 and the lower insert in Fig. 6 the average

Parameters of the box-shaped lens.

Electrode	Potential, kV	Electrode length, mm	Ellipsis radii, the left butt-end of the electrode		Ellipsis radii, the right butt-end of the electrode	
			r_x, mm	r_z, mm	r_x, mm	r_z, mm
1	17.5	165			85	80
2	10	135	87.5	82.5	150	125
3	12.398	144	152.5	127.5	152.5	152.5
4	0	135	150	150		

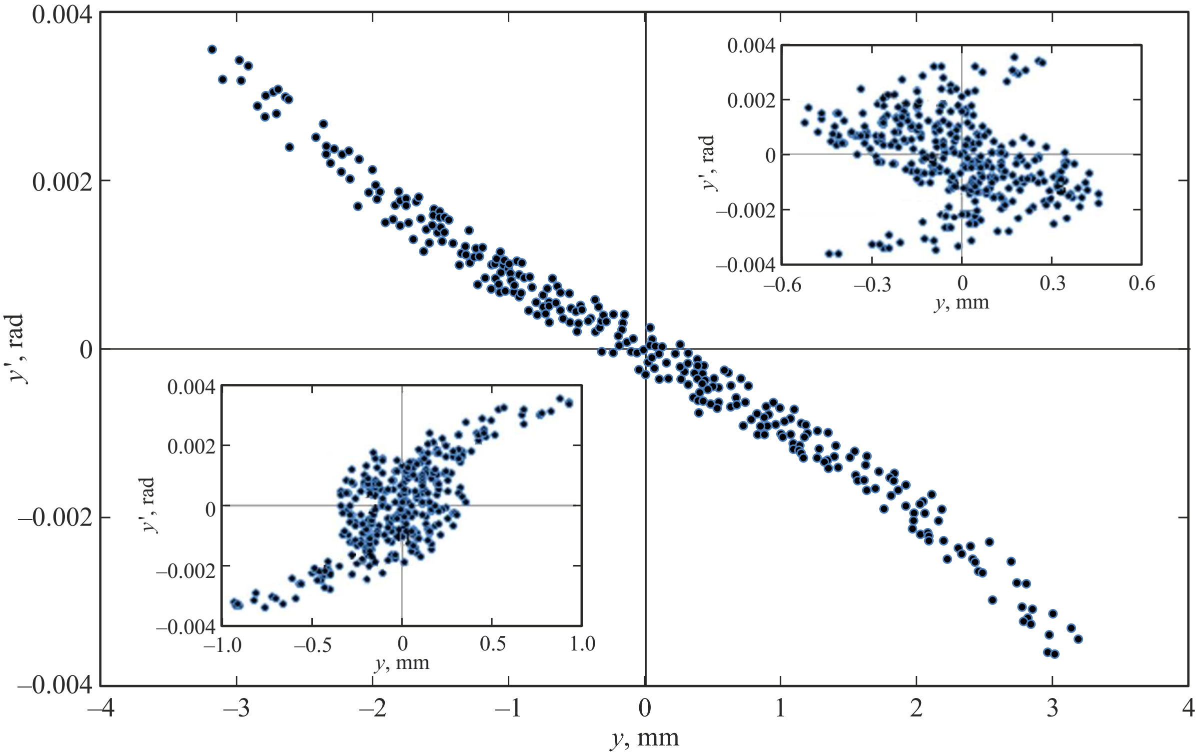

Figure 6. Phase diagram of the beam in the vertical plane. The right insert includes a diagram in front of the crossover, while the left insert includes a diagram in the crossover point.

lines of the phase diagrams in these cases resemble the third-order curves.

Conclusion

Eight aberration coefficients are known in the aberration theory [2], whose combinations determine aberration types, for example, spherical, astigmatism, distortion, etc. In our case, aberrations manifest themselves integrally and a method of their elimination is reduced to local attenuation (or enhancement) of focusing.

Since a density of the particles in the beam is constant, with an increase of the horizontal width of the beam it becomes narrower in the vertical plane and the aberration contributions to vertical motion is reduced. In the crossovers, the size ratio is as follows: $z = 40$ mm and $y = 0.7$ mm.

Let us note that the linear ribbon beam is obtained in the quite compact lens. Its sizes are 587 by 150 mm with an internal space's height of 40 mm.

The box lens has the same linear phase diagram of the beam as in the lens [1]. As shown in [6], with such a lens resolution of the mass-separator increases in several times as compared to operating installations.

In order to increase the beam width above the obtained value of 40 mm, it is necessary to stitch the ellipse arc with a curve that has a decreasing curvature. Such a combined shape of the gap has not been sought for. Another method is to increase the horizontal size of the lens.

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Conflict of interest

The author declares that he has no conflict of interest.

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