Conditions and mechanisms for growth of large-scale magnetic field in outer parts of galactic disks

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The generation and evolution of magnetic fields in spiral galaxies is traditionally described by the dynamo mechanism. Most studies focus on magnetic fields at moderate distances from the center of the galactic disk. However, the existence of a galactic magnetic field in the peripheral regions of the galactic disk is confirmed by numerical models. To estimate the scale of the magnetic field in the outer regions of a spiral galaxy, an eigenvalue problem can be solved based on previously used equations. The first eigenvalues and eigenfunctions for this problem are presented. The eigenvalues can be found both theoretically and numerically.

Keywords: dynamo, galaxies, magnetism, perturbation theory.

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Introduction

Today, the existence of magnetic fields of the value of about $1-3\,\mu$G in some spiral galaxies, such as the Milky Way, is practically undoubted [1]. The earliest evidence of existence of these fields was associated with study of a synchrotron radiation spectrum and spatial distribution of cosmic rays. Presently, the galactic magnetic fields are studied mainly by measuring the Faraday rotation of a radiowave polarization plane. The first studies dedicated to the Milky Way included analysis of data on radiation from several dozen pulsars [2], while to data more than a thousand sources of the magnetic field for our Galaxy [3] and millions of extragalactic objects [4] are known.

Occurrence of the large-scale magnetic fields is usually described by a dynamo mechanism that is closely related to helicity of turbulent motions of an interstellar medium and differential rotation of the galactic disk, whose simultaneous presence can result in an exponential increase of the field [5]. At the same time, turbulent diffusion, being the main dissipative process, tends to destroy regular field structures. Thus, the magnetic field can be generated only when the dynamo effect is quite intense to resist turbulent diffusion [6]. Usually, these conditions are fulfilled at a relatively short distance to the galaxy center (up to 10 kpc) and most studies anyhow assumed that the magnetic fields existed only in internal parts of the galaxy, whereas it was still unclear whether the field existed at the large distance from the center of the galactic disk.

It was evidently shown by computation research related to investigation of generation and evolution of the magnetic fields at the distances of up to $15-20$ kpc from the galaxy center that despite the fact that the field had a substantially lower value in those parts, it still could be present at such peripheries [7]. The magnetic field can also increase even when a value of the dynamo number is below or comparable with its critical values, so that at first sight the field increase shall be suppressed by dissipative effects according to estimates. In these studies, the main attention is focused on the increase of the magnetic field due to nonlinear transfer of the magnetic field by means of a mechanism similar to one in the Kolmogorov–Petrovsky–Piskunov [8].

At the same time, though it may seem strange, generation of the magnetic field in the outer parts can be described by means of linear effects as well. The matter is that it is necessary to consider a problem of generation of the magnetic field not in a local meaning, but by means of differential operators that have smooth eigenfunctions, which correspond to profiles of the magnetic fields. Although, they shall decrease in the outer parts, where the dynamo effect is attenuated, they are still finite there. The main subject of the present study is an issue of how they can look like at the large distance from the center.

1. Equations for the magnetic field

Let us consider the galaxy in a thin disk approximation: since a half-thickness of the galactic disk is significantly less than its radius, it can be considered that the galaxy is a quite flat disk in order to reduce the problem to a plane problem. Let us also consider an axisymmetrical case since large-scale field structures tend to get to an axisymmetrical form as previously shown [9]. The equations for the magnetic field will be the following [10]:

$$\partial Br/\partial t = -\,\Omega(r)l^2B_\varphi/h^2(r) - \eta\pi^2Br/4h^2(r) + \eta(\partial^2Br/\partial r^2 + \partial Br/r\partial r - Br/r^2),$$

$$\partial B_\varphi/\partial t = B_r r d\Omega/dr - \eta\pi^2B_\varphi/4h^2(r) + \eta(\partial^2B_\varphi/\partial r^2 + \partial B_\varphi/r\partial r - B_\varphi/r^2),$$

where $\Omega(r)$ — differential rotation, η — turbulent diffusion that reflects dissipation in the disk plane, $h(r)$ — its half-thickness with taking into account disk expansion [11] and a function $\Omega(r) = V_0/r$ is also introduced.

In order to study excitability of the magnetic field in the outer parts, we considered the following initial and boundary conditions:

$$B_{r|r=0} = B_{\varphi|r=0} = B_{r|r=r_{\max}} = B_{\varphi|r=r_{\max}} = 0,$$

where $r_{\max}$ is a certain quite large value of the distance from the center (for example, for the galaxies that are similar to the Milky Way, it can be assumed that $r_{\max} = 20\,\text{kpc}$).

First of all, for spectral analysis, it is necessary to formulate the problem as two separate, mutually uncoupled equations. Let us introduce symmetrical replacements for this [12]:

$$y = B_r(-rd\Omega/dr)^{1/2} - B_\varphi(\Omega(r)l^2/h^2(r))^{1/2},$$

$$z = B_r(-rd\Omega/dr)^{1/2} + B_\varphi(\Omega(r)l^2/h^2(r))^{1/2}.$$

Let us note that the field components are easily expressed by means of the following relationships:

$$B_r = (y+z)(-rd\Omega/dr)^{-1/2}/2,$$

$$B_\varphi = (z-y)(\Omega(r)l^2/h^2(r))^{-1/2}/2.$$

By substituting the said replacements into the initial equations and after some algebraic transformations, one can obtain two indistinctly interrelated equations, which can be solved separately within the framework of the present problem due to proportionality of the field components B_r and B_φ:

$$\partial z/\partial t = -z(A_1(r) + A_2(r)) + \eta(\partial^2z/\partial r^2 + \partial z/r\partial r - z/r^2),$$

$$\partial y/\partial t = y(A_1(r) - A_2(r)) + \eta(\partial^2y/\partial r^2 + \partial y/r\partial r - y/r^2),$$

where the functions $A_1(r) = (-rd\Omega/dr)^{1/2}(\Omega(r)l^2/h^2(r))^{1/2}$ and $A_2(r) = \eta\pi^2/4h^2(r)$ are introduced for conveniently and shortly writing the equations.

For each of the two equations, a dependence on the variable t is exponential, i.e.

$$z(r,t) = z_0(r)\exp(p_z t),$$

$$y(r,t) = y_0(r)\exp(p_y t).$$

Then, it can be stated that $z'(r,t) = p_z z(r,t)$ and, similarly, $y'(r,t) = p_y y(r,t)$ and the eigenproblems will be written as

$$p_z z = -z(A_1(r) + A_2(r)) + \eta(\partial^2z/\partial r^2 + \partial z/r\partial r - z/r^2),$$

$$p_y y = y(A_1(r) - A_2(r)) + \eta(\partial^2y/\partial r^2 + \partial y/r\partial r - y/r^2).$$

2. Approximations for the eigenvalues

We will seek for the eigenvalues by means of the perturbation theory. To begin with, we will consider a differential operator, which will be assumed to be unperturbed, and find exact expressions for its unperturbed eigenvalues. For example, we will consider the equation with the variable z, then we will do the same for the equation with the variable y. The problem with the unperturbed operator (we will take a radial part of the Laplace vector operator for this) will be written as

$$\partial z/\partial t = \eta(\partial^2z/\partial r^2 + \partial z/r\partial r - z/r^2).$$

Let us substitute $\partial z/\partial t = p_z z_0(r)\exp(p_z t)$ and perform a number of simple algebraic transformations. In this case, we obtain the equation

$$r^2z_0''(r) + rz_0'(r) + z_0(r)(-p_z r^2/\eta) - z_0(r) = 0.$$

Let us introduce the substitution $x = r(-p_z/\eta)^{1/2}$, then it can be expressed $r = x(-\eta/p_z)^{1/2}$. Therefore,

$$z_0'(r) = dz_0/dx \cdot dx/dr = (-p_z/\eta)^{1/2} \cdot dz_0/dx;$$

$$z_0''(r) = -p_z/\eta \cdot d^2z_0/dx^2.$$

And after substitution we obtain

$$x^2 \cdot d^2z_0/dx^2 + x \cdot dz_0/dx + (x^2-1)z_0 = 0.$$

A solution of this equation is the first-order Bessel function. In this case, the exact solution of the unperturbed problem will be written as follows

$$z_0(r) = ZJ_1(r(-p_z/\eta)^{1/2}).$$

Zeros of the Bessel function are determined from the relationship

$$J_{1,n}(x) = (2/\pi x)^{1/2}\cos(x - 3\pi/4) = 0,$$

$$r_{\max}(p_{zn}/\eta)^{1/2} = \pi/4 + \pi n,$$

$$p_{zn} = -\eta(\pi/4 + \pi n)^2/r_{\max}^2.$$

For the second problem, the unperturbed eigenvalues and the eigenfunctions will be similar

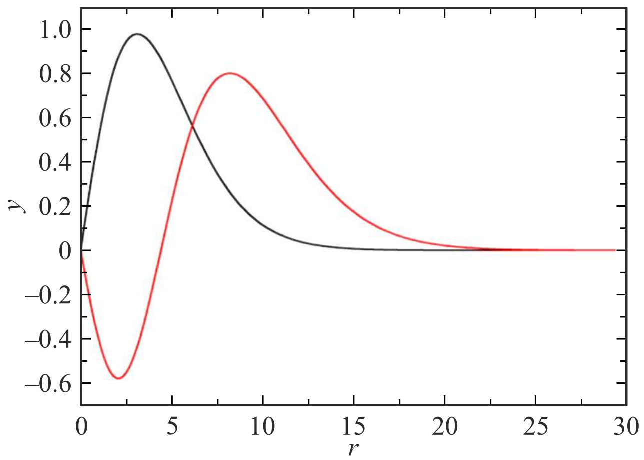
$$y_0(r) = YJ_1(r(-p_y/\eta)^{1/2}),$$

$$p_{yn} = -\eta(\pi/4 + \pi n)^2/r_{\max}^2.$$

3. Perturbation theory

The other parts of the equations will act as perturbations. In order to calculate the first-order perturbations, it is necessary to calculate the integrals [13]:

$$\Delta p_{zn}^{(1)} = \Big(-\int_r (J_1(r(-p_{zn}/\eta))^2(A_1(r) + A_2(r))rdr\Big) \Big/ \Big(\int_r (J_1(r(-p_{zn}/\eta))^2 rdr\Big),$$



Eigenfunctions corresponding to the following eigenvalues: $p1 = 2.34$ (black), $p2 = 1.62$ (red).

$$\Delta p_{yn}^{(1)} = \Big(\int_r (J_1(r(-p_{yn}/\eta))^2(A_1(r) - A_2(r))rdr\Big) \Big/ \Big(\int_r (J_1(r(-p_{yn}/\eta))^2 rdr\Big).$$

Thus, for each equation, with taking into account the first-order perturbations, a senior eigenvalue can be calculated as follows:

$$P_{z1} = p_{z1} + \Delta p_{zn}^{(1)},$$

$$P_{y1} = p_{y1} + \Delta p_{yn}^{(1)}.$$

Based on the obtained eigenvalues, it can be concluded that the equation with the variable z corresponds to a decreasing field component and is undescriptive at the large distances. Further on, we will focus our attention on solving the second equation.

4. Numerical study

Let us use a reciprocal power method for solving the eigenproblem. Its essence is to multiply affecting an arbitrary function y_0 by the operator $(L - pI)^{-1}$, while having approximate eigenvalues. After obtaining the respective eigenfunction in this way, we will find the eigenvalue by the formula

$$P = < y, Ly > / < y, y >$$

The eigenvalues obtained in this way are: $P_{y1} = 2.34$, $P_{y2} = 1.62$ and $P_{y3} = -0.84$. The results of calculations are shown in the figure.

Results

Thus, it can be assumed that the third and subsequent harmonics will not contribute to field generation at the periphery of the galactic disk, thereby allowing us further on limiting ourselves with the two first harmonics for further describing the weak magnetic fields that originate in the galactic disk. Nevertheless, in addition to the dynamo, there can also be other mechanisms contributing to generation of the magnetic field at the remote distances from the center of the galactic disk.

Conflict of interest

The authors declare that they have no conflict of interest.

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