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On the stability of orbital dynamics of exoplanet satellite candidates

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By calculating the Lyapunov characteristic exponents for a number of exoplanetary systems in which satellites (exomoons) potentially exist or have already been identified, orbital dynamics stability diagrams were constructed and studied in detail. Estimates of the Lyapunov times (the time of predictable dynamics) in the orbital dynamics of exomoons and possible values of the orbital parameters of candidate exomoons are obtained.

Keywords: exoplanets, planetary satellites, celestial mechanics, stability of motion, Lyapunov exponents.

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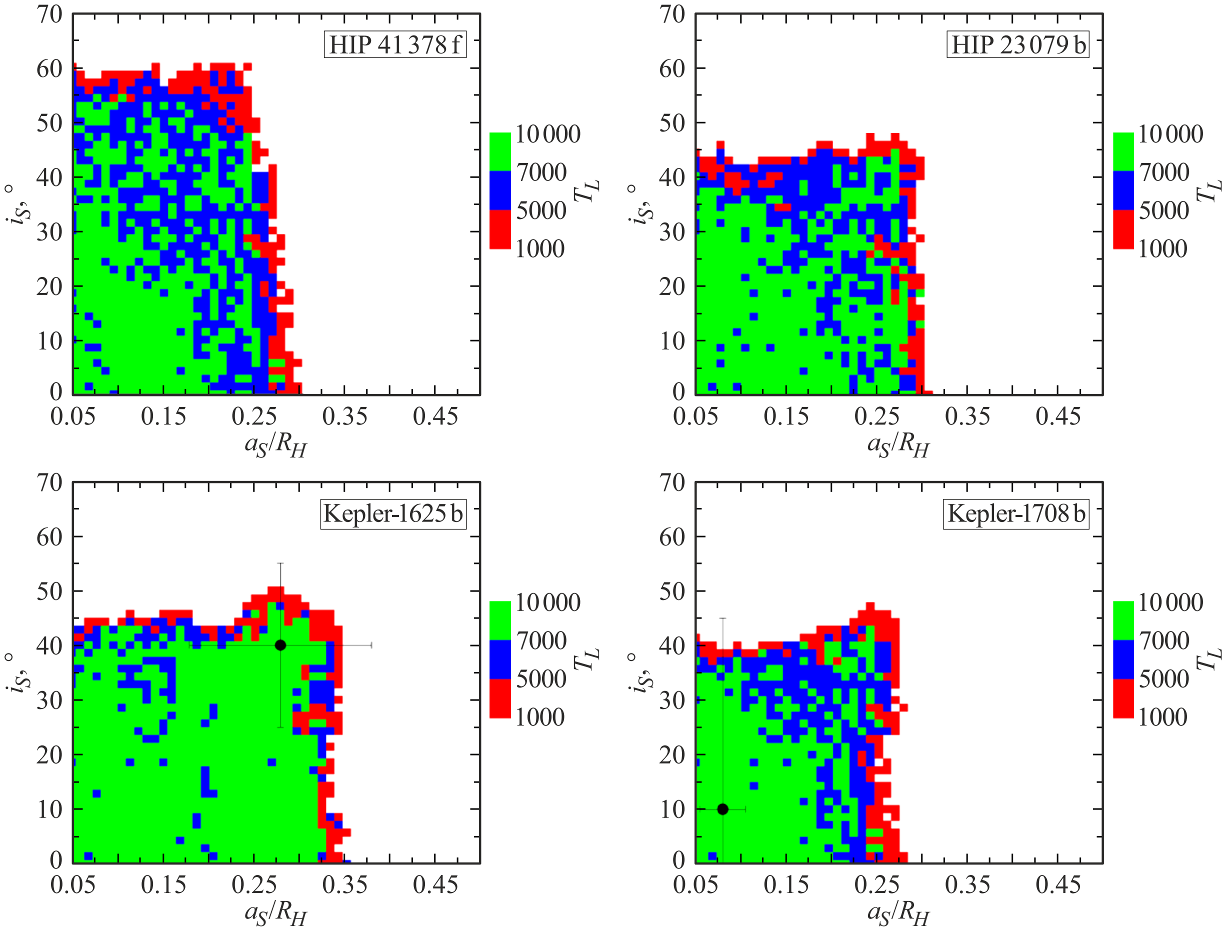
All the planets of the Solar System, with the exception of Mercury and Venus, have satellites, and the number of satellites in the giant planets ranges from several dozen to two hundred. Currently, more than 6,000 planets (exoplanets) have been discovered other stars. There is no doubt that most exoplanetary systems have satellite subsystems [1,2]. A significant part of the exoplanets are gas giants. A number of them are located in the region of potential habitability near the parent star, therefore, the natural satellites of exoplanets (exomoons) may have conditions suitable for the existence of life [2,3]. The search and study of the dynamics of exomoons is a laborious, but very important and urgent task. The active development of observation methods in the last decade has made it possible to identify a number of candidates for exomoons. The first candidate for an exomoon of the planet Kepler-1625b was presented in Ref. [4]. Later, eight planetary systems were identified in Ref. [5] in which there are observational signs of the presence of satellites. Recently, the possibility of the existence of a satellite of the planet Kepler-1708b was noted Ref. [6]. The possibility of the existence of satellites in the planets HD 23079b and HIP 41378f is considered in Ref. [7,8].

The use of numerical methods for studying the stability of the motion of celestial bodies (see for more details [9]) makes it possible to evaluate the possibility of detecting exomoons from the analysis of observations and to obtain/refine information about the orbital and physical parameters of the identified candidates for exomoons. The long-term orbital dynamics for a satellite that actually exists in a planetary system must be stable. Having determined the boundaries of the regions of its stable dynamics for a planetary system based on the set of possible parameter values and initial conditions for the proposed satellite subsystem (by constructing stability diagrams), it is possible to evaluate the possibility of detecting exomoons using modern observational means. By noting the position of the exomoon candidates detected from the analysis of observations on the stability diagrams, it is possible to establish the reliability and estimate the error of the values of the orbital parameters assumed for it.

The stability of the dynamics of a number of the above-mentioned candidates for exomoons was studied in Ref. [10–13]. The stability of the long-term orbital dynamics of the exomoons is studied in this paper by calculating the maximum Lyapunov characteristic exponents (MLCE). This method, unlike others widely used in the problem under consideration, [10,12,13] methods of stability analysis (calculation of MEGNO, estimation of the maximum attainable value of the eccentricity of the exomoon during evolution, etc.), allows us to strictly estimate Lyapunov times [14] and has already been successfully used by us earlier [11]. We would like to remind that the LCE represent the average exponential divergence rate of the phase space trajectories of a dynamical system that are close (according to initial conditions) (see for more details [9,14]). A Hamiltonian system with N degrees of freedom has $2N$ LCE. A non-zero value of the maximum LCE (L) indicates a chaotic, and a zero value indicates a stable character of the motion. We made a conclusion about the stability of the motion of the candidates for the exomoons listed in the table based on the results of numerical integration and estimates of the values

Parameters of planetary systems used in numerical modeling: M is the mass of the star in the masses of the Sun, M_p is the mass of the planet in the masses of Jupiter, a_p, e_p is the semi-major axis and the eccentricity of the orbit of the planet, m_S is the mass of the proposed satellite of the planet (exomoon) in the masses of the Earth. According to [7,8,13]

The planet	M	M_p	a_p, AU	e_p	m_S
HIP 41378f	1.16	0.038	1.370	0.004	0.15
HD 23079b	1.01	2.41	1.586	0.087	1.0
Kepler-1625b	1.04	11.6	0.863	0.0	10.22
Kepler-1708b	1.09	4.6	1.640	0.40	17.15



Lyapunov times (in years) for the orbital dynamics of the considered exomoons (see table), calculated on the set of possible values of the semi-major axis a_S (in the radii of the Hill sphere R_H of the parent planet) and the inclination of the orbit i_S. The white color corresponds to unstable orbits. Black dots with error bars, according to Ref. [13], the position of candidates for exomoons is marked.

of the Lyapunov time (the time of predictable dynamics of the system), for which [15] $T_L = 1/L$ is assumed. For more information on methods for obtaining numerical and analytical estimates of the MLCE in problems on the dynamics of satellite systems and in general in celestial mechanics, see [9, 14].

The integration of the equations of motion describing the dynamics of three bodies (a star, a planet, and an exomoon) was carried out on a set of initial values of the orbital parameters of the exomoon, set at nodes of a uniform grid of size 50×50 on the plane (a_S, i_S), where a_S and i_S is the semi-major axis and the inclination of the exomoon's orbit to the plane of the planet's orbit. Three values of the eccentricity of the exomoon's orbit were considered: $e_S = 0, 0.05$ and 0.1, it was accepted: $0.05 \le a_S/R_H \le 0.5$ and $0^\circ \le i_S \le 70^\circ$, where the radius of the Hill sphere of the planet R_H was determined by $R_H = a_p((M_p + m_S)/(3M))^{1/3}$. The accepted values for the parameters of planetary systems are given in the table. It was assumed that at the initial moment of time the exomoon and the planet were located in the pericenters of their orbits, zero values were assumed for unknown orbital elements (arguments of the pericenters and longitude of the ascending nodes) (see also [7,8]). Numerical integration was performed over a time interval of 10^5 years. Double precision numbers were used. During integration, the DOP853 [16] integrator was used, which implements an explicit Runge-Kutta method of the 8th order with an automatically variable value of the integration step. The maximum integration step was assumed to be $\Delta t_{\max} = 10^{-2}$ years, the value of the local (at one step) error was set to $\varepsilon = 10^{-12}$. Based on the value of ε, the DOP853 integrator automatically selects the required integration step value, which does not exceed $\Delta t_{\max}$ (see for more information [16]).

The orbit of the exomoon was considered unstable, and numerical integration stopped if there was a close approach/collision with the planet, or the exomoon left the Hill sphere of the planet. The fact of a close approach/collision of an exomoon with a planet was recorded when a_S decreased to a value equal to the radius of the planet, or when the relative energy of the „planet-satellite“ $\Delta E \ge 10^{-7}$ system changed (see the discussion in Ref. [11]). It should be noted that the change in energy $\Delta E \le 10^{-12}$ in the case of stable dynamics during integration over the time interval of 10^5 years. If the integration was successfully completed when the specified time interval was reached, then the conclusion about the stability of the orbital dynamics of the exomoon was made based on the Lyapunov time T_L determined for it. It was believed that the orbits of exomoons with $T_L \geq 5000$ years are stable (see details in Ref. [11]). Next, the boundaries of the stability region were determined on the plane (a_S, i_S), the position of candidates for exomoons (if known) on it was considered, and the relative proportion of trajectories occupied by them on the constructed two-dimensional diagram was estimated.

The figure shows examples of stability diagrams obtained at $e_S = 0$ for exomoon candidates in planetary systems listed in the table. In all cases, the relative proportion of stable trajectories in the constructed diagram was about 25–40 %, which is consistent with the results in Ref. [8,11,13]. With an increase in e_S, the proportion of stable orbits of exomoons decreased and did not exceed 15–25 % at $e_S = 0.1$. For $a_S/R_H > 0.4{-}0.5$ and $i_S > 60°$, stable orbits were not found in the diagrams we constructed (see the discussion in Ref. [10,13]). This can be explained by the fact that at high inclinations of the exomoon's orbit, the Lidov-Kozai resonance begins to play an essential role in its dynamics, leading to unstable motion. If we compare the sizes of the stability regions shown in the figure with the estimates obtained in Ref. [10,13], it should be noted (see also [11]) that when using the stability criterion based on the value of the MLCE, the upper limit in terms of a_S is less for all the considered exomoons. From the diagrams shown in the figure, it can be seen that in the Kepler-1625b system, the candidate for an exomoon is located in the region of stable motion near the stability boundary, and in the Kepler-1708b system, the exomoon is located far enough away from areas with unstable dynamics. No exomoons have yet been identified in the HIP 41378f and HD 23079b systems, but the diagrams show that the stability regions on the plane (a_S, i_S) are quite large (especially for i_S in the case of HIP 41378f), and in these planetary systems there may be satellites with the considered parameters (masses m_s).

So, the method proposed in Ref. [11] is developed in this paper for constructing stability diagrams of satellite subsystems of exoplanets by calculating the LCE. Stability diagrams with significantly (several times) higher resolution than in Ref. [11] have been constructed and analyzed for a number of planetary systems (HIP 41378f, HD 23079b, Kepler-1625b, Kepler-1708b) in which satellites potentially exist or have already been identified. It is shown that the dynamics of exomoon candidates in the planetary systems Kepler-1625b and Kepler-1708b is stable. The possibility of exomoons in HIP 41378f and HD 23079b systems has been confirmed and estimates of their possible orbital parameters have been obtained.

Conflict of interest

The author declares that he has no conflict of interest.

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