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Theoretical model for photonic topological insulator based on a cubic lattice of bianisotropic resonators

© *A.D. Rozenblit, N.A. Olekhno*

ITMO University,
197101 St. Petersburg, Russia
e-mail: alina.rozenblit@metalab.ifmo.ru

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In the present work, we propose a theoretical model to describe a cubic lattice of resonators with bianisotropic response based on dyadic Green's function approach. The effects of the bianisotropy parameter value on the dispersion diagram are investigated and topological properties of the system are considered by calculating the Berry curvature distributions for three different planes.

Keywords: topological insulators, bianisotropy, dyadic Green's function, Berry curvature.

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Introduction

Photonic topological insulators are arrays of coupled resonators that ensure the existence of edge states — the propagation or localization of electromagnetic energy along the boundaries of the structure. The existence of such a wave regime is governed by symmetry properties of the system bulk and is characterized by unidirectionality and the absence of scattering by geometric defects [1].

One of the ways to implement a photonic topological insulator is based on the use of resonators, the geometry of which is characterized by the absence of inversion center. A breaking of the spatial inversion symmetry of the resonator leads to the formation of two hybrid modes corresponding to the bianisotropic response instead of the electric and magnetic dipole modes of the original resonator with unperturbed symmetry [2,3]. In turn, the introduction of bianisotropy leads to the opening of a band gap in which pseudospin-polarized topological edge states exist, with the direction of propagation of which is strictly related to the sign of the pseudospin, which is an analog of the spin-orbit interaction [2,4]. For example, topological insulators based on a triangular lattice have been proposed that are characterized by Dirac dispersion [3,5], as well those based on structures with symmetry C_{4v}, with quadratic degeneracy of eigenmodes [6].

This paper is devoted to the development of a theoretical model based on the dyadic Green's function method for describing a topological insulator consisting of bianisotropic particles located at the nodes of a cubic lattice. We calculated the Berry curvature for three orthogonal directions of the lattice to study the topological properties of the system under consideration. Previously, a theoretical description of such systems was proposed within the framework of perturbation theory [7], applicable only in the vicinity of high symmetry points and not including the study of topological properties, which requires the calculation of the Berry curvature in the entire Brillouin zone.

1. Derivation of the effective Bloch Hamiltonian

Let us consider a cubic lattice with a period of a, at the nodes of which point electric $p_{x(y)}$ and magnetic $m_{x(y)}$ dipoles oriented along the $x(y)$ axis (Fig. 1) are located, corresponding to the simultaneous excitation of electric and magnetic dipoles in the xy plane of a dielectric resonator consisting of two concentric cylinders of different sizes, the axis of which coincides with the z axis [3,6,8]. At the same time, the geometry of these resonators ensures the equality of magnitudes of the electric and magnetic responses necessary for the degeneration of eigenmodes at high symmetry points in the absence of bianisotropy [2], therefore, we will consider the amplitudes of electric and magnetic fields at a given node to be equal.

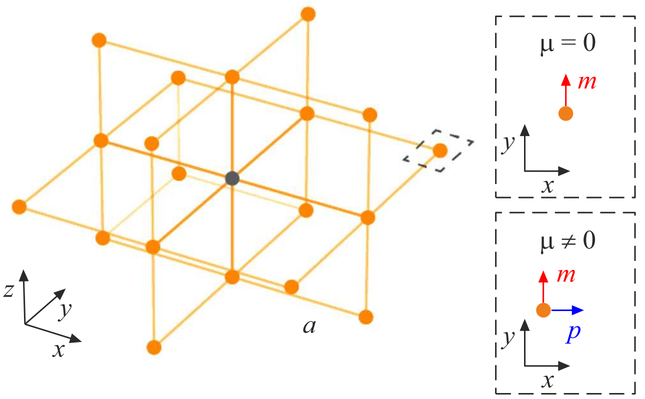

Figure 1. Cubic lattice with a period of a, with bianisotropic particles located in the nodes. The parameter μ characterizes the coupling strength of the electric p and magnetic m dipole moments.

The components of the electric E and magnetic H fields at the node with coordinates (ia, ja, ka) are related to dipole moments by the polarizability tensor $\hat{\alpha}$ (in the CGS symmetric system of units):

$$\begin{pmatrix} p_x^{ijk} \\ p_y^{ijk} \\ m_x^{ijk} \\ m_y^{ijk} \end{pmatrix} = \hat{\alpha} \begin{pmatrix} E_x^{ijk} \\ E_y^{ijk} \\ H_x^{ijk} \\ H_y^{ijk} \end{pmatrix} = \begin{pmatrix} \beta & 0 & 0 & i\chi \\ 0 & \beta & -i\chi & 0 \\ 0 & i\chi & \beta & 0 \\ -i\chi & 0 & 0 & \beta \end{pmatrix} \begin{pmatrix} E_x^{ijk} \\ E_y^{ijk} \\ H_x^{ijk} \\ H_y^{ijk} \end{pmatrix},$$

where β is polarizability and χ is electromagnetic coupling. Then the components of the electric and magnetic fields at a given node are determined by the following expression:

$$\begin{pmatrix} E_x^{ijk} \\ E_y^{ijk} \\ H_x^{ijk} \\ H_y^{ijk} \end{pmatrix} = \hat{\alpha}^{-1} \begin{pmatrix} p_x^{ijk} \\ p_y^{ijk} \\ m_x^{ijk} \\ m_y^{ijk} \end{pmatrix} = \begin{pmatrix} u & 0 & 0 & -iv \\ 0 & u & iv & 0 \\ 0 & -iv & u & 0 \\ iv & 0 & 0 & u \end{pmatrix} \begin{pmatrix} p_x^{ijk} \\ p_y^{ijk} \\ m_x^{ijk} \\ m_y^{ijk} \end{pmatrix}, \tag{1}$$

where $u = \beta/(\beta^2 - \chi^2)$ and $v = \chi/(\beta^2 - \chi^2)$. Let there be only one resonant state with a frequency ω_0 in the frequency range where hybridization of electric and magnetic dipole moments is observed. Then the parameter u can be represented as an approximating function $u = (\omega - \omega_0)/C$, where C is a constant. To make the parameters u and v dimensionless, we introduce the parameters $\mu = va^3$ and $\lambda = a^3(\omega - \omega_0)/C$ [8].

On the other hand, the field amplitudes at a given node can be found as the sum of the fields created by all other point dipoles expressed using the dyadic Green's function $G(r, k_0) = G$:

$$\begin{pmatrix} E_x^{ijk} \\ E_y^{ijk} \\ H_x^{ijk} \\ H_y^{ijk} \end{pmatrix} = \sum_{\substack{m \neq i, \\ n \neq j, \\ l \neq k}} \begin{pmatrix} G_{xx}^{ee} & G_{xy}^{ee} & G_{xx}^{em} & G_{xy}^{em} \\ G_{yx}^{ee} & G_{yy}^{ee} & G_{yx}^{em} & G_{yy}^{em} \\ G_{xx}^{me} & G_{xy}^{me} & G_{xx}^{mm} & G_{xy}^{mm} \\ G_{yx}^{me} & G_{yy}^{me} & G_{yx}^{mm} & G_{yy}^{mm} \end{pmatrix} \begin{pmatrix} p_x^{mnl} \\ p_y^{mnl} \\ m_x^{mnl} \\ m_y^{mnl} \end{pmatrix}. \tag{2}$$

The dyadic Green's function is determined by the distance between the point dipoles

$$r = a\sqrt{(m-i)^2 + (n-j)^2 + (l-k)^2}$$

and the wave number k_0. According to the assumptions introduced, the electric and magnetic components of the dyadic Green's function coincide, $G^{ee} = G^{mm}$. In addition, the electromagnetic and magnetoelectric components are related as ratio $G^{em} = -G^{me}$. The components of the dyadic Green's function are defined by the following expressions (in the CGS symmetric system of units):

$$G_{\xi\eta}^{ee} = (\partial_\xi \partial_\eta + k_0^2 \delta_{\xi\eta}) \frac{e^{ik_0 r}}{r},$$

$$G_{\xi\eta}^{em} = \operatorname{rot} G_{\xi\eta}^{ee} = ik_0 \varepsilon_{\xi\eta z} \partial_z \frac{e^{ik_0 r}}{r},$$

where $\varepsilon_{\xi\eta z}$ is the Levi–Civita symbol, $\delta_{\xi\eta}$ is the Kronecker delta and $\partial_\xi = \partial/\partial_\xi$. The expressions obtained for the components of the dyadic Green's function in the quasi-static approximation ($k_0 = 0$) have the following form:

$$G_{xx}^{ee} = \left(3a^2(m-i)^2/r^2 - 1\right)/r^3,$$

$$G_{yy}^{ee} = \left(3a^2(n-j)^2/r^2 - 1\right)/r^3,$$

$$G_{xy}^{ee} = 3a^2(m-i)(n-j)/r^5,$$

$$G_{xx}^{em} = G_{xy}^{em} = G_{yx}^{em} = G_{yy}^{em} = 0.$$

To simplify the derivation, we will take into account only the couplings between neighbors in the first and second coordination spheres. According to Bloch theorem, the dipole moments at the nodes of a cubic lattice with coordinates (ia, ja, ka) and (ma, na, la) are related by a phase multiplier

$$\left(p_x^{ijk}, p_y^{ijk}, m_x^{ijk}, m_y^{ijk}\right)^T = e^{-ik_x(i-m) - ik_y(j-n) - ik_z(k-l)} \times \left(p_x^{mnl}, p_y^{mnl}, m_x^{mnl}, m_y^{mnl}\right)^T,$$

where k_x, k_y, and k_z are the wave vectors along the axes x, y, and z. Combining the equations (1) and (2), rewriting the result as an eigenvalue problem $\hat{H}|\psi\rangle = \lambda|\psi\rangle$, and taking into account Bloch theorem and the expressions obtained for dyadic Green's function, we obtain the effective Bloch Hamiltonian $\hat{H}$ in the basis $|\psi\rangle = (p_x, p_y, m_x, m_y)^T$. Let us change the basis to pseudospin states $|\psi'\rangle = (p_x + m_x, p_y + m_y, p_x - m_x, p_y - m_y)^T$ [3] by converting $\hat{H}' = U\hat{H}U^\dagger$, where the matrix U has the following form:

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}.$$

Finally, the effective Bloch Hamiltonian in the pseudospin basis $|\psi'\rangle$ is defined as:

$$\hat{H}' = \begin{pmatrix} b & i\mu - d & 0 & 0 \\ -i\mu - d & c & 0 & 0 \\ 0 & 0 & b & -i\mu - d \\ 0 & 0 & i\mu - d & c \end{pmatrix} = \begin{pmatrix} \hat{H}^\uparrow & 0 \\ 0 & \hat{H}^\downarrow \end{pmatrix},$$

$$b = 4\cos k_x - 2\cos k_y - 2\cos k_z + \frac{1}{\sqrt{2}}(\cos k_x \cos k_y + \cos k_x \cos k_z - 2\cos k_y \cos k_z),$$

$$c = -2\cos k_x + 4\cos k_y - 2\cos k_z + \frac{1}{\sqrt{2}}(\cos k_x \cos k_y + \cos k_y \cos k_z - 2\cos k_x \cos k_z),$$

$$d = \frac{3}{\sqrt{2}} \sin k_x \sin k_y,$$

where $\hat{H}^\uparrow$ and $\hat{H}^\downarrow$ denote the pseudospin parts of the Bloch Hamiltonian with opposite polarization.

2. Dispersion curves and topological properties

The dispersion curves of the obtained Bloch Hamiltonian $\widehat{H}'$ for three different values of the bianisotropy parameter μ are shown in Fig. 2 and are described by the following expression:

$$\lambda_{1(2)}^{\uparrow(\downarrow)} = \cos k_x + \cos k_y - 2\cos k_z + \frac{1}{2\sqrt{2}}\left(2\cos k_x \cos k_y - \cos k_x \cos k_z - \cos k_y \cos k_z\right) \pm \frac{1}{4\sqrt{2}}\Big(32\mu^2 + 342 + 117\cos(2k_y) + 144\sqrt{2}\cos k_z + 72\sqrt{2}\cos(2k_y)\cos k_z + 18\cos(2k_z) + 9\cos(2k_y)\cos(2k_z) - 36\cos k_x \cos k_y\left(17 + 8\sqrt{2}\cos k_z + \cos(2k_z)\right) + 9\cos(2k_x)\left(13 + 4\cos(2k_y) + 8\sqrt{2}\cos k_z + \cos(2k_z)\right)\Big)^{1/2}.$$

In the absence of bianisotropy ($\mu = 0$), the dispersion is characterized by quadratic degeneracy at high symmetry points $\Gamma(0,0,0)$ and $M(\pi,\pi,0)$. In addition, degeneracy is observed at point $A(\pi,\pi,\pi)$, between points $\Gamma(0,0,0)$ and $Z(0,0,\pi)$ and between points $M(\pi,\pi,0)$ and $A(\pi,\pi,\pi)$. The degeneracy is removed with the introduction of bianisotropy, and an increase in the value of the parameter μ is accompanied by an increase in the band gap. Indeed, at the point $\Gamma(0,0,0)$, the expression for eigenvalues takes the form $\lambda_{1(2)}^{\uparrow(\downarrow)} = \pm\mu$, which emphasizes the crucial role of bianisotropy in the opening of the band gap.

To study the topological properties of a cubic lattice of bianisotropic particles, the distributions of the Berry curvature (which is an analog of the magnetic field in the reciprocal space [9]) were calculated for three different planes using the formula

$$\Omega_\xi(k_\zeta, k_\eta, k_\xi = 0) = \frac{\partial}{\partial k_\zeta}\left\langle \psi_{1(2)}^{\uparrow(\downarrow)} \middle| \frac{\partial}{\partial k_\eta} \middle| \psi_{1(2)}^{\uparrow(\downarrow)} \right\rangle - \frac{\partial}{\partial k_\eta}\left\langle \psi_{1(2)}^{\uparrow(\downarrow)} \middle| \frac{\partial}{\partial k_\zeta} \middle| \psi_{1(2)}^{\uparrow(\downarrow)} \right\rangle,$$

where $\xi \neq \zeta \neq \eta$ take values to $[x; y; z]$, the upper index of the eigenfunction ψ indicates the pseudospin part of the Hamiltonian $\widehat{H}^{\uparrow(\downarrow)}$, and the lower index indicates the branch of the eigenvalues. The Berry curvature for all planes vanishes in the absence of bianisotropy ($\mu = 0$). When bianisotropy is introduced ($\mu = 10$), a nonzero local and opposite-sign distribution of Berry curvature is observed Ω_z in the vicinity of points with coordinates (0,0) and (π,π), as shown in Fig. 3. In this case, the values of the Berry curvature distribution change their sign when the direction of the pseudospin or branch of the eigenstates change. The Berry curvature in the other two planes Ω_x and Ω_y retains its trivial properties even with a nonzero bianisotropy parameter μ. Thus, the cubic lattice under consideration is an example of a weak topological insulator with nontrivial topological properties along the z axis.

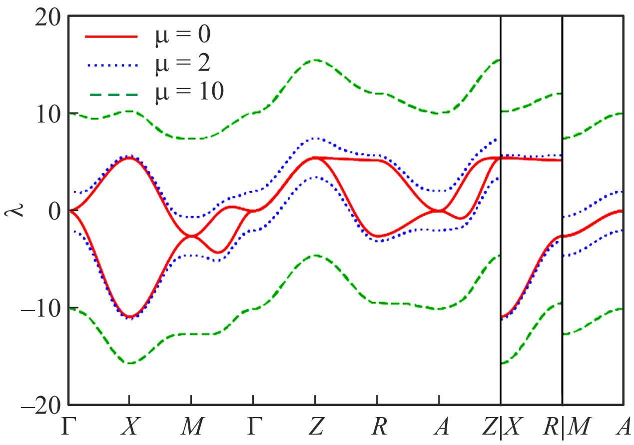

Figure 2. Dispersion curves of the effective Bloch Hamiltonian $\widehat{H}'$ in the first Brillouin zone of a cubic lattice. The solid, dotted, and dashed curves correspond to three different values of the bianisotropy parameter.

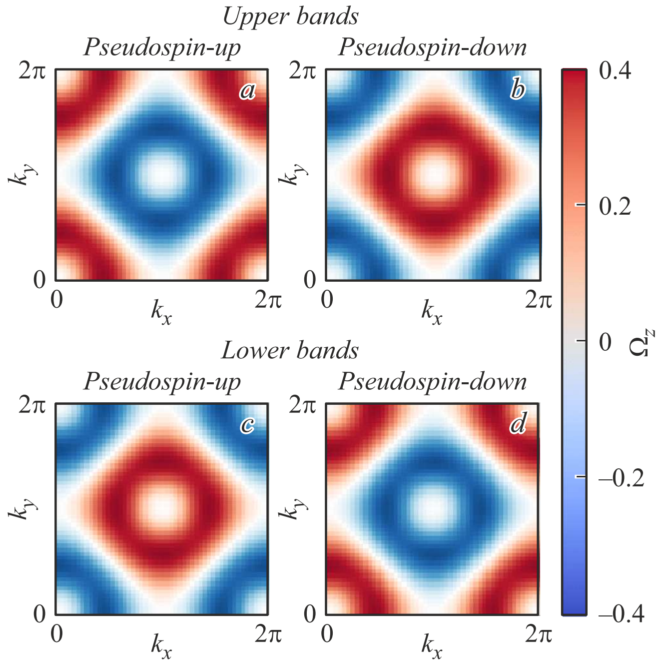

Figure 3. Berry curvature distributions Ω_z in the xy plane for the upper (*a*,*b*) and lower (*c*,*d*) branches of eigenstates corresponding to pseudospin-up $\widehat{H}^{\uparrow}$ and pseudospin-down $\widehat{H}^{\downarrow}$ parts of the Hamiltonian $\widehat{H}'$ for the value of the bianisotropy parameter $\mu = 10$.

Conclusion

A theoretical model is proposed for describing a cubic lattice of bianisotropic resonators based on the dyadic

Green's function and taking into account the couplings between nodes in the first and second coordination spheres. The crucial role of bianisotropy in the opening of the band gap and the emergence of nontrivial topological properties is demonstrated, as follows from Berry curvature distributions. The proposed model can be used to describe arrays of bianisotropic resonators with the size, as well as the distance between the nearest resonators, being much smaller than the wavelength of electromagnetic radiation in the considered frequency range.

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Conflict of interest

The authors declare that they have no conflict of interest.

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