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Theory of Flash Memory Based on a Two-Dimensional Wigner Cluster

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Modern flash memory operates through the confinement (trapping) of electrons at defect sites within a dielectric layer, which modulates the resistance of a two-dimensional semiconductor channel. In this work, we numerically investigate structural ordering — Wigner crystallization of localized electrons in deep trapping sites. The angular distribution function of these electrons confirms their Wigner localization.

Keywords: Wigner cluster, flash memory, localized electrons, angular distribution function.

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1. Introduction

Wigner crystallization of free electrons was theoretically predicted in 1934 by Wigner [1]. It is caused by Coulomb repulsion of free electrons and is observed in conditions when a potential energy of the electrons exceeds their kinetic energy. Wigner crystallization of free electrons above the surface of liquid helium was predicted in [2] and experimentally observed in [3,4]. Wigner crystallization of free electrons and holes in an inversion layer of a semiconductor was theoretically predicted in [5].

Wigner crystallization of electrons was experimentally observed in the heterostructure $GaAs/Al_xGa_{1-x}As$ [6]. Wigner crystallization of one-dimensional free electrons was observed in carbon nanotubes [7]. Two-dimensional (2D) Wigner crystals were studied in a 2D gas of free electrons in the magnetic fields [8,9] and observed in moire superlattices of transition metal dichalcogenides [10–14]. The two-dimensional Wigner crystals of free electrons were theoretically investigated in (see [15] and references therein), in particular, finite-dimension crystals [16–22].

Amorphous silicon nitride Si_3N_4 has a high concentration of electron and hole traps and is used as a storage medium in modern Charge Trap Flash Memory (CTFM) [23,24]. Carriers that are localized on the traps in Si_3N_4 induce (logical 1) or do not induce (logical 0) a conduction inversion channel in silicon.

The higher the concentration of the charged traps in Si_3N_4, the higher a difference between the logical zero and unit — a memory window. The study [25] has put forward a hypothesis on Wigner crystallization of electrons that are localized on the traps in a dielectric. By means of computational modeling in a periodic lattice [26], it was shown that electrons that are localized on the traps in the two-dimensional dielectric formed an ordered structure.

Due to miniaturization of the flash memory, it is interesting to consider two-dimensional Wigner clusters (2DWC) with a small number of electrons. It is important to note that neutral traps that are responsible for a memory effect in Si_3N_4 are randomly distributed in space. However, due to presence of Coulomb repulsion of the localized electrons, the latter form an ordered structure and, when taking into account a disorder, they form a Wigner glass. Competition between Coulomb interaction and the disorder that is induced by random distribution of impurities was previously investigated in the study [27]. Similar systems were studied within the framework of an Efros–Shklovskii Coulomb-gap model [28–30]. Pinning of one-dimensional and two-dimensional Wigner crystals with the disorder was investigated in several studies (see [30] and references therein). Unlike the mentioned studies, our task is to determine a ground state of 2DWC of electrons arranged on the traps. We pay attention to a transition between an ordered state at the low electron concentration and random arrangement of electrons on the traps, when their concentrations are commensurable.

Electrons in a flash-memory dielectric are localized on deep traps. The number of these traps is high (in 100 times) as compared to the number of captured electrons. When electrons are arranged, they tend to minimize Coulomb interaction energy. Thus, a cluster structure is determined by minimality of the Coulomb energy provided that the electrons are randomly arranged on the traps. It creates competition of the disorder in arrangement of the traps and ordering due to Coulomb interaction. A disorder degree is determined by a density ratio of neutral and electron-charged traps. Moreover, finity of a cluster size results in the influence of the full number of electrons on their ordering [18–22].

In the present paper we study the 2DWC of electrons that are localized on random traps in the dielectric. We

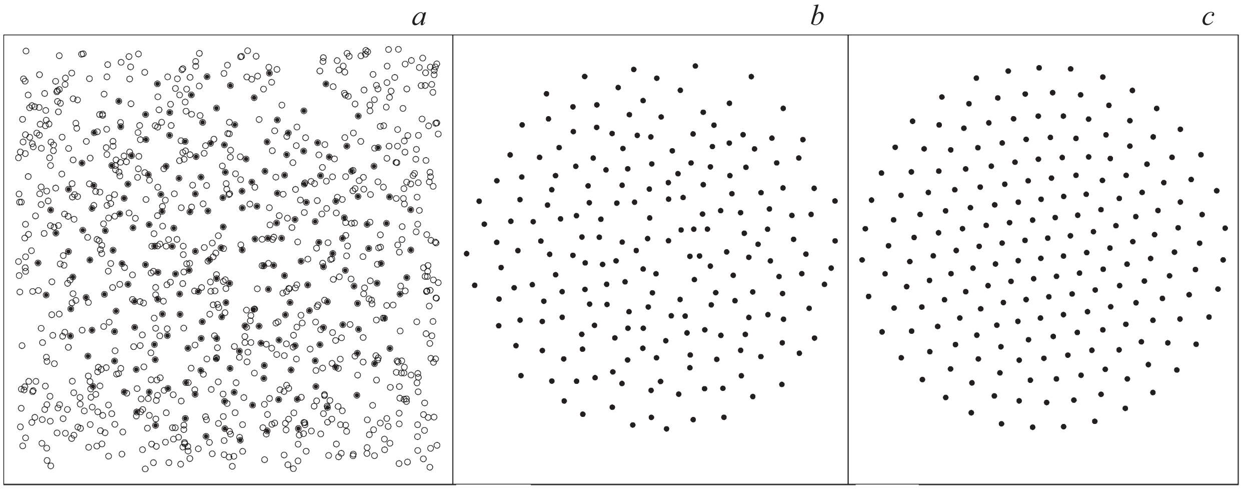

Figure 1. a — the system of 1000 traps (circles) with 200 localized electrons (dots), whose position corresponds to a minimum of the full energy of the system; b — the cluster formed by 200 localized electrons; c — the 2DWC of 200 free electrons.

use computational modeling for the study. Further on, we will consider the problem classically. Quantum states on the traps can be reduced to this problem when neglecting a size of the state and potential multi-charged traps.

The main task is to investigate equilibrium configurations of the localized electrons, their spatial distribution and phase state (the Wigner crystal, glass) in a dependence on system parameters: stiffness of a parabolic potential, the concentration of the traps and electrons. An important role is played by competition between a tendency of the parabolic potential to focus electrons in the system center and Coulomb repulsion, which, on the contrary, facilitates their uniform (homogeneous) distribution. This competition results in formation of inhomogeneous charge distribution that can demonstrate various types of ordering in a dependence on the parameters.

2. 2DWC of electrons localized on the traps

The dielectric is a two-dimensional system in the plane (x, y), and it has neutral traps randomly distributed, which are free locations for arranging electrons therein. The electrons introduced into the system are localized at random positions of the traps $\mathbf{r}_i$, while the size of the state is neglected. At the same time, both multicharged traps and uncaptured electrons are excluded (the latter are quickly thermalized and captured). Electron interaction is determined by Coulomb repulsion as well as by an external parabolic potential created by an electrode field. The parabolic potential is written as $U(\mathbf{r}) = kr^2/2$, $(k > 0)$, k is stiffness of the parabolic potential, r is a distance from the system center. Coulomb interaction between the electrons is described by the expression $V = \sum_{i>j} e^2/\varepsilon|\mathbf{r}_i - \mathbf{r}_j|$, where e is the electron charge, ε is permittivity of the surrounding medium. The energy of the electrons in the system, which is measured in energy units $E_0 = (ke^4/2\varepsilon^2)^{1/3}$, is written as $E = \sum_{i>j} 1/|\mathbf{r}_i - \mathbf{r}_j| + \tilde{k} \sum_i r_i^2$, where $\tilde{k} = (\sqrt{2}\varepsilon/e^2)^{2/3}$.

The equilibrium configurations of the electrons localized on the traps, which correspond to the minimum of their full energy, are found by means of computational modeling. The results are analyzed by constructing angular electron distribution f as well as a spatial configuration of the electrons, thereby making it possible to clearly observe formation of ordered or disordered structures.

The results of computational modeling, which demonstrate capturing of the electrons in random traps, are shown in Figure 1, *a*. Figure 1, *b* highlights a cluster formed by these electrons. The 2DWC of the free electrons is shown in Figure 1, *c*. In case of capturing of the electrons in the random traps and for the case of the free electrons, parameters of the holding potential are set identically. Radii of the clusters in the first and second cases turn out to be the same, although the ordered structure is lost due to random arrangement of the traps.

Figure 2, *a–c* shows structures of the 2DWC of free electrons. Our calculations have showed that the electrons localized on the traps form similar ordered structures (Figure 2, *d–f*). The higher the ratio of trap number to the captured electron number, the closer the obtained structures to the structure of the 2DWC. In our case, this ratio is 100 : 1. Unlike the cluster of free electrons, the electrons localized on the traps are slightly disordered (Figure 2, *e*). With an increase of the number of the electrons, there is less freedom for their arrangement and minimization of the full energy results in formation of the disordered 2DWC (Wigner glass) (Figure 1, *a, b* and Figure 2, *f*).

Figure 3 demonstrates an angular correlation in the electron arrangement. It shows distribution of mutual angles („valence angles") between vectors that connect neighboring electrons (up to 6). It is clear to see an angle periodicity with a period of $\pi/3$, which corresponds to

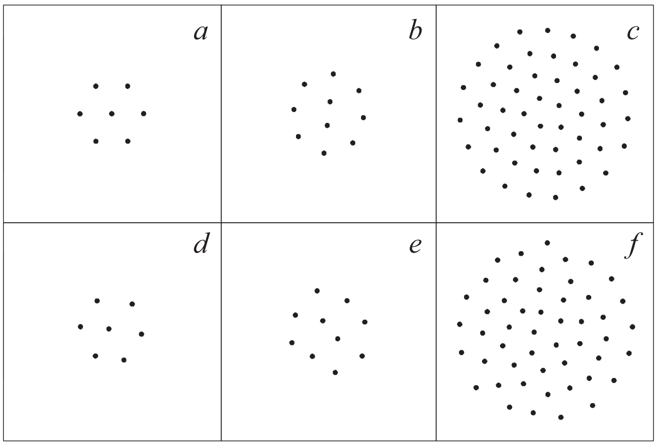

Figure 2. 2DWC structures formed by 7 (*a*), 10 (*b*) and 50 (*c*) free electrons and the 2DWC structures formed by 7 (*d*), 10 (*e*) and 50 (*f*) electrons localized on the 5000 randomly arranged traps.

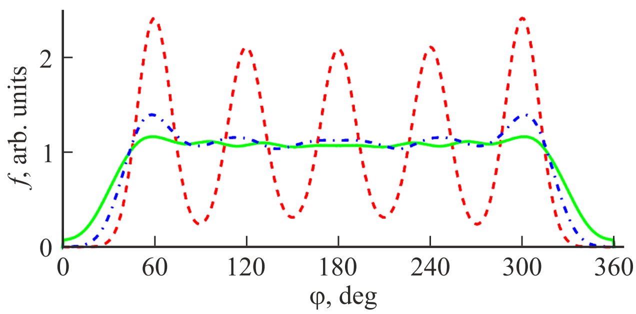

Figure 3. Angular distribution function f for the nearest neighbors of the electrons of the 2DWC of 200 free electrons (the red dashed line) and the system of 200 electrons captured in 1000 (the green solid line) and 5000 (the blue dash-dotted line) traps. An angle in degrees is along the horizontal axis, while arbitrary units are along the vertical axis.

the 6 nearest neighbors. Usually, due to dense packing of the electrons they have a hexagonal surrounding. We note that with introduction of the traps the angular correlation is preserved better than the spatial one. It is due to insensitivity of the angular correlation to the cluster density.

3. Discussion

The obtained equilibrium configurations of the localized electrons indicate formation of structures that are similar to the Wigner cluster of free electrons that form the hexagonal lattice. At the same time, a similarity degree increases with the increase of the ratio of the number of the traps to the number of the captured electrons. Unlike the Wigner cluster of free electrons, with the low ratio of these numbers, the electrons that are localized on the traps form an unstrictly periodic, disordered lattice — the Wigner glass.

4. Conclusion

We have traced the transition from the state of the Wigner cluster to the disordered Wigner glass, in which the correlation is preserved only in the first coordination sphere. We have obtained the dependence of the angular distribution function and a Coulomb interaction force.

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Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] E. Wigner. Phys. Rev. **46**, 1002–1011 (1934).
[2] H.M. Van Horn. Phys. Rev. **157**, 342–349 (1967).
[3] R.S. Crandall, R. Williams. Phys. Lett. A **34**, 404–405 (1971).
[4] C. Grimes, G. Adams. Phys. Rev. Lett. **42**, 795–798 (1979).
[5] A.V. Chaplik. Sov. Phys. JETP **35**, 395 (1972).
[6] E.Y. Andrei, G. Deville, D.C. Glattli, F.I.B. Williams, E. Paris, B. Etienn. Phys. Rev. Lett. **60**, 2765–2768 (1988).
[7] V.V. Deshpande, M. Bockrath. Nat. Phys. **4**, 314–318 (2008).
[8] V. Goldman, M. Santos, M. Shayegan, J. Cunningham. Phys. Rev. Lett. **65**, 2189–2192 (1990).
[9] H. Zhou, H. Polshyn, T. Taniguchi, K. Watanabe, A.F. Young. Nat. Phys. **16**, 154–158 (2020).
[10] H. Li, Sh. Li, E.C. Regan, D. Wang, W. Zhao, S. Kahn, K. Yumigeta, M. Blei, T. Taniguchi, K. Watanabe, S. Tongay, A. Zettl, M.F. Crommie, F. Wang. Nature **597**, 650–654 (2021).
[11] E.C. Regan, D. Wang, Ch. Jin, M.I.B. Utama, B. Gao, X. Wei, S. Zhao, W. Zhao, Z. Zhang, K. Yumigeta, M. Blei, J.D. Carlström, K. Watanabe, T. Taniguchi, S. Tongay, M. Crommie, A. Zettl, F. Wang. Nature **579**, 359–363 (2020).
[12] Y. Xu, S. Liu, D.A. Rhodes, K. Watanabe, T. Taniguchi, J. Hone, V. Elser, K.F. Mak, J. Shan. Nature **587**, 214–218 (2020).
[13] X. Huang, T. Wang, Sh. Miao, Ch. Wang, Z. Li, Z. Lian, T. Taniguchi, K. Watanabe, S. Okamoto, D. Xiao, S.-F. Shi, Y.-T. Cui. Nat. Phys. **17**, 715–719 (2021).
[14] H. Li, Z. Xiang, A.P. Reddy, T. Devakul, R. Sailus, R. Banerjee, T. Taniguchi, K. Watanabe, S. Tongay, A. Zettl, L. Fu, M.F. Crommie, F. Wang. Science **385**, 86–91 (2024).
[15] M. Mazars. EPL **110**, 26003 (2015).
[16] C. Yannouleas, U. Landman. Phys. Rev. Lett. **82**, 5325 (1999).
[17] R. Egger, W. Häusler, C.H. Mak, H. Grabert. Phys. Rev. Lett. **82**, 3320 (1999).
[18] V.M. Bedanov, F.M. Peeters. Phys. Rev. B **49**, 2667 (1994).
[19] V.A. Schweigert, F.M. Peeters. Phys. Rev. B **51**, 7700 (1995).
[20] Mahmood M. Mahmoodian, Mehrdad M. Mahmoodian & M.V. Entin. Theory of a Two-Dimensional Rotating Wigner Cluster. JETP Letters **115**, *10*, 608–614 (2022).
[21] Mehrdad Mahmoodian, M.V. Entin. J. Phys.: Conf. Ser. **2227**, 012012 (2022).

[22] Mahmoodian Mehrdad. M., Mahmoodian M.M., Entin M.V. Spin structure and spin magnetic susceptibility of two-dimensional Wigner clusters. Physics of the Solid State **65**, *10*, 1694–1700 (2023).
[23] V.A. Gritsenko. In: Thin Films on Silicon: Electronic and Photonic Applications. World Scientific Press (2016). Ch. 6, p. 273.
[24] A. Goda. Electronics **10**, 3156 (2021).
[25] V.A. Gritsenko. JETP Lett. **64**, 525–530 (1996).
[26] S.S. Shaimeev, V.A. Gritsenko, H. Wong. Appl. Phys. Lett. **96**, 263510 (2010).
[27] R. Chitra, T. Giamarchi, P. Le Doussal. Phys. Rev. B **65**, 035312 (2001).
[28] A.L. Efros, B.I. Shklovskii. J. Phys. C: Solid State Phys. **8**, *4*, L49 (1975).
[29] A.L. Efros. J. Phys. C: Solid State Phys. 9, 2021 (1976).
[30] B.I. Shklovskii. Low Temperature Physics 50 (12), 1101–1112 (2024).

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