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Integral cross sections of quasi-elastic scattering of neutrinos on nucleons

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Within the framework of the Geant4 software package, the cross section of quasi-elastic scattering of neutrinos and antineutrinos from nucleons was simulated. The simulation results are compared with experimental data. The parameters of the nucleon form-factors used in the formalism of quasi-elastic neutrino scattering from nucleons are discussed.

Keywords: neutrino, antineutrino, quasi-elastic scattering, cross section, simulation, Geant4.

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The currently conducted and planned neutrino experiments aimed at exploring physics beyond the Standard Model need including simulation of neutrino interactions with electrons and nuclei. Studying quasi-elastic reactions and their partial contributions to the total neutrino-nuclear cross section is of great importance for precise determination of the neutrino oscillation parameters in investigation with muon neutrino beams having energies of about several GeV in both the long-baseline MINOS/MINOS+ [1], NOvA [2], and T2K [3] experiments and short-baseline MiniBooNE experiments [4]. This is also important for the next-generation neutrino experiments DUNE [5], Hyper-Kamiokande [6] whose potential defines the horizons of searching for new physics in the Standard Model neutrino sector [7].

At present, there are available several software packages that describe the generation of quasi-elastic neutrino interactions, such as Fluka, GENIE, NEUT or GiBUU (see review [8] and references therein). However, those packages allow modeling interactions of neutrinos with matter only without accounting for the processes of neutrino generation. Package Geant4 possesses this capability.

Software package Geant4 [9] is designed to simulate the passage of penetrating radiation through materials organized into complex geometric structures. Geant4 is applicable in areas such as high-energy physics, nuclear physics, accelerator technology, and also in medicine and space research. Historically, Geant4 was intended to simulate electromagnetic and strong interactions, while simulation of weak interactions was limited to describing weak decays (muons, pions, kaons, etc.) and weak electron capture by nuclear protons (nuclear neutronization). Neutrinos generated in these processes propagated further without interaction.

The advantage of including neutrino interactions in Geant4 is the ability to perform full-scale (rather than fragmentary) simulation of neutrino propagation from the generation point to that of interaction. Recently, the first cases of considering neutrino interactions within the Geant4 package have been reported [10,11].

In this paper, we consider cross sections of quasi-elastic interactions between neutrinos and nucleons by using more accurate parameterizations of the electric and magnetic nucleon form-factors than those in [11]. We also present the results of comparing the calculated cross sections with experimental data.

In calculating cross sections of quasi-elastic neutrino-nucleon interactions, the following reactions are considered, $l = (e, \mu, \tau)$:

$$\nu_l + n \to l^- + p, \qquad (1)$$

$$\bar{\nu}_l + p \to l^+ + n. \qquad (2)$$

Description of the cross section of quasi-elastic neutrino-nucleon interactions complies with the formalism developed in [12]. It is used in many software packages, e.g. in Fluka [13]. Differential cross section $d\sigma_{qe}/dQ^2$ for reactions (1) and (2) is represented by the following expression ($\hbar = c = 1$) [12,14]:

$$\frac{d\sigma_{qe}}{dQ^2} = \frac{G_\mathrm{F}^2 M^2 \cos^2\theta_c}{8\pi E_\nu^2} \times \left[A(Q^2) \pm B(Q^2)\,\frac{s-u}{M^2} + C(Q^2)\,\frac{(s-u)^2}{M^4}\right], \qquad (3)$$

where superscripts correspond to reaction (1) and subscripts — to reaction (2). Square of the 4-vector of energy-momentum transfer is $q^2 = -Q^2$, G_F is the weak interaction Fermi constant, M is the nucleon mass, E_ν is the neutrino energy, s and u are the invariant Mandelstam variables, and θ_c is the Cabibbo angle ($\cos\theta_c = 0.97427$) [15]. Terms in square brackets are

$$A(Q^2) = (m_l^2 + Q^2)/M^2\{(1+\xi)G_A^2 - (1-\xi)f_1^2$$
$$+\xi(1-\xi)f_2^2 + 4\xi f_1 f_2 - [(f_1+f_2)^2$$
$$+(G_A + 2G_P)^2 - 4(1+\xi)G_P^2]m_l^2/M^2\}, \quad (4)$$

$$B(Q^2) = G_A(f_1+f_2)Q^2/M^2, \quad (5)$$

$$C(Q^2) = (G_A^2 + f_1^2 + \xi f_2^2)/4. \quad (6)$$

Here $\xi = Q^2/4M^2$, m_l is the charged lepton mass. Expressions for nucleon form-factors $f_1(Q^2)$, $f_2(Q^2)$, $G_A(Q^2)$ and $G_P(Q^2)$ are discussed below. Elastic cross section for neutral currents is formed in a similar way with another set of form-factor parameters [16].

The integral cross section of quasi-elastic neutrino scattering from nucleons is

$$\sigma_{qe} = \int_{Q^2_{\min}}^{Q^2_{\max}} \frac{d\sigma_{qe}}{dQ^2}\, dQ^2. \quad (7)$$

The integration limits are determined by the scattering kinematics [17]:

$$sQ^2_{\max\,/\,\min} = 2E_\nu^2 M^2 - M^2 m_l^2 \pm E_\nu^2 M m_l^2$$
$$\pm E_\nu^2 M \sqrt{(s-m_l^2)^2 - 2(s+m_l^2)M^2 + M^4}, \quad (8)$$

where the square of the neutrino and nucleon total energy in the center-of-mass system is $s = M^2 + 2E_\nu M$.

Simulation in the Geant4 package was performed as follows. Pre-computed integral cross sections meeting the above-mentioned formalism were realized within the Geant4 package in the following classes of the C++ programming language:

G4ElNeutrinoNucleusTotXsc,

G4MuNeutrinoNucleusTotXsc,

G4TauNeutrinoNucleusTotXsc.

These classes are descendants of the base class of the hadron integral cross sections *G4VCrossSectionDataSet*. Since neutrino interactions are characterized by very small cross sections, Geant4 realizes a special user interface in the *G4EmMessenger* class. This interface defines a scaling of neutrino cross sections to values that allow efficient modeling of neutrino interactions. Consider an example of a set of interface commands:

```
/physics_lists/factory/addNeutrino

/physics_lists/em/NeETotXscActivation true

/physics_lists/em/NuNucleusBias 1.0e12

/physics_lists/em/NuDetectorName det

...
/run/initialize
```

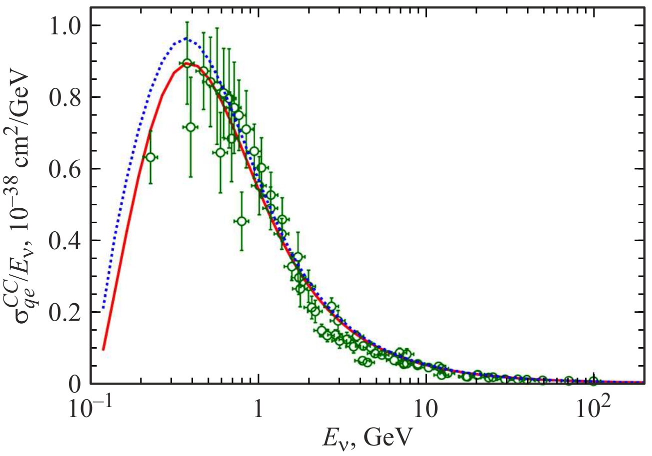

Figure 1. Cross section of quasi-elastic muon neutrino scattering from a neutron. The solid line represents calculations via relation (7), the dotted line represents the results of [11]. Circles are the compiled experimental data from [16,18]

The first command activates neutrino interactions with creating objects of the *G4(El, Mu, Tau) NeutrinoNucleusProcess* classes and relates them with the second command to the relevant integral cross sections in the *G4EmExtraPhysics* constructor. This constructor is used by default in most Geant4 physics lists (sets of processes and particles). The third command introduces the neutrino cross section scaling factor (in this case it is $1 \cdot 10^{12}$) inside region *G4Region* (registration area) specified in the setup geometry by name *det* (the last command). This factor is to be selected so as to ensure the probability of observing a neutrino event in the detector's registration area named *det* at the level of ≤ 0.1. In this case,

$$1 - \exp(-\sigma_{bias} n_{at} L) \sim \sigma_{bias} n_{at} L, \quad (9)$$

where n_{at} and L are the detector's material atomic density and length along the neutrino beam, respectively. Then the spatial distribution of neutrino events in the detector is approximately uniform, which meets the experimental conditions. In the remaining part of the setup geometry, the neutrino cross section shift factor is set to unity by default. Within the Geant4 package, this scheme enables simulating the neutrino propagation along the trajectory from the generation point to neutrino detector registration area. It is available for users in the versions after 11.1.

Fig. 1 demonstrates the neutrino-energy dependence of the ratio between the cross section of quasi-elastic muon neutrino scattering from a neutron and neutrino energy (charged current). The solid curve is the calculations via (7), dotted line represents the results of [11], and circles are the compiled experimental data from [16,18]. A similar dependence for quasi-elastic muon antineutrino scattering from a proton is shown in Fig. 2.

The algorithm for calculating cross sections of quasi-elastic scattering of neutrinos from nucleons represented

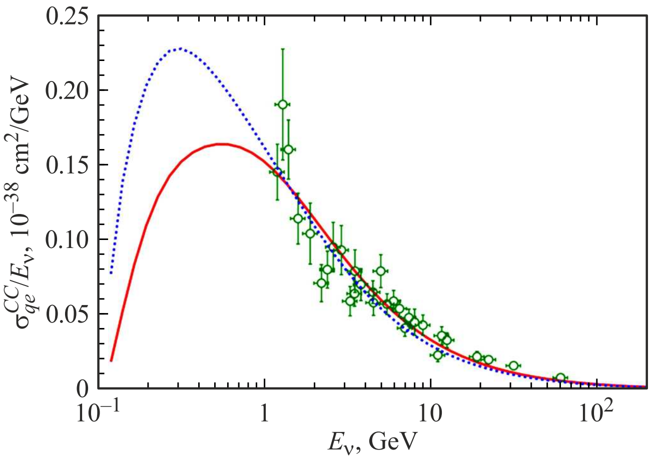

Figure 2. Cross section of quasi-elastic muon neutrino scattering from a proton. The solid line represents calculations via (7), the dotted line represents the results of [11]. Circles are compiled experimental data from [16,18].

in relations (3)−(6) is determined by parameterization of the nucleon form-factors. Form-factors $f_1 = F_1^p - F_1^n$ and $f_2 = F_2^p - F_2^n$ are expressed in terms of Pauli-Dirac form-factors F_1 and F_2 for protons (superscript p) and neutrons (superscript n). In turn, the Pauli-Dirac form-factors are expressed through the nucleon electric and magnetic form-factors G_E^N and G_M^N ($N = p, n$):

$$F_1^N(Q^2) = (G_E^N + \xi G_M^N)/(1+\xi), \quad (10)$$

$$F_2^N(Q^2) = (G_M^N - G_E^N)/(1+\xi). \quad (11)$$

In [11] we used a simplified dipole form of these form factors:

$$G_E^p(Q^2) = G_M^p(Q^2)/\mu_p = G_M^n(Q^2)/\mu_n = G_D(Q^2), \quad (12)$$

$$G_D(Q^2) = 1/(1+Q^2/M_V^2)^2, \quad (13)$$

where μ_p and μ_n are the magnetic moments of the proton and neutron, respectively, and $M_V = 0.843$ GeV [14]. In this work, the electric and magnetic form-factors of nucleons are parameterized as follows:

$$G_{E,M}^N = \frac{1+\Sigma_{k=1}^{n_1} a_k \xi^k}{1+\Sigma_{k=1}^{n_2} b_k \xi^k}, \quad \xi = \frac{Q^2}{4M}, \quad (14)$$

where $n_1 \sim n_2 \leq 4$; coefficients a_k and b_k have been fitted to the experimental data in [19]. To represent the axial and pseudoscalar form-factors, the dipole form is used:

$$G_A(Q^2) = -\frac{g_A}{(1+Q^2/M_A^2)^2}, \quad (15)$$

$$G_P(Q^2) = \frac{2M^2}{m_\pi^2 + Q^2} G_A(Q^2), \quad (16)$$

where m_π is the pion mass, $M_A = 1.0282$ GeV (differs by only 0.2% from the value of 1.026 ± 0.021 GeV recommended in [20]), $g_A = -1.267$ (signs of $B(Q^2)$, G_A and g_A are selected so as to ensure $\sigma_{qe}^{\nu n} > \sigma_{qe}^{\bar{\nu} p}$).

Empirical parameterization of the electric and magnetic form-factors of nucleons allows slightly improving the statistical description accuracy for experimental data on the integrated cross sections of the neutrino quasi-elastic scattering from a nucleon. In the case of the „neutrino from neutron“ scattering, the reduced (divided by the number of freedom degrees) quantity χ^2 is 1.5, whereas in [11] it is 1.8. For quasi-elastic scattering of an antineutrino from a proton, those values appear to be 2.7 and 3.1, respectively.

In conclusion, note that the improved but not free of empiric corrections algorithm for calculating cross sections of quasi-elastic neutrino scattering from nucleons, which was proposed for Geant4, provides a satisfactory description of the experimental data. Further improvement of the accuracy of the experimental data description needs more data at low energies of about 0.1−1 GeV where the curves in Figs. 1 and 2 pass through the broad maximum and the difference in the predictions of different parameterizations at the maximum is more pronounced. This is especially noticeable in the case of quasi-elastic muon antineutrino scattering from a proton.

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Conflict of interests

The authors declare that they have no conflict of interests.

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