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Modeling the electron distribution profile on a luminescent screen during the studying a semiconductor field cathode

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Presents an algorithm for modeling the profile of the emission current density distribution over the surface of a flat conductive anode during cold field emission from a pyramid-shaped semiconductor cathode with a hemispherical tip. The development includes constructing electron trajectories as they move from the cathode to the anode and processing the data obtained using integral approximation calculations.

Keywords: field emission from semiconductors, modeling of electron trajectories in COMSOL, current density distribution, field emission projector.

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Elongated field emitters demonstrate the scattering of electrons after emission from their tops. This spread is caused by the focusing of the lines of force at the top of the emitter [1].

This distribution of the emission current across the anode is observed using a field projector in which a luminescent screen acts as a flat anode (usually a luminescent layer applied to glass with a transparent conductive coating).

The light response of the emitter formed on the fluorescent screen is related to the magnitude of its current. This relationship is often used to analyze the characteristics of individual emitters [2], as well as to estimate the distribution of the total emission current over the surface of multi-pointed cathodes [3].

The processing of the glow patterns obtained at the anode is complicated by the presence of a number of concomitant effects [4]: these are the scattering of electrons, and secondary emission from the anode with the formation of a halo ring around the main response in the field projector images, and the unevenness of the phosphor along the anode plane, and the non-linearity of the dependence of its luminosity on the emission current.

The effect of secondary emission can significantly change the brightness distribution of reflections and cause errors in estimating the emission characteristics of the corresponding emission centers. We conducted a detailed study of the halo effect and obtained an experimental brightness profile (current density) of both the central response and the surrounding ring in Ref. [5]. The main purpose of studying the response structure is to find the dependence of the brightness of the central peak on the brightness of the halo in order to deconvolute the responses of individual emitters in the field projector image and obtain correct estimates of their local emission parameters.

To connect this experimental dependence with the theory of field emission, including the use of adequate formulas for its approximation, it is necessary to model the profile of the light response to obtain a theoretical distribution of the emission current over the surface of the anode (we will call this distribution for short — distribution IA).

Modern software packages such as COMSOL Multiphysics allow calculating electron trajectories for cathodes of various shapes and materials. For example, the electron trajectories for field emission from a carbon nanotube were calculated in [6] using computer modeling, and an IA distribution profile was constructed, which has the form of a power dependence.

Since the time of Spindt, the classical shape of emitters produced by lithography methods has the form of a cone [7], the tip of which is as a result of chemical etching [8,9], as well as under the action of emission currents causing reflow and self-diffusion, and even as a result of vacuum discharge [10] acquires a hemispherical shape. Sometimes such structures are created on the tops of pedestals to further enhance the field [11].

The purpose of this work is to develop an algorithm for calculating the distribution profile of IA using computer modeling in the COMSOL Multiphysics environment of hemispherical top emitters.

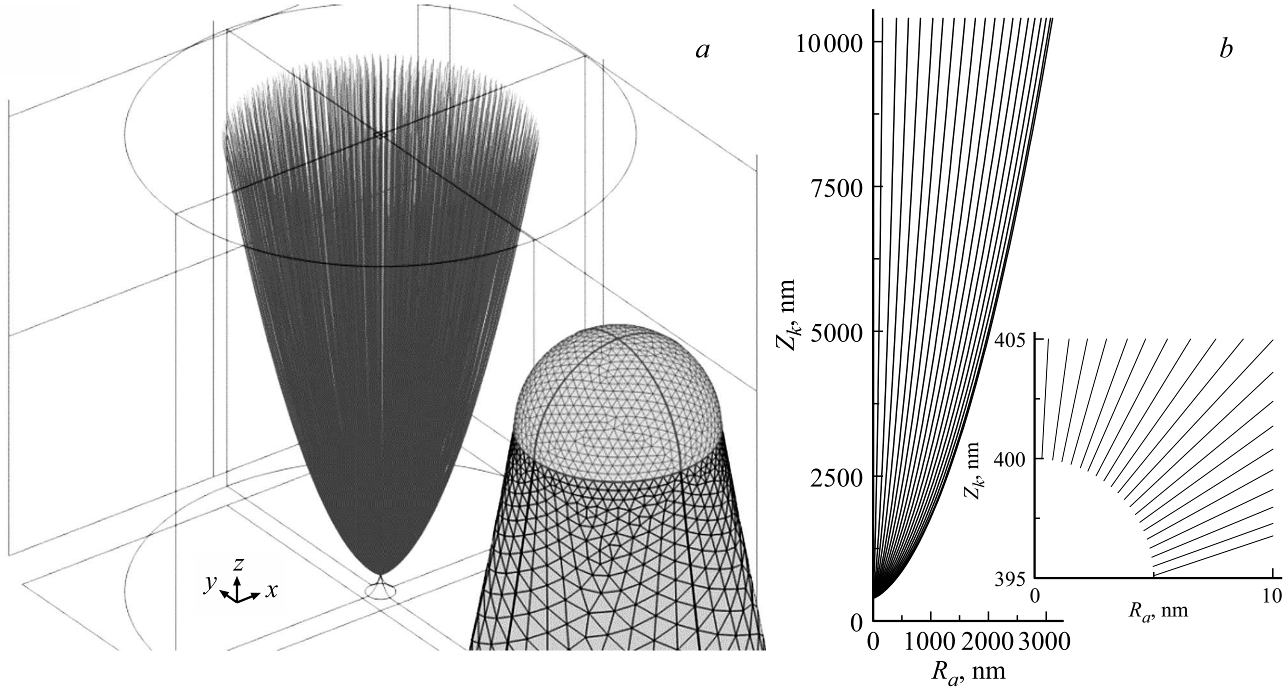

Figure 1. Modeling of field emission from a cone-shaped emitter: *a* — emitter model with a calculated grid and trajectories; *b* — electron trajectories with a uniform step along the azimuth of the vertex surface.

Figure 1, *a* shows a COMSOL model of a silicon emitter having a conical shape with a hemispherical tip. Model parameters: height from base of cathode to vertex $h = 400$ nm, radius of vertex $R_0 = 5$ nm, distance from vertex to plane of anode $d = 10\,\mu$m.

Figure 1, *b* shows the electron trajectories obtained as a result of modeling are shown (the trajectories are shown with a uniform step along the azimuth of the hemisphere surface). It should be noted that the trajectories have an increased density not in the center of the anode, but closer to its edges, so if the density of emitting electrons on the cathode were the same over the surface, then a ring-shaped current density would appear on the anode.

It is known that the software packages use a computational grid in the form of triangles and cannot provide coverage of a hemisphere with triangles of absolutely equal area. This leads to large errors in obtaining the IA distribution by plotting a histogram of electron fluxes corresponding to these triangles. Therefore, we used an original method to eliminate this error.

An electron is placed in the center of each cell on the cathode and the corresponding points E and J_k are calculated. Then two calculated dependencies are constructed: $J_k(Z_k)$ and $Z_k(R_a)$, where Z_k is the coordinate of the cell node on the cathode surface (the axis is directed downward with the origin at the vertex level), R_a is the distance from the center of the anode to the point of arrival of the electron (the end of the trajectory). Figure 2, it a shows the dependence $J_k(Z_k)$, which is well approximated by the cubic dependence. The dependence $Z_k(R_a)$ (symmetric with respect to the center of the anode) is best approximated by a sixth-degree polynomial (Figure 2, *b*). We obtain the formula for calculating the current density at the anode (distribution IA) using the approximating polynomials $Z_k(R_a)$ and $J_k(Z_k)$.

Let's divide the surface of the anode into annular segments of equal width ΔR_a, but with a different radius R_a. Current density at each point of the segment:

$$J_a = \Delta I/\Delta S_a, \quad (1)$$

where ΔI is the emission current entering the segment, $\Delta S_a = 2\pi R_a \Delta R_a$ is the area of the segment. Each such segment of the anode corresponds to an annular segment of the cathode with a height of ΔZ_k. The emission current coming from it is equal to ΔI and can be found by the formula

$$\Delta I = J_k \cdot \Delta S_k, \quad (2)$$

where J_k is the current density on the cathode surface at a height of Z_k from the top, $\Delta S_k = 2\pi R_0 \Delta Z_k$ is the area of the sphere segment.

The sought-for relationship is obtained from formulas (1) and (2), which, as the intervals ΔR_a decrease, takes the differential form:

$$J_a = \frac{J_k R_0}{R_a}\frac{\mathrm{d}Z_k}{\mathrm{d}R_a}. \quad (3)$$

It is necessary to obtain the values of the functions J_k and $\frac{\mathrm{d}Z_k}{\mathrm{d}R_a}$ to build the sought-for dependency $J_a(R_a)$ according to the formula (3). These values can be found using approximating polynomials $J_k(Z_k)$ and $Z_k(R_a)$. Figure 3, *a*

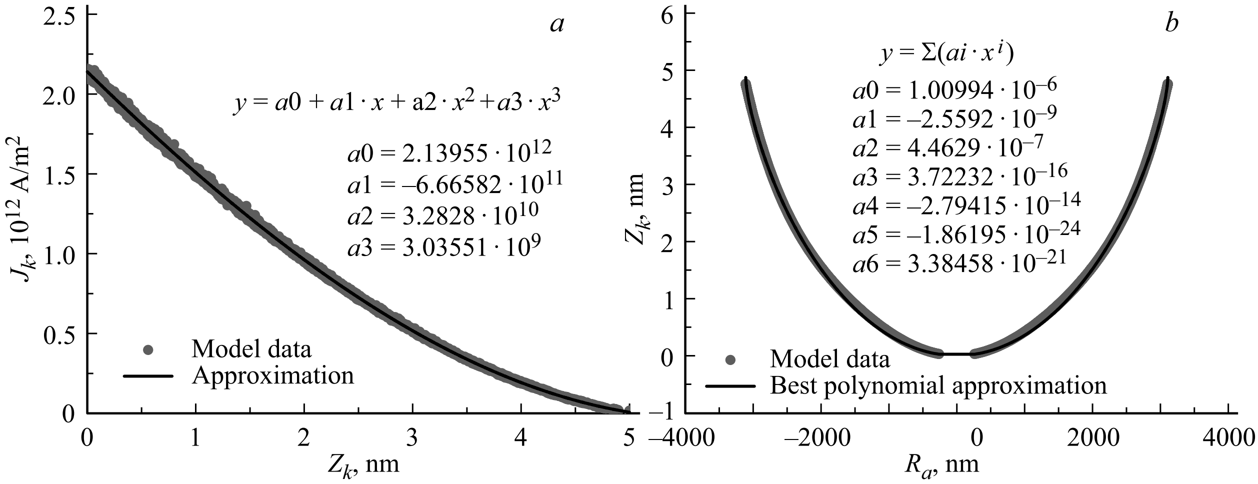

Figure 2. The result of the analysis of electron trajectories: a — dependence of the current density in the node of the computational grid on the surface of the emitter on the coordinate of the node Z; b — dependence of the coordinate Z the beginning of the trajectory of the electron that flew out of the node, from the coordinate R_a of the end of the trajectory at the anode.

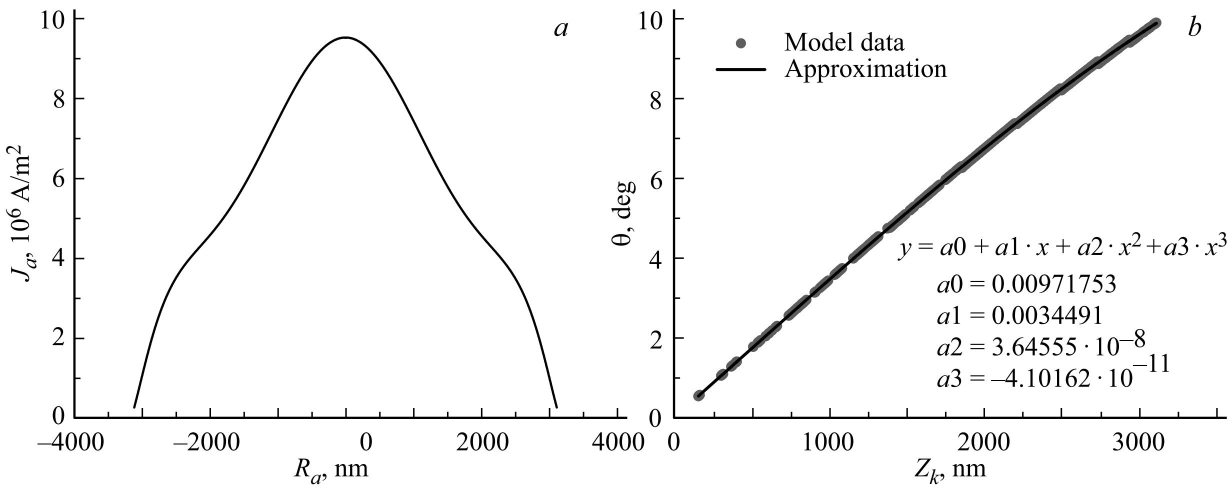

Figure 3. Characteristics of the electron flow incident on the anode: a — dependence of the current density at the anode J_a on the radial coordinate R_a; b — dependence of the angle of incidence of an electron on a plane the anode is from R_a.

shows the result of the described calculation for a given model. The kinks observed on the graph are related to the inaccuracy of the approximation of the dependence $Z(R_a)$.

The emission current was calculated using the Stratton formula [12], optimized for the range of fields on the surface of the emitter tip: $F = 5 \cdot 10^9 - 10^{10}$ V/m. The current density J has a dependence in the form of two multipliers:

$$J = j_e \cdot S_e, \tag{4}$$

$$j_e = a_{FN} \cdot \frac{F^2}{X_F} \cdot \exp\left(-\frac{b_{FN} \cdot X_F^{1.5} \cdot \nu}{F}\right), \tag{5}$$

where $a_{FN} = e^3/8\pi h$, $b_{FN} = (8\pi/3eh) \cdot (2m_e)^{1/2}$ is the first and second Fowler-Nordheim constants, the function $X_F = X - a \cdot F^{4/5}$, where X is the chemical affinity (eV), a is a constant equal to $\left(\frac{15^2 \cdot h^6 \cdot \varepsilon_0^2}{2^{13}\eta^2(m_e^*)^3}\right)^{1/5} \cdot \varepsilon^{-2/5}$ (here ε is the chemical permeability, m_e^* is the effective mass of an electron), a special function $\nu = 1 - u + u \cdot \ln(u)/6$, where the parameter is entered

$$u = \left(\frac{c_s}{e^2} \cdot \frac{\varepsilon - 1}{\varepsilon + 1}\right) \cdot \frac{F}{X_F^2},$$

$c_s = \frac{e^3}{4\pi\varepsilon_0}$ is the Schottky constant;

$$S_e = 1 - (1 + C) \cdot \exp(-C), \tag{6}$$

where parameter $C = b\frac{\sqrt{X_F}}{F^{1/5}}$, b is a constant equal to $\sqrt{2m_e^*/e}\left(\frac{15^2 h\pi^3\varepsilon_0^2}{2^3(m_e^*)^3}\right)^{1/5} \cdot \varepsilon^{-2/5}$.

Values e, m_e, h (J · s), k_B (eV/K), ε_0 are known physical constants.

The material parameters in the calculation were set for a real cone-shaped silicon cathode: $\varepsilon = 11.9$, $x = 4.04\,\text{eV}$, $T = 300\,\text{K}$, $m_e^* = 1.09m_e$, $E_g = 1.12\,\text{eV}$.

The plotting and approximation by polynomials were performed in the LabVIEW software environment.

The obtained distribution IA has a power-law dependence in the vertex region, similar to that obtained in Ref. [6]: $J = J_0 - x^n$, where J_0 is the current density in the center of the anode.

Figure 3, *b* shows a separate calculation of the dependence of the angle of incidence of electrons on the surface of the anode $\alpha(R_a)$. To determine the angle for each of the calculated trajectories, a linear approximation of the last few points of the trajectory was constructed ($\sim 1000\,\mu$m). The dependence best approximates a 4th-order polynomial, but in general it has an almost linear form. The angles of incidence do not exceed 10° from the normal. This means that the elastic bounce of electrons from the anode cannot lead to the appearance of the experimentally observed halo effect on the phosphor coating (a uniformly illuminated circle with a diameter of $4d$ [13], as well as a circle with a bright ring around the perimeter [14]).

So, we performed a numerical simulation of the electron fluxes of field emission from a silicon tip and, using an original algorithm, obtained the current density distribution on the surface of the anode. In the future, this distribution will be used for numerical analysis of light responses on the field projector screen in order to correct data on the characteristics of individual emitters in a multi-pointed cathode.

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Conflict of interest

The authors declare that they have no conflict of interest.

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