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Prospects for electron acceleration with not-so-high-power lasers

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The capabilities of femtosecond terawatt laser systems of not very high power level (~ 100 TW) have been analyzed for the sake of electron beam generation using laser wakefield acceleration method in relativistic laser plasmas. A self-consisted analysis of the optimal parameters of laser pulses and plasma was performed, allowing electron beam energies of the order of 0.5 GeV to be achieved. It was shown that laser pulses with duration of 40 fs from a 60-TW power system, allow electron acceleration in a plasma with a density of $1.9 \cdot 10^{18}\,\mathrm{cm}^{-3}$ to energies of ~ 500 MeV over an acceleration length of about 1 cm. The calculations were compared with the results obtained in previous experiments employing laser systems of comparable power.

Keywords: laser wakefield acceleration, laser electron acceleration, terawatt lasers, compact accelerator.

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Introduction

Obtaining high-quality electron beams is an important task in modern applied and fundamental physics. High beam quality refers to certain combinations of beam parameters, such as relatively high values of electric charge (from 10^{-11} to 10^{-8}C), low beam emittance (both small size and low angular divergence), as well as high electron energy at the level of $\sim$ GeV. Currently, high-quality electron beams are routinely produced using synchrotrons [1] and free electron lasers (FELs) [2]. The resulting electron beams are mainly used to generate radiation in the ultraviolet, extreme ultraviolet (EUV), soft X-ray (SXR) and hard X-ray spectral regions [3].

At the same time, the cost of building and commissioning a synchrotron or FEL is quite high, which limits researchers' access to such unique radiation sources. However, in recent years, these difficulties have been gradually overcome through the use of compact laser-plasma accelerators based on multi-terawatt and petawatt laser systems emerging all over the world [4]. Unlike petawatt laser systems, not only outstanding research centers can afford femtosecond lasers of the order of ~ 100 TW, but also higher educational institutions with research laboratories.

The concept of compact plasma accelerators was formulated in general terms back in the 1950s in the works of G.J. Budker, V.I. Veksler and I.B. Fainberg [5–7]. For many years after that, solutions were proposed in the form of plasma accelerators based on the excitation of plasma waves using first microwave generators, and then also high-current relativistic electron beams [8]. Those solutions made it possible to create accelerating electric fields in plasma waves with characteristic intensity values of 0.5 MV/cm, that were already approaching the values of accelerating fields in synchrotrons existing by that time.

The same historical period saw the heyday of the era of laser development, when new laser wavelengths were appearing from year to year, as well as an unprecedented increase in the power of laser systems, in particular due to shorter laser pulse durations. The idea of creating a compact plasma accelerator based on ultrashort laser pulses was first proposed by T. Tajima and J. Dawson in 1979 [9]. In this fundamental work, it was shown that ultrashort high-intensity laser pulses excite, under certain conditions, longitudinal Langmuir waves in plasma, in which electric fields can reach values of $\sim$ GV/cm, that is several orders of magnitude greater than the maximum possible accelerating fields in traditional accelerators based on radio frequency resonators. Later, these conclusions were confirmed by deeper theoretical analysis and computer modeling [10,11].

The first experiments with the electron acceleration by plasma wakefields excited by ultrashort laser pulses, which is now referred to as Laser WakeField Acceleration (LWFA), became possible after the invention of the chirped laser pulse amplification method by D. Strickland and G. Mourou in 1985 [12], that made it possible the creation of laser systems of unprecedented power. First multi-terawatt lasers with a power of tens or even hundreds of TW began their operation already in the mid-1990s.

The first successful experiments on laser wakefield acceleration made it possible to obtain electrons with energies up to 30–40 MeV and a wide, almost continuous spectrum [13–15]. Those electron beams were not quite useful for further radiation generation. However, three independent groups in 2004 almost simultaneously reported the experimental generation of electron beams with energy of ~ 100 MeV and energy dispersion at the level of $< 10\%$ [16–18] by wakefields in plasma, which allowed talking about quasi-monoenergetic beams.

It can be said that from this moment the development of the era of compact laser plasma accelerators began.

Accelerated energy of the accelerated electron beams increased rapidly at first to 1 GeV [19], then there was an almost ten-year break until the energy values of the received electron beams reached 2 GeV [20], and then 4 GeV [21]. A paper published in 2019 [22] reported an electron beam generation with an energy of 8 GeV produced in a compact laser accelerator after the propagation of a petawatt laser pulse through a pre-formed plasma channel, and for a long time this was the best result in laser-plasma electron acceleration. However, two groups reported reaching beam energies of 10 GeV in 2024 [23,24].

Along with attempts to increase the energy of electron beams produced in laser plasma accelerators, research is underway to improve the parameters of these generated electron beams. The desired characteristics of the electron beams usually comprise a relatively large charge (at the level from 10^{-11} to 10^{-9} C), low beam divergence (from 1 to 10 mrad) and low emittance, which is initially easily achieved due to the small ($\sim\mu$m) initial dimensions of the electron beam, but then the emittance degrades very quickly while the divergent electron beam propagates in free space after leaving the plasma [25]. High attention should also be paid to the reproducibility of the properties of the electron beam, namely, the maximum preservation of its characteristics from shot to shot. The most impressive successes in this regard were demonstrated by the team of the German compact accelerator LUX [26], which showed stable generation of electron beams with an energy of 368 MeV and a charge of 25 pC at a repetition frequency of 1 Hz during 28 h operation of the accelerator [27].

While many more laser complexes are emerging all over the world that deliver to experimental chambers laser pulses with a power from several hundred terawatts to 10 petawatts [28], there is also a need to understand the capabilities of less powerful ($\sim$ 100 TW) laser systems for conducting experiments, including electron acceleration by wakefields in low-density laser plasmas.

A consistent calculation of the requirements for a low-power laser system (less than 100 TW) has been carried out in this paper, which makes it possible to obtain electron beams with energies of the order of 0.5 GeV in a compact laser plasma accelerator. The characteristics of a gas-plasma target for such a compact accelerator and the areas of its potential applications are discussed. In particular, the possibility of using a combination of such a laser system with a compact laser-plasma accelerator to create a compact free electron laser is being discussed, a goal that about a dozen scientific groups around the world are currently moving towards [29].

1. Equations of the electron acceleration method by wakefields in plasma

1.1. The normalized vector-potential of the electromagnetic wave of the laser field

The first key factor for effective acceleration of electrons by a laser field in a plasma is the relativistic intensity of the electromagnetic laser wave, which is determined by the value of the normalized vector potential a, expressed as the ratio of the momentum that a free electron acquires in the field of the laser wave over a quarter of its oscillation period, to the momentum $p = m_e c$:

$$a = eE_{las}/m_e c\omega_{las}. \tag{1}$$

Here ω_{las} is the angular frequency of the laser radiation, E_{las} is the maximum value of the electric field strength in the laser wave, e and m_e is the charge and mass of an electron, c is the speed of light in vacuum.

The value $a = 1$ corresponds to the case, where a free electron is being accelerated from the rest state to a momentum of $m_e c$ over a quarter of the oscillation period of the laser wave. Accordingly, the value of the normalized vector potential a should be $a \sim 1$ for the laser field to be relativistic. The relationship of the parameter a with the intensity of the laser field will be clearly shown below, and the optimal range of the a values for effective electron acceleration by the traveling wake method in plasma is indicated.

1.2. The relationship between the duration of the laser pulse and the plasma density

When a powerful ultrashort laser pulse propagates through a plasma, longitudinal Langmuir waves are generated in it [30]. Unlike transverse waves, the electric field vector in such waves oscillates in the direction of wave propagation, and the frequency is equal to the plasma frequency and depends only on the plasma density n_e:

$$\omega_{pl} = \sqrt{e^2 n_e/\varepsilon_0 m_e} \approx 56\sqrt{n_e\,[\mathrm{m}^{-3}]}. \tag{2}$$

In the formula (2), the plasma frequency ω_{pl} is measured in [s^{-1}], and the electron density is expressed in the number of electrons per cubic meter.

A laser beam passing through a plasma is a wave packet moving at a group velocity determined by the refractive index of the plasma:

$$v_{gr\ las} = c\sqrt{1 - (\omega_{pl}/\omega_{las})^2} < c, \tag{3}$$

where ω_{las} is the angular frequency of laser radiation, c is the speed of light in vacuum. It is assumed here that the plasma has low density, i.e. $\omega_{pl} < \omega_{las}$.

Passing through the plasma, an ultrashort laser pulse pushes electrons out of the region of the intense electromagnetic field into the surrounding plasma under the action of ponderomotive extrusive forces, which generates a wave of electron density compressions and stretchings following the laser pulse (Fig. 1). This is the relativistic Langmuir wave, the phase velocity of which turns out to be equal to the group velocity of the laser pulse exciting this wave (3):

$$v_{phase\ pl} = \omega_{pl}/k_{pl} = v_{gr\ las} = c\sqrt{1 - (\omega_{pl}/\omega_{las})^2} < c. \tag{4}$$

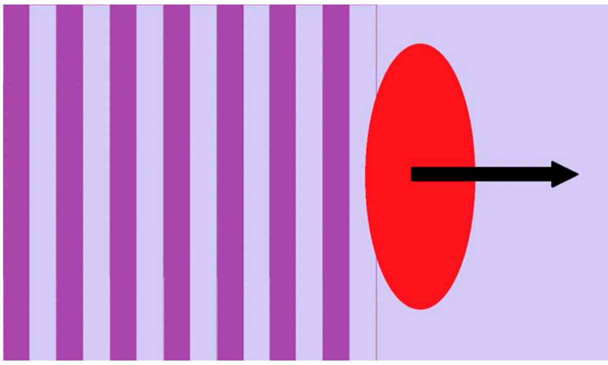

Figure 1. A schematic illustration of a one-dimensional model, where a laser pulse passing through a plasma excites a Langmuir wave traveling behind it with periodic modulations of the electron density (compression-and-stretching wave).

Here k_{pl} is the wave vector of the propagating Langmuir wave in plasma, which is related to the wave period by the classical relation $k_{pl} = 2\pi/\lambda_{pl}$. Such a longitudinal relativistic wave excited by a laser pulse and traveling behind it is called a relativistic wakefield in a low-density plasma.

As it was shown by T. Tajima and J. Dawson in Ref. [9], two conditions are necessary for the most effective excitation of such a wakefield. The first condition is the relativistic intensity of the laser wave, and the second condition indicates the optimal ratio of the duration of the laser pulse and the period of plasma oscillations in the Langmuir wave, namely, that the length of the laser pulse in the optimal case should be equal to half the plasma wavelength:

$$L_{las\ imp} = \lambda_{pl}/2 = \pi c/\omega_{pl}, \tag{5}$$

or, what is the same, the expression (5) should also be true for the ratio of the duration of the laser pulse and the period of plasma oscillations in the Langmuir wakefield:

$$\tau_{las\ imp} = T_{pl}/2 = \pi/\omega_{pl}. \tag{6}$$

The formulas (5) and (6) use the approximation $\omega_{pl} \ll \omega_{las}$, which is most often encountered in experiments, where the phase velocity of the Langmuir wave (4) is already quite close to the speed of light in vacuum c, and the problem itself is essentially relativistic.

Thus, from the equations (2) and (6), a relation appears that relates the duration of the laser pulse, the plasma frequency, and the electron density in the plasma:

$$\tau_{las\ imp} = \pi/\omega_{pl} = \pi\sqrt{\varepsilon_0 m_e/e^2 n_e} \approx 0.56 ps \Big/ \sqrt{n_e[10^{16}\,\text{cm}^{-3}]}. \tag{7}$$

It can be seen from the formula (7) that subpicosecond laser pulses are needed to effectively excite the wakefields in gaseous plasma targets at electron densities in the range from 10^{16} to $10^{20}\,\text{cm}^{-3}$, which is most often implemented in experiments.

1.3. Accelerating field and required intensity values

As mentioned above, the first important condition described in Refs. [9,10] says that the intensity of laser radiation depends on the parameter a from the formula (1) and must be on the order of or above the relativistic threshold defined by the expression

$$I = 2\pi^2\varepsilon_0 m_e^2 c^5/e^2 (a/\lambda_{las})^2 = 1.37\cdot 10^{18}\,\text{W/cm}^2 (a/\lambda_{las}[\mu\text{m}])^2. \tag{8}$$

Intensity value $1.37 \cdot 10^{18}\,\text{W/cm}^2$ from formula (8) is considered as the limit, above which the light electromagnetic field is relativistic. This limit depends on the radiation wavelength λ_{las}, and at a value of $a = 1$ corresponds to the case when the electric field strength of the light wave is sufficient to accelerate a free electron, which was initially at rest, to the momentum value of $p = m_e c$ over a quarter of the laser wave oscillation period. Accordingly, at the wavelength of the laser radiation $\lambda_{las} = 800\,\text{nm}$, this relativistic limit will be even higher, since the intensity depends on the laser wavelength as $I \propto (1/\lambda_{las})^2$. The value of the parameter a from formulas (1) and (8) largely determines the properties of the wakefield excited by a high-power laser pulse and the physics of the processes, including injection of electrons into the cavity and their acceleration.

The intensity of the relativistic laser wave and the value of the normalized vector potential a from formulas (1) and (8) determine how quickly and efficiently electrons will be removed under the action of ponderomotive forces from the region of space, where the laser pulse passes through (Fig. 2). Consequently, this will determine the depth of the electron density modulation in the relativistic wakefield following the femtosecond laser pulse.

Papers [9,10] have also shown that effective electron acceleration is possible with high modulation of the electron density in the relativistic wakefield in a low-density plasma, since the depth of electron density modulation is directly related to the ratio of the real accelerating field and the theoretically possible one. This can be illustrated by an example of the interaction of weakly relativistic laser pulses ($a < 1$) with a low-density plasma (Fig. 3). At $a < 1$, the perturbation of the electron density in a one-dimensional traveling wakefield can be considered to be harmonic:

$$\delta n_e(x, t) = -\delta n_{e\_\max}\cos(k_{pl}x - \omega_{pl}t). \tag{9}$$

Substituting the dependence (9) into the common Poisson equation for the electric field $\Delta\varphi = -\nabla E = -\rho_e/\varepsilon_0 = -\delta n_e e/\varepsilon_0$, we obtain after integration the expression

$$\delta E(x, t) = (\delta n_{e\_\max} e/\varepsilon_0 k_{pl})\sin(k_{pl}x - \omega_{pl}t)$$
$$= E_{\max}\alpha\sin(k_{pl}x - \omega_{pl}t). \tag{10}$$

Here $E_{\max}$ is the maximum accelerating longitudinal field, and the coefficient $\alpha = \delta n_{e\_\max}/n_e$ is the depth of modulation of the electron density in the wakefield, which depends on the normalized vector potential a of the laser

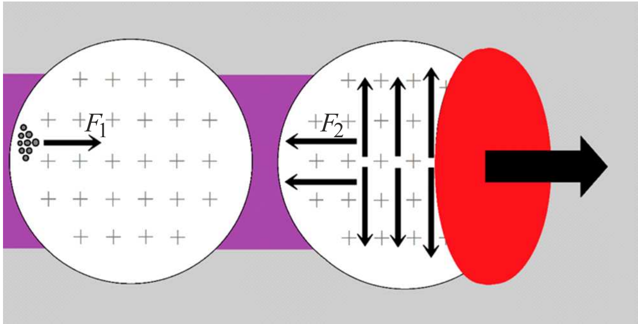

Figure 2. An illustration of a three-dimensional model. A femtosecond laser pulse generates ponderomotive forces F_2, that push electrons out of the region of high intensity of the laser field, resulting in the formation of an ion cavity. F_1 denotes the force accelerating an electron beam trapped inside the ion cavity.

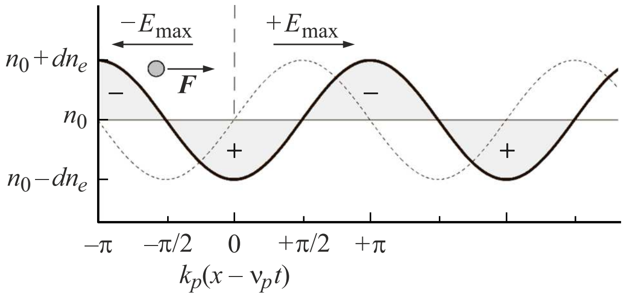

Figure 3. Dependence of the electron density (solid curve) and the longitudinal accelerating electric field (dotted curve) in a harmonic wakefield excited by a weakly relativistic laser pulse with $a < 1$. The dependences are given on the normalized running coordinate $k_{pl}(x - v_{phase\ pl}t)$, where the 0 mark is the point, where the longitudinal electric field changes sign from „–“ to „+“.

field. Equation (10) shows that the behavior of the longitudinal accelerating field in the weakly relativistic case will also have a harmonic character (Fig. 3), and the value of E_{max} is found by the following formula:

$$E_{max} = m_e c\omega_{pl}/e \approx 0.96\,\text{V/cm}\sqrt{n_e[\text{cm}^{-3}]}. \quad (11)$$

The ratio $k_{pl} = \omega_{pl}/c$ and the expression (2) were used here.

It can be immediately seen from formula (11) that at a plasma electron density of the order of $n_e = 10^{18}\,\text{cm}^{-3}$, the longitudinal accelerating field strength can reach values of $\sim$ GV/cm at a modulation depth of the electron density of $\alpha \approx 1$. It should be noted that even the modulation depth of $\alpha = 0.1$ in a plasma with a density of $n_e = 10^{19}\,\text{cm}^{-3}$ is sufficient to obtain the peak value of the accelerating field strength at the level of 0.3 GV/cm.

The interaction of a femtosecond laser pulse with a low-density plasma becomes significantly more nonlinear in case of higher values of the normalized vector potential of the laser field $a > 1.5$, which changes the shape of the dependence of the accelerating longitudinal field on the coordinate and time $\delta E(x, t)$, which, unlike the expression (10), now ceases to be harmonic. However, a deeper modulation of the electron density can be obtained at values of $a > 2$ [31], which makes it possible to achieve accelerating fields close to the maximum value (11).

If we substitute the value of the normalized vector potential of the laser field $a = 2$ into formula (8), we obtain for laser radiation with a wavelength of $\lambda_{las} = 800$ nm the characteristic value of the required intensity of the focused laser pulse:

$$I = 1.37\cdot 10^{18}\,\text{W/cm}^2(a/\lambda_{las}[\mu\text{m}])^2 \approx 8.6\cdot 10^{18}\,\text{W/cm}^2. \quad (12)$$

To achieve this value of laser radiation intensity in focus, it is necessary to assemble a focusing system capable of making a focal spot diameter in the region of $d \sim 20\,\mu$m in full width at half maximum (FWHM), and at the same time having a good quality beam profile. The results of calculations and computer simulations in Ref. [31] show that, in one of the most optimal versions of a consistent set of parameters for laser-plasma electron acceleration, the diameter of the focal spot, into which the femtosecond laser pulses exciting the wake are focused, should be close to the value of the plasma wavelength of Langmuir waves:

$$d \approx \lambda_{pl} = 2\pi c/\omega_{pl} \approx 33\,\mu\text{m}\Big/\sqrt{n_e[10^{18}\,\text{cm}^{-3}]}. \quad (13)$$

It should be noted that the required laser system power depends on the selected value of the plasma electron density according to the law

$$P = I\pi d^2/4 \approx 7.4\cdot 10^{13}\,\text{W}\Big/n_e[10^{18}\,\text{cm}^{-3}]. \quad (14)$$

If we make a primary estimate of the required power of the laser system, knowing the values of expressions (12) and (13) for the electron density of the plasma $n_e = 10^{18}\,\text{cm}^{-3}$, then we get from (14) the required laser power at the level of $P = 7\cdot 10^{13}\,\text{W} = 70\,\text{TW}$.

1.4. Maximum electron beam energy

By analogy with conventional relativistic mechanics, two important parameters are introduced in the physics of laser-plasma accelerators based on a traveling wakefield:

$$\gamma_{pl} = \omega_{las}/\omega_{pl}, \quad \beta_{pl} = \sqrt{1 - (1/\gamma_{pl})^2}. \quad (15)$$

The correctness of the analogy can be verified by substituting formulas (15) into (3) and (4) and obtaining the classical expressions

$$\gamma_{pl} = 1\Big/\sqrt{1 - (v_{phase\ pl}/c)^2}$$

and

$$\beta_{pl} = v_{phase\ pl}/c = v_{gr\ las}/c.$$

Having the expressions of the parameters (15), it is possible to obtain the value of the maximum energy limit of an electron beam accelerated in a wakefield, following the simple formula from Ref. [9,10] and taking into account that in most cases $\beta_{pl} \approx 1$:

$$W_{\max} = 2\beta_{pl}\gamma_{pl}^2 m_e c^2 \approx 2\gamma_{pl}^2 m_e c^2. \quad (16)$$

Formula (16) will serve as the basis for our calculation, since it gives the value of the maximum possible energy of an electron beam accelerated at optimal parameters of the laser system for a given value of the coefficient γ_{pl}, which is determined by formula (15) from the ratio of the plasma frequency and the frequency of oscillations of the laser field. This means that it depends solely on two parameters — the average electron density in the plasma, and the wavelength of the laser radiation.

1.5. Optimal acceleration length

Nevertheless, in order to obtain electron beam energy values at the level of (16) in a real experiment, it is necessary to ensure that optimal acceleration conditions are kept at macroscopic lengths of the order of several millimeters or even centimeters, depending on the average plasma density [9,10].

The primary estimate of the required minimum acceleration length can be obtained by simply dividing the formula (16) by $eE_{\max}$ from the expression (11):

$$l_{accel\ \min} = W_{\max}/eE_{\max} = 2\gamma_{pl}^2 c/\omega_{pl}. \quad (17)$$

The formula (17) immediately makes it possible to see how fast the acceleration length increases with increase of the maximum electron beam energy: both $W_{\max}$ and l_{accel} increase proportionally to γ_{pl}^2.

Fig. 3 shows that two different phases are present in the region of reduced electron density inside the wakefield traveling behind a femtosecond laser pulse: the acceleration phase, where the longitudinal electric field is directed against the motion of the laser pulse, and the deceleration phase, where the longitudinal electric field is directed along the direction of propagation of the laser pulse. The trapped electron beam is being accelerated in the first phase, and is being slowed down in the second phase. If we draw a detailed phase diagram of the dependence of the electron beam energy on its position inside the accelerating cavity [32,33], it will be seen that the phase trajectories of the electrons are closed: first, the electron beam is being accelerated and gradually overtakes the wakefield following the laser pulse, then passes the point, at which the longitudinal electric field changes its sign, and then the beam is being slowed down (Fig. 4). The electron beam will have the highest energy precisely after passing the point of the zero accelerating field corresponding to the point 0 in Fig. 4. If we choose the length of the plasma medium so that the time of passage of the laser pulse through it would be equal to the acceleration time from the energy of the electron injection into the low-density region to $W_{\max}$ (16), then this length of the plasma medium can be considered

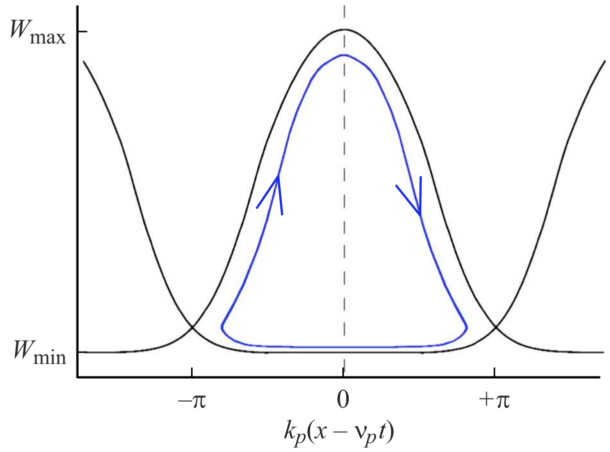

Figure 4. The phase trajectory of an electron beam trapped in the ion cavity. The phase trajectories of the particles are closed and located in a region bounded by separatrices. As long as the electrons are in the region $k_{pl}(x - v_{phase\ pl}t) < 0$, the beam is being accelerated, and after passing the point 0 and acquiring maximum energy not exceeding the maximum theoretical value $W_{\max}$, the deceleration begins.

as the optimal acceleration length. This length is called the dephasing length and is written as:

$$l_{accel\ optim} = l_{dephas} = \gamma_{pl}^2 \lambda_{pl}. \quad (18)$$

If we substitute the value of the electron density of $n_e = 10^{18}\,\text{cm}^{-3}$ into (13) and (15), then we get the value of the dephasing length (18) at the level of 60 mm. This is quite a lot, and in order to experimentally implement this large acceleration length, it will be necessary to fulfill the condition of self-focusing of the laser pulse in order to ensure its propagation in a channel with a reduced electron density over long distances of the order of 1 cm and more. This puts an additional limitation on the power of the laser pulse, arising from the theory of relativistic self-focusing [34]:

$$P_{\min} = P_{self-focus} = \gamma_{pl}^2 2m_e^2 c^5/e^2 \approx \gamma_{pl}^2 \cdot 17.4\,\text{GW}. \quad (19)$$

At an electron density of $n_e = 10^{18}\,\text{cm}^{-3}$, the expression (19) gives the value of 30 TW, which is the minimum necessary to create a self-focusing channel for the propagation of a laser pulse in a plasma medium at a given density without using additional laser pulses to prepare the plasma medium.

Two aspects should be noted here. First, the power value for self-focusing (19) of a laser pulse in plasma is inversely proportional to the plasma electron density $P_{\min} \propto 1/n_e$, therefore, $P_{\min}$ can be reduced simply by increasing the electron density (however, it should be remembered that this will also decrease both the value of $W_{\max}$ (16), and the value of the dephasing length (18) l_{dephas}). Second, modern installations never operate under the $P \approx P_{\min} = P_{self-focus}$ relation, since the self-focusing conditions necessary for effective acceleration of the electron beam can be quickly lost due to fluctuations in the laser pulse parameters, so the following ratio [35,36] is commonly used, which we will adhere to as well:

$$P = (3-5)P_{self-focus} \approx (3-5)\gamma_{pl}^2 \cdot 17.4\,\text{GW}. \quad (20)$$

The last limitation on the plasma medium length required to accelerate the trapped electron beam is related to the length, at which the laser pulse loses enough energy and stops effectively exciting the wakefield. This length is called the depletion length of the laser pulse and is calculated using the formula [32,37]:

$$l_{depl\ las} = \gamma_{pl}^2 \cdot c\tau_{las}. \quad (21)$$

It can be seen that, when the ratio (6) is fulfilled, the depletion length of the laser beam, calculated according to the formula (21), will be two times less than the dephasing length (18). Thus, due to the limitation (21), it will not be possible to achieve the beam energies of $W_{\max}$ from formula (16) in a real experiment, but it might be possible to obtain up to 60–70 % of this value.

2. Calculation of the not-so-high-power laser system parameters

Based on what was described in the previous section, here let's try to collect and summarize all the necessary requirements for the laser and the experimental system in a general form in the following table. First, let's choose the operating range of the electron density values n_e. On the one hand, as shown above, the value of $n_e = 10^{18}\,\text{cm}^{-3}$ leads to a relatively high value of $P_{self-focus}$ from (19), equal to 30 TW, which, according to the requirement (20), imposes a limitation on the laser system power $P > 90$ TW. And since $P_{\min} \propto 1/n_e$, it is impractical to consider a lower plasma density for our task.

The next step is to put the upper limit for the range of the n_e values we are interested in. To do this, let's plot the dependence of $W_{\max}$ (16) on the electron density n_e (Fig. 5) and agree that we are interested in electron beams with energy of $W_{\max} > 200$ MeV. Then the range of the plasma electron density of our interest n_e will correspond to the shaded area in Fig. 5. In this case, the density values of interest n_e must satisfy the ratio $10^{18} < n_e < 8.8 \cdot 10^{18}\,\text{cm}^{-3}$, or $n_e \sim 10^{18}\,\text{cm}^{-3}$.

Next, we will limit the length of the plasma medium to some reasonable value, which is quite easy to implement in modern experiments. Many modern experimental studies on laser wakefield acceleration of electron beams use gas nozzles or gas capillary cells with a plasma channel length of about 1 cm. Therefore, we will use the reference value of $l_{accel} \sim 1$ cm.

Let's take the laser radiation wavelength of $\lambda_{las} = 800$ nm, for our calculations, because it is typical for Ti:Sa laser systems, as the laser radiation wavelength for our calculations. Then the laser angular frequency value

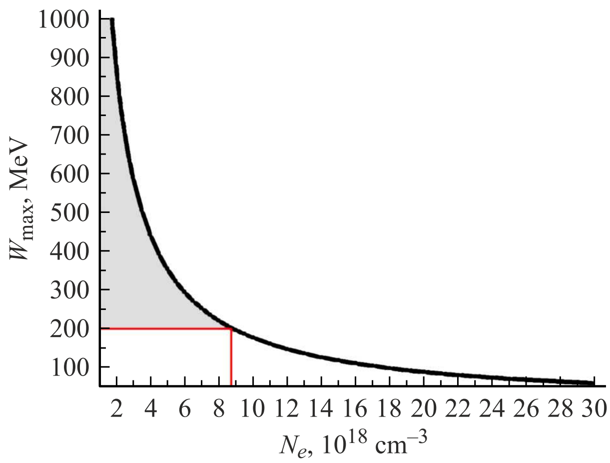

Figure 5. Dependence of the maximum possible theoretical value of the accelerated beam energy $W_{\max}$ on the plasma electron density. The shaded area corresponds to the range of values n_e, for which $W_{\max} > 200$ MeV and the required power of the femtosecond laser system does not exceed 100 TW.

will be $\omega_{las} = 2\pi c/\lambda_{las} = 2.36 \cdot 10^{15}\,\text{s}^{-1}$. We need the ratio of $\gamma_{pl} = \omega_{las}/\omega_{pl} \approx \lambda_{pl}/\lambda_{las}$ (15) to be as high as possible, preferably $\gamma_{pl} > 20$, which increases the restriction on n_e and sets a new upper limit for the electron density: $n_e < 4.4 \cdot 10^{18}\,\text{cm}^{-3}$. At the same time, $W_{\max}$, calculated using the formula (16), becomes higher: $W_{\max} > 400$ MeV.

Since the total electron beam acceleration length will be determined by three factors (the presence of self-focusing, the dephasing length (18), and the laser pulse depletion length (21) in the plasma medium), it makes sense to consider a relatively big duration of the laser pulse. Let's use $\tau_{las\ imp} = 40$ fs for the calculation. As the remaining experimental parameters, let's write in the second column of the table the requirement for $E_{\max}$ from (11): $E_{\max} > 1$ GV/cm, as well as the requirement for the power of the laser system: $P < 100$ TW.

Then let's calculate the consistent electron density by expressing n_e from (6) and (7) for $\tau_{las\ imp} = 40$ fs. We obtain the value of $n_{e\_opt} = 1.9 \cdot 10^{18}\,\text{cm}^{-3}$. This value corresponds to the value of $\gamma_{pl} = 30.5$, which, following the formula (16), gives an energy value of $W_{\max} = 930$ MeV. Substituting n_{e_opt} into formula (11) yields the value of $E_{\max} = 1.3$ GV/cm, which meets the stated requirements.

The next step is to ensure that the self-focusing condition for the laser pulse in the plasma is met according to condition (20). Substituting the value of $\gamma_{pl} = 30.5$ into formula (19), we obtain the minimum required laser system power to meet the self-focusing condition: $P_{self-focus} = 15.8$ TW. Then, according to the formula (20), we need to have a laser system with a power of $P \approx 4P_{self-focus} \approx 63$ TW.

Now let's check that the intensity of the laser radiation in the focus exceeds the value of (12), when the power of the laser system is at the level of 60 TW. Taking the diameter of the focal spot equal to λ_{pl}, according to relation (13), we obtain from expression (14) that the intensity in focus will be equal to $\approx 1.2 \cdot 10^{19}\,\text{W/cm}^2$, which exceeds the value

Calculation of experimental parameters for laser wakefield electron acceleration using a laser system with a power of less than 100 TW

Parameter, Units of Measurement	Target value	Formula	Calculation result
Duration $\tau_{las\ imp}$, fs	> 30	–	40
Optimal electron density $n_{e\_opt.}$, cm^{-3}	$\sim 10^{18}$	$(56\,\text{fs}/\tau_{las\ imp})^2 \cdot 10^{18}$	$1.9 \cdot 10^{18}$
ω_{pl}, s^{-1}	$< 2.4 \cdot 10^{14}$	$56\sqrt{n_e[\text{m}^{-3}]}$	$7.7 \cdot 10^{13}$
γ_{pl}	> 20	ω_{las}/ω_{pl}	30.5
$W_{\max}$ (theoretical), MeV	> 400	$2\gamma_{pl}^2 \cdot m_e c^2$	930
$E_{\max}$, GV/cm	> 1	$m_e c\omega_{pl}/e$	1.3
λ_{pl}, μm	~ 20	$2\pi c/\omega_{pl}$	24.4
Minimum self-focusing power $P_{self-focus}$, TW	< 20	$\gamma_{pl}^2 2m_e^2 c^5/e^2$	15.8
Laser power P, TW	< 100	$4P_{self-focus}$	63
Radiation intensity in focus I, W/cm^2	$> 8.6 \cdot 10^{18}$	$4P/\pi d^2$	$1.2 \cdot 10^{19}$
Normalized vector potential a	> 2	$\sqrt{I/I_{rel}}$	2.4
Dephasing length l_{dephas}, mm	~ 10	$\gamma_{pl}^2 \cdot \lambda_{pl}$	22.7
Laser depletion length $l_{depl\ las}$, mm	~ 10	$\gamma_{pl}^2 \cdot c\tau_{las}$	11
Acceleration length l_{accel}, mm	~ 10	$\min\{l_{dephas}, l_{depl\ las}\}$	11

of $8.6 \cdot 10^{18}$ W/cm^2 from (12). In this case, the normalized vector potential a, determined from expressions (1) and (8), will be equal to 2.4.

As for the total acceleration length, it will be determined by the minimum value of the dephasing length l_{dephas} (18) and the depletion length of the laser pulse $l_{depl\ las}$ (21). In the case of our calculation parameters, the dephasing length of l_{dephas} is 22.7 mm, and the length depletion of the laser pulse $l_{depl\ las}$ is 11 mm. This makes it possible to assemble a compact laser-plasma accelerator based on relativistic wakefields with a characteristic plasma channel length of 11 mm, which corresponds to the reference value of $l_{accel} \sim 1$ cm.

The geometry required for such experiments can be implemented, in particular, with the aid of gas-filled capillaries, as in Refs. [19–22,26]. The processes of establishing the required density profile before propagation of a femtosecond laser pulse through a gaseous medium in a capillary are also actively being modeled [38].

Due to the fact that the ratio $l_{depl\ las} < l_{dephas}$ is true for the calculated set of parameters given in the table, in reality it will not be possible to achieve the values of $W_{\max}$, that were calculated using formula (16). However, modeling confirms that the achievable energy values of the accelerated electron beams can be relatively high and amount to 60–70% of the maximum theoretical values [39]. Example spectra of the electron bunches at the output of an accelerating channel with a length of 11 mm for the calculated experimental parameters are shown in Fig. 6.

The curves depicted in Fig. 6 were calculated using formulas for a one-dimensional model from Ref. [32]. The shape of the spectra is determined by the fact that different electrons from the bunch were trapped in the accelerating cavity at different points of time, with different energies and in different phases. Accordingly, they fall onto different phase-space trajectories (Fig. 4). In this case, most of the electrons in the bunch reach the maximum energy corresponding to their phase-space trajectory, while some of the surrounding electrons have already entered the deceleration phase and possess energy less than the

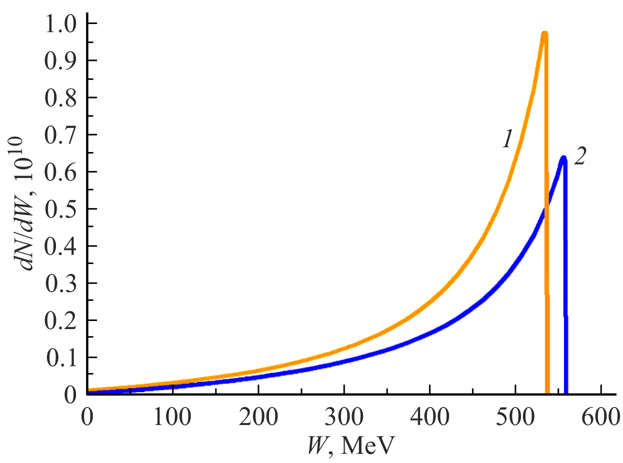

Figure 6. Theoretically calculated particle energy distribution spectra obtained via modeling of the laser wakefield acceleration of electrons under the propagation of a laser pulse ($\tau_{las\ imp} = 40$ fs, $a = 2.4$) through a sparse plasma with an average density of $n_{e\_opt} = 1.9 \cdot 10^{18}$ cm^{-3}. The curve *1* corresponds to the total charge of the captured electron beam at the level of 15 pC, curve *2* corresponds to 10 pC.

maximum, and some other electrons did not have time enough to be accelerated to maximum energies during their stay in the accelerating cavity. Moreover, the higher the total charge of the trapped electrons, the more strongly this charge affects the longitudinal accelerating field, and this process reduces the acceleration efficiency and the maximum achievable energy of the accelerated beam.

It should be noted that laser systems with a power of ~ 100 TW are used for laser acceleration of electrons in several laboratories around the world. SIOM in China can be pointed to as an example, where a 200-TW laser system produced electron beams with an energy of 580 MeV in a compact laser plasma accelerator, and then generated beams with energies in the range of 780–840 MeV and a relative energy spread in the bunch at the level of 0.2–0.4 % [29]. An experimental facility LUX is currently operating in Germany, where the Angus laser system with a power of 69 TW is used to generate electron beams with energy values of 258 MeV and a relative energy spread of 0.7 % [26,27]. Another example is the COXINEL laboratory in France, where a 50-TW laser produces electrons with energy values of 250 MeV, and the project is designed to generate electron beams with energies up to 400 MeV [29]. Theoretical calculations and simulation results show that the correct choice of a gas target can make it possible to obtain electron beams with energies of 500–600 MeV using laser systems with a power of the order of 50 TW [39]. In Russia, electron beams with energies of about 0.5 GeV were generated by the PEARL laser facility in Nizhny Novgorod [40–42], however, this facility is more powerful than the laser systems considered in this paper.

The aforementioned data show that multi-terawatt laser systems of not very high power (~ 100 TW) can be successfully used to produce electron beams with energies of the order of 0.5 GeV and a relatively small energy spread. Many of the laboratories mentioned above are pursuing the ambitious goal of creating a compact FEL using a laser-driven plasma wakefield accelerator in a low-density plasma. Calculations show that, depending on the chosen undulator design, it will be possible to achieve a brightness of soft X-ray radiation at the level of 10^{21} photon/mrad^2/mm^2 within 0.1% spectral bandwidth for incoherent radiation [43], and up to 10^{29} photon/mrad^2/mm^2 within 0.1% spectral bandwidth for coherent SXR radiation [44].

Conclusion

The paper presents a self-consistent calculation of the parameters of a relatively low-power laser system (~ 100 TW), which makes it possible to produce electron beams with an energy of the order of 0.5 GeV using the relativistic wakefield acceleration method in a compact laser-plasma accelerator. It was shown that, for laser pulses with a duration of 40 fs, at the optimal electron density value of $1.9 \cdot 10^{18}\,\mathrm{cm}^{-3}$, the experimental parameter γ_{pl} is 30.5, and relativistic self-focusing in plasma takes place at the laser power of $P_{self-focus} = 15.8$ TW. Bearing in mind a fourfold power margin, it was demonstrated that a 63-TW laser system is capable of producing electron beams with energies of ~ 550 MeV in a compact gaseous-plasma accelerator with an acceleration length of 11 mm, while the value of the maximum theoretically achievable electron beam energy is 930 MeV for the calculated experimental parameters. A comparison of the results obtained, with data from various experiments employing laser systems of comparable power, suggests that the calculation may be useful for creating a soft X-ray FEL based on a compact laser-plasma accelerator using a low-power laser system.

Conflict of interest

The author declares that he has no conflict of interest.

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