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Estimation of the effect of electron momentum parallel to the heterojunction boundary on the height of the first subband bottom in transistor superlattices with thin high barriers.

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Based on the analytical solution of the dispersion equation describing the allowed states in superlattices with thin high barriers, the influence of an electron momentum parallel to the heterojunction boundary on the height of the dimensional subband bottom in such superlattices was estimated for the transistor heterostructures, and, accordingly, the efficiency of those heterostructures located on the channel edges was evaluated. The study has shown that such sets of barriers prevent the transition from a narrow-band transistor channel to a wide-band material confining the channel (at least until intense transitions to the semiconductor upper valleys start in the channel itself).

Keywords: superlattice, potential barrier, dispersion equation, transistor heterostructure.

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As shown by experiments [1] and theoretical evaluation [2–4], the use of short-period GaAs/AlAs superlattices (or, more precisely, groups of barriers) allows for significant improvement of characteristics of heterostructure field-effect transistors. Here a natural question arises: will such a group of barriers always be efficient? In particular, at what momenta parallel to the heterojunction this system will stop hindering intense transitions from the channel to the strong-scattering region [5]? The maximum coefficient of transmission through a superlattice fragment (group of barriers) cannot occur lower than the first subband bottom in the relevant superlattice. Essentially, this is a question about the dimensional subband positions in the given superlattices. On the one hand, for Bloch functions, which are the solutions to such problems, namely for

$$\psi(x) = u(x)\exp(iKx), \tag{1}$$

the equation describing the dimensional subbands with the Kroning–Peni potential (Fig. 1) has long been known (see, e.g. [6,7]):

$$\cos\big(K(a+b)\big) = \cos(ka)\mathrm{ch}(\gamma b) - 0.5\left[\frac{k}{\gamma} - \frac{\gamma}{k}\right]\sin(ka)\mathrm{sh}(\gamma b). \tag{2}$$

It is just what defines the states allowed for all the existing cases. On the other hand, this is a rather complex transcendental equation with respect to three unknowns: wave vectors K, k and damping decrement γ; this makes the analytical examination extremely difficult even in the simplest cases, unless significant simplifications are made. The situation does not get changed significantly by the fact that two unknowns, wave vector k and damping decrement γ, are interrelated:

$$\gamma = \sqrt{\frac{2m_2}{\hbar^2}(U-E)}, \quad k = \sqrt{\frac{2m_1}{\hbar^2}E}. \tag{3}$$

Here U is the barrier height, b is the barrier thickness, a is the inter-barrier distance, E is the electron energy, $\hbar$ is the Planck constant, m_1, m_2 are the electron effective masses in the narrow-band and wide-band materials (well and barrier).

Thus, various approaches have been developed (see, e.g. [7]), which have allowed making a number of general conclusions, for instance, about the conduction band bottom reduction relative to the quantum level in weakly coupled wells. However, it seems that the problem of a superlattice with a set of strongly coupled wells separated by thin high barriers (δ-barriers), which is similar to that described above, earlier either has not arisen at all, or has not been studied analytically. At the same time it is known that in other problems the very use of the δ-barrier approximation [8] often strongly simplifies the situation thus allowing for obtaining their solutions, albeit approximate, in the analytical form (see, e.g. [9–11]).

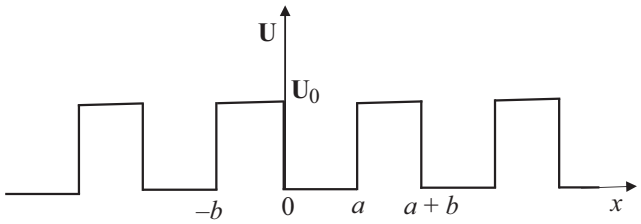

Figure 1. One-dimensional periodic Kroning–Peni potential.

Consider a periodic potential (Fig. 2) consisting of a set of δ barriers located at distance a from each other.

As in [6,7], it is easy to obtain for wave functions (1) a dispersion equation (equation for characteristic numbers) with respect to two independent variables, which is much simpler than initial equation (2):

$$2k\big(\cos(Ka)-\cos(ka)\big)-y\sin(ka)=0. \tag{4}$$

Here parameter y defines „power" of the δ barrier and, under the assumption that transmission coefficients through the δ barrier and rectangular barrier are the same, has the following form:

$$y^2=\frac{2m_1}{\hbar^2}\frac{m_1}{m_2}\frac{\left[U-E\left(1-\left(\frac{m_1}{m_2}\right)\right)\right]^2}{U-E} \times \mathrm{sh}^2\left(\sqrt{\frac{2m_2}{\hbar^2}(U-E)b^2}\right). \tag{5}$$

In the limiting case of just a very „thin and high" barrier, obtain

$$y=\frac{2m_1}{\hbar^2}Ub. \tag{6}$$

Note that (6) includes right the effective mass m_1 in the narrow-bandgap material. Let us consider the first miniband and, accordingly, equation (4) in various limiting cases.

High thick barriers are characterized by

$$y\gg k, \qquad ka\approx\pi. \tag{7}$$

For such a superlattice, solution of (4), as well as of (2), will be close to that for an infinitely deep quantum well.

For medium-thickness barriers,

$$ya\approx\pi, \qquad ka\approx\pi/2. \tag{8}$$

For thin barriers

$$ya<\pi/2. \tag{9}$$

Expanding the trigonometric functions included in (4) relative to points π, $\pi/2$ and 0 into Taylor series, obtain for high thick barriers the following wave vector

$$k_{\min}=\frac{\pi y}{4+ya}, \tag{10}$$

that for medium-thickness barriers is

$$k_{\min}=\frac{-4+2x+\pi ya+2\sqrt{4-4\pi+x^2+ya(8-2x)+2y^2a^2}}{2(4a+ya^2)}, \tag{11}$$

that for thin barriers is

$$k_{\min}=\sqrt{\frac{6y}{a(6+ya)}}. \tag{12}$$

For the miniband top, wave vector in all the three cases is

$$k_{\max}=\pi/a. \tag{13}$$

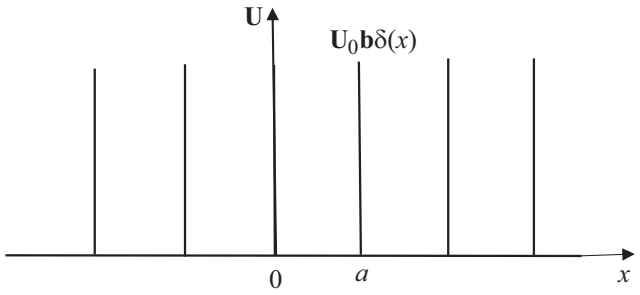

Figure 2. One-dimensional periodic potential of the δ-barriers.

The following estimations may be made. Typical thickness of AlAs barriers in the GaAs/AlAs lattice does not exceed three monatomic layers. The inter-barrier distance is four to eight GaAs monolayers. In this case, the first level height in an infinite-wall GaAs well of such a width is 6 to 1.5 eV, while the barrier height on the heterojunction boundary is only 1.04 eV. Thus, the top of already the first miniband passes through the tops of potential barriers. At such a barrier height and GaAs thickness of four to eight monolayers, product ay at low electron energies is about 3.8−7.6, which corresponds to the case of thick barriers. At the same time, ay is approximately 0.7−1.4 already at half barrier height; this is indeed the case of thin barriers. Taking this into account ($ay\ll 6$), derive from (12) and (5), by expanding the hyperbolic sine in a series up to the second term, a quadratic equation for the miniband bottom energy $E_{\min}$:

$$aE_{\min}=b(U-E_{\min})\left(1+\frac{m_2}{3\hbar^2}(U-E_{\min})b^2\right). \tag{14}$$

Electron momentum in the channel $K_{\|}$ parallel to the heterojunction boundary is known [5] to reduce the barrier height by

$$\Delta U=\frac{\hbar^2K_{\|}^2}{2m_1}\left(1-\frac{m_1}{m_2}\right). \tag{15}$$

Fig. 3 presents the dependences of the miniband bottom energy position (solution of equation (14)) on the distance between AlAs barriers for two cases: the longitudinal motion energy is zero and the longitudinal motion energy is 0.3 eV which gives $\Delta U\approx 0.27$ eV as per (15).

Evidently, when the barrier height decreases by 0.27 eV, the miniband bottom drops significantly less: by approximately 0.17−0.13 eV depending on the inter-barrier distance. Even when the inter-barrier distance becomes 2 nm, the miniband bottom height does not fall below 0.3 eV. As noted earlier, the lower quantum level position in a separate superlattice fragment always lies above the miniband bottom. However, since at the inter-barrier distance of about 2 nm or more and longitudinal motion energy equal to the intervalley gap the height of the miniband bottom appears to be at the level of the L valley in the channel material, the efficiency of using such structures in devices

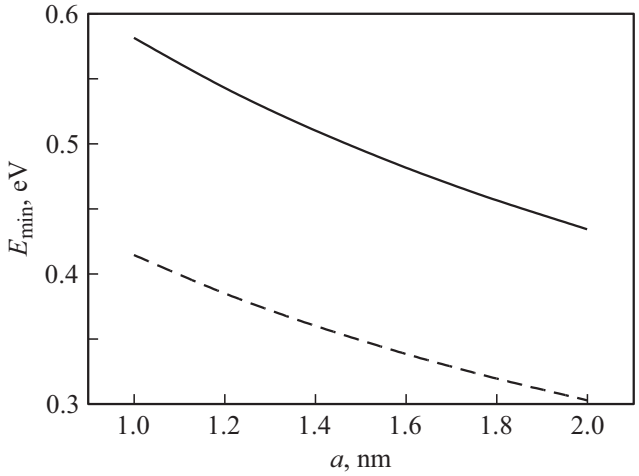

Figure 3. The first miniband bottom energy versus the distance between AlAs barriers. The solid line represents the case when the electron longitudinal motion energy is zero, the dashed line is for the case when the energy is 0.3 eV.

should preferably be assessed via more accurate models (e.g. [12]).

Thus, in this study there was obtained a dispersion equation describing the allowed states in superlattices with thin high barriers, as well as its analytical solutions. The case of transistor structures was analyzed. The obtained results allow a conclusion that, in transistor heterostructures with a channel confined by a set of AlAs barriers, the inter-barrier distance should preferably be less than 2 nm. Such superlattice fragments will prevent electron transitions into the wide-bandgap material (the barrier system) at least until the beginning of intense transitions into the channel-material upper valleys.

Conflict of interests

The author declares that he has no conflict of interests.

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