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Control of permanent magnet material characteristics and magnetic field calculation based on demagnetization factor value

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The article shows that demagnetization factor of a permanent magnet, defined as the ratio of magnetic field strength to magnetization value, depends on hysteresis loop parameters, in addition to the shape, the relationships between magnet sizes, and the direction of magnetization. The analytical expression for calculating the demagnetization factor is obtained. It allows constructing the load line of a magnet. It is shown that the magnetization value obtained from the load line makes it possible calculating field parameters close to observed. Graphical dependencies to determine demagnetization factor values of ring-shaped axially and radially magnetized permanent magnets are constructed.

Keywords: demagnetization factor, magnetic field strength, magnetization, load line, permanent magnet.

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Introduction

In practice, it is often necessary to check compliance of magnetic properties of permanent magnets exemplars with nominal parameters of a said material grade. This task is solved by one of the two methods:

• by measuring demagnetization of witness samples that are usually discs with a strictly determined size ratio;

• by measuring magnetic field strength values in a number of observation points and comparing the measured values with the calculated ones.

An advantage of the second method is advantageous is universality: it can evaluate a quality of the permanent magnet of any form and sizes.

The present study is dedicated to developing a calculation part of this method.

The permanent magnets with often used forms, which are axially magnetized disc-shaped magnets and axially and radially magnetized ring-shaped magnets, have axes at which the magnetic fields can be calculated by quite simple formulas.

1. The axially magnetized disc magnet.

The axial z-component of the magnetic field strength along an axial axis of the magnet outside a magnet region [1]:

$$H_z(p) = -\frac{M_{zq}}{2}\left(\frac{p}{\sqrt{\frac{D^2}{4}+p^2}} - \frac{p+h}{\sqrt{\frac{D^2}{4}+(p+h)^2}}\right), \quad (1)$$

where M_{zq} — magnetization of the magnet in a certain point q that belongs to the magnet region Ω, $q \in \Omega$, p — the distance of an observation point to a magnet surface, D — the magnet diameter, h — the magnet height in a magnetization direction.

2. The axially magnetized ring-shaped magnet.

The axial z-component of the magnetic field strength along the axial axis of the magnet [1,2]:

$$H_z(z) = -\frac{M_{zq}}{2}\left(\frac{z-\frac{h}{2}}{\sqrt{\frac{D_2^2}{4}+\left(z-\frac{h}{2}\right)^2}} - \frac{z-\frac{h}{2}}{\sqrt{\frac{D_1^2}{4}+\left(z-\frac{h}{2}\right)^2}} - \frac{z+\frac{h}{2}}{\sqrt{\frac{D_2^2}{4}+\left(z+\frac{h}{2}\right)^2}} + \frac{z+\frac{h}{2}}{\sqrt{\frac{D_1^2}{4}+\left(z+\frac{h}{2}\right)^2}}\right), \quad (2)$$

where M_{zq} — magnetization of the magnet in a certain point $q \in \Omega$ of the magnet region, z — the coordinate of the observation point, D_1 — the internal diameter of the magnet, D_2 — the external diameter of the magnet, h — the magnet height in the magnetization direction.

3. The radially magnetized ring-shaped magnet.

The axial z-component of the magnetic field strength along the axial axis of the magnet [1,2]:

$$H_z(z) = -\frac{M_{rq}}{2}\left(\frac{\frac{D_1}{2}}{\sqrt{\frac{D_1^2}{4}+\left(z-\frac{h}{2}\right)^2}} - \frac{\frac{D_1}{2}}{\sqrt{\frac{D_1^2}{4}+\left(z+\frac{h}{2}\right)^2}} - \frac{\frac{D_2}{2}}{\sqrt{\frac{D_2^2}{4}+\left(z-\frac{h}{2}\right)^2}} + \frac{\frac{D_2}{2}}{\sqrt{\frac{D_2^2}{4}+\left(z+\frac{h}{2}\right)^2}} + \ln\frac{\left(\frac{D_2}{2}+\sqrt{\frac{D_2^2}{4}+\left(z-\frac{h}{2}\right)^2}\right)\left(\frac{D_1}{2}+\sqrt{\frac{D_1^2}{4}+\left(z+\frac{h}{2}\right)^2}\right)}{\left(\frac{D_2}{2}+\sqrt{\frac{D_2^2}{4}+\left(z+\frac{h}{2}\right)^2}\right)\left(\frac{D_1}{2}+\sqrt{\frac{D_1^2}{4}+\left(z-\frac{h}{2}\right)^2}\right)}\right), \quad (3)$$

where M_{rq} — magnetization of the magnet in the radial direction in a certain point $q \in \Omega$ of the magnet region, z — the coordinate of the observation point, D_1 — the internal diameter of the magnet, D_2 — the external diameter of the magnet, h — the magnet height.

The relationships $(1)-(3)$ were obtained when assuming that magnetization is permanent. However, as shown in the present study, M_{zq} and M_{rq} shall be values of magnetization in a certain point of the magnet region, whose position depends not only on magnetic properties of the magnet material, but on a position of the observation point z as well. It is also shown that admissibility itself of using the given relationships for calculating the permanent magnets fields is determined by fulfilment of a number of conditions.

The aim of the present study is

• to determine conditions, at which the known formulas for calculating the magnetic field strength $(1)-(3)$ are valid and, therefore, can evaluate the quality of parameters of a hard magnetic materials, from which a specific magnet is made of;

• to show that these formulas can use a single value of magnetization for calculating the magnetic fields in any points both within the magnet region and outside it;

• to present tabular and graphic data and clarifications for performing the calculations, which are acceptable when controlling compliance of the material to the nominal parameters;

• using examples that cover various kinds of practically used hard magnetic materials, to show proximity of the calculated and measured values of the magnetic magnitudes.

Many researchers of the permanent magnets properties focus their attention on a demagnetization factor [3–6]. The demagnetization factor is a parameter which characterizes a load line and, along with the demagnetization curve, comprehensively describes a specific permanent magnet.

It is shown in the present study that the demagnetization factor can be decisive in determining the magnetization values to be used in the known relationships $(1)-(3)$.

1. Calculation of the magnetic field strength of the permanent magnet

We write an expression for calculating the strength of the magnetic field that is created by the magnetized permanent magnet in any point of space: within the magnet region or outside it [1,2,7,8]:

$$\mathbf{H}_q = -\,grad_q \iiint\limits_{\Omega} \frac{\mathbf{M}_p\mathbf{R}_{pq}}{4\pi R_{pq}^3}\,dV_p = -\iiint\limits_{\Omega} grad_q \frac{\mathbf{M}_p\mathbf{R}_{pq}}{4\pi R_{pq}^3}\,dV_p, \quad (4)$$

where $\mathbf{M}_p$ — magnetization of the permanent magnet in the point p of the magnet region, $\mathbf{R}_{pq}$ — the radius vector from the point p of the magnet region into the observation point q, Ω — the space area occupied by the permanent magnet.

Let the magnet be magnetized along the axis x, then

$$\mathbf{M}_p = M_p\mathbf{e}_x.$$

As it is differentiated under the integral sign by q, in the right hand system of coordinates the expression (4) for the vector components of the magnetic field strength will take the following form [8]:

$$H_{xq} = \iiint\limits_{\Omega} M_pK(p,q)dV_p;$$

$$H_{yq} = \iiint\limits_{\Omega} M_pL(p,q)dV_p;$$

$$H_{zq} = \iiint\limits_{\Omega} M_pP(p,q)dV_p. \quad (5)$$

The functions $K(p,q)$, $L(p,q)$, $P(p,q)$ are nuclei of the integral relationships (5).

All further relationships will be constructed for H_{xq} being a component that determines distribution of magnetization and, therefore, is of the greatest interest.

We consider the point $q \notin \Omega$. Let the nucleus $K(p,q)$ is sign-constant by q. Then, using the generalized theorem about the average [9], it can be written

$$H_{xq} = M_{\xi}\iiint\limits_{\Omega} K(p,q)dV_p. \quad (6)$$

In the expression (6) M_{ξ} is a value of magnetization of the permanent magnet in a certain point $\xi \in \Omega$. The position of this point is determined both by functions that describe behavior of the nuclei and the function of distribution of magnet magnetization as well. For the points $q \in \Omega$ the nucleus $K(p,q)$ is sign-constant by its nature [8].

It follows from the expression (6) that for the permanent magnet made of any hard magnetic material, the field in the observation point q, for which the nuclei of the integral relationships (5) are sign-constant, can be calculated by integrating the nuclei $K(p,q)$, $L(p,q)$, $P(p,q)$ at the known value of magnetization in several points of the magnet ξ_i.

The position of the points ξ_i is determined, in particular, by the position of the observation point q and a nature of magnetization distribution.

The values of the integral from the nucleus in (6) do not depend on the magnetic parameters of the permanent magnet. It is this circumstance that allowed obtaining so simple expression for the fields of some forms of the permanent magnets $(1)-(3)$.

Due to the fact that magnetization M_p is sign-constant within the magnet region, then using the generalized theorem about the average it is possible to write an expression for the component H_{xq} of the magnetic field strength as

$$H_{xq} = \iiint\limits_{\Omega} M_p K(p,q) dV_p = K(\alpha,q) \iiint\limits_{\Omega} M_p dV_p = K(\alpha,q) M_{aver} \Omega = M_{aver} K(\alpha,q) \Omega, \quad (7)$$

where the point α belongs to the magnet region.

According to the theory about the average [9,10], there is a point t, for which the expression (7) can be written as

$$H_{xq} = M_{aver} \iiint\limits_{\Omega} K(p,t) dV_p. \quad (8)$$

It follows from the foregoing that the magnetic field strength in any observation point q can be calculated using a single value of magnetization that is equal to the average magnetization and the integrals from the nuclei, which are calculated in some points different from the observation point. This statement is true for the observation points, in which the nuclei of the integral relationships (5) are sign-constant.

It follows from the expressions (6) and (8) that the exact value of magnetization M_ξ is determined as

$$M_\xi = \left(\iiint\limits_{\Omega} K(p,t) dV_p \Big/ \iiint\limits_{\Omega} K(p,q) dV_p \right) \cdot M_{aver}. \quad (9)$$

In the obtained relationship the point t does not coincides with the point q. Therefore, it is necessary to determine how M_ξ differs from M_{aver}. We will show that the relationship (9) depends not only on the values of the nucleus of the integral relationship (6), but on the magnetic parameters of the hard magnetic material.

Using a solution of the integral equation that describes distribution of magnetization of the magnetized magnet [11], we write

$$M_p = M_{rfic} \Big/ \left(1 + \chi \left| \iiint\limits_{\Omega} K_{\varsigma,\xi} dV_\xi \right| \right), \quad (10)$$

where M_{rfic} — the value of residual magnetization of the linearized curve of demagnetization, χ — reverse susceptibility of the linearized curve of demagnetization.

Then, the average value of magnetization is

$$M_{aver} = \frac{1}{\Omega} \iiint\limits_{\Omega} \left(M_{rfic} \Big/ \left(1 + \chi \left| \iiint\limits_{\Omega} K_{\varsigma,\xi} dV_\xi \right| \right) \right) dV_p = M_{rfic} \Big/ \left(1 + \chi \left| \iiint\limits_{\Omega} K_{t,\xi} dV_\xi \right| \right), \ t \in \Omega. \quad (11)$$

The exact value of magnetization

$$M_\xi = \iiint\limits_{\Omega} M_p K_{p,q} dV_p \Big/ \iiint\limits_{\Omega} K_{p,q} dV_p = M_{rfic} \Big/ \left(1 + \chi \left| \iiint\limits_{\Omega} K_{r,\xi} dV_\xi \right| \right), \quad (12)$$

where the point q can both belong to the magnet region and be outside it and $r \in \Omega$.

From the relationships (11) and (12) we obtain

$$M_\xi = M_{aver} \left(1 + \chi \left| \iiint\limits_{\Omega} K_{t,\xi} dV_\xi \right| \right) \Big/ \left(1 + \xi \left| \iiint\limits_{\Omega} K_{r,\xi} dV_\xi \right| \right). \quad (13)$$

The magnetization determined in this way allows exactly describing the fields both within the magnet region and outside it.

It is obvious that the position of the point r is determined via the observation point q, although it belongs to the magnet region. We note that r, $t \in \Omega$. Therefore, estimate of admissibility of replacing the exact value of M_ξ with the approximate one M_{aver} and its error can be made for any point with taking into account a magnet size ratio, the magnetization direction and, of course, the magnetic parameters of the hard magnetic material.

From expression (13) we get

$$\text{if } \chi \left| \iiint\limits_{\Omega} K_{r,\xi} dV_\xi \right| \ll 1 \text{ then } M_\xi \approx M_{aver},$$

$$\text{if } \chi \left| \iiint\limits_{\Omega} K_{r,\xi} dV_\xi \right| \approx 1 \text{ then } M_\xi \approx M_{aver},$$

$$\text{if } \chi \left| \iiint\limits_{\Omega} K_{r,\xi} dV_\xi \right| \gg 1 \text{ then}$$

$$M_\xi \approx M_{aver} \left| \iiint\limits_{\Omega} K_{t,\xi} dV_\xi \right| \Big/ \left| \iiint\limits_{\Omega} K_{r,\xi} dV_\xi \right|.$$

Thus, in any point of space it is possible to calculate the strength of the magnetic field that is created by any permanent magnet made of the hard magnetic material via the single value of magnetization, namely, the average value of magnetization within the magnet region.

2. Demagnetization factor and determination of the average value of magnetization

The average value of magnetization within the magnet region is determined by the parameters of the permanent

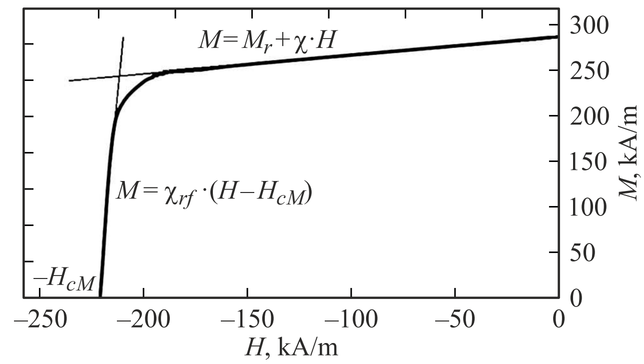

Figure 1. Linearization of the demagnetization curve.

magnet material, which are presented by a dependence of magnetization on a demagnetization fields, the so-called demagnetization curve, as well as by the form, the magnet size ratio and the magnetization direction of the magnet. In turn, the distribution of strength of the demagnetization magnetic field in the magnet can be determined by means of the relationship (4) via the distribution of magnetization. The average value of magnetization can be determined by solving a magnetostatic problem.

The integral equation that describes the linearized magnetostatic problem is as follows [11]:

$$M_p - \chi \iiint_\Omega M_\xi K_{p,\xi} dV_\xi = M_{rfic}.$$

According to the study [11], a solution of this equation is described by the expression (10).

At this, the value of magnetization in the considered point $p \in \Omega$ is calculated according to the expression (10) in the point $\varsigma \in \Omega$ that is related to the point p by the relationship

$$K_{p,\delta}\Omega = \iiint_\Omega K_{\varsigma,\xi} dV_\xi.$$

For an initial portion of the demagnetization curve (Fig. 1):

$$M_q = M_r + \chi_r H_q \tag{14}$$

and

$$M_{rfic} = M_r, \quad \chi = \chi_r, \quad H_q = \frac{M_q - M_r}{\chi}.$$

Then

$$M_{aver} = \frac{1}{\Omega} \iiint_\Omega M_p dV_p$$

$$= \frac{1}{\Omega} \iiint_\Omega \left(M_r \Big/ \left(1 + \chi \left| \iiint_\Omega K_{\varsigma,\xi} dV_\xi \right| \right) \right) dV_p.$$

At the same time

$$H_{Maver} = \frac{M_{aver} - M_r}{\chi}$$

$$= \frac{1}{\Omega_\chi} \iiint_\Omega \left(M_r \Big/ \left(1 + \chi \left| \iiint_\Omega K_{\varsigma,\xi} dV_\xi \right| \right) \right) dV_p$$

$$- \frac{M_r}{\chi}.$$

Thus, the demagnetization factor [12,13], which determines the average value of magnetization is equal

$$N_{Maver} = -\frac{H_{Maver}}{M_{aver}}$$

$$= \frac{1}{\chi} \left(\left(\Omega \Big/ \iiint_\Omega \left(dV_p \Big/ \left(1 + \chi \left| \iiint_\Omega K_{\varsigma,\xi} dV_\xi \right| \right) \right) \right) - 1 \right).$$

Since it is integrated by all the points $p \in \Omega$, it can be expressed as follows

$$N_{Maver} = \frac{1}{\Omega}$$

$$\times \left(\left(\Omega \Big/ \iiint_\Omega \left(dV_p \Big/ \left(1 + \chi \left| \iiint_\Omega K_{p,\xi} dV_{zeta} \right| \right) \right) \right) - 1 \right), \tag{15}$$

i.e. via integration by the points p.

For the portion of the demagnetization curve, which is pre-defined by the relationship (Fig. 1):

$$M = \chi_{rf}(H - H_{cM}) \tag{16}$$

and

$$M_{rfic} = -\chi_{rf} H_{cM}, \quad \chi = \chi_{rf}, \quad H = \frac{M}{\chi_{rf}} + H_{cM},$$

where H_{cM} is a coercive force by magnetization.

Then

$$H_{Maver} = \frac{M_{aver}}{\chi_{rf}} + H_{cM},$$

$$H_{Maver} = \frac{1}{\chi_{rf}\Omega}$$

$$\times \iiint_\Omega \left(M_{rfic} \Big/ \left(1 + \chi_{rf} \left| \iiint_\Omega K_{\varsigma,\xi} dV_\xi \right| \right) dV_p \right) + H_{cM}$$

and

$$N_{Maver} = \frac{1}{\chi_{rf}}$$

$$\times \left(\Omega \Big/ \iiint_\Omega \left(dV_p \Big/ \left(1 + \chi_{rf} \left| \iiint_\Omega K_{\varsigma,\xi} dV_\xi \right| \right) \right) - 1 \right). \tag{17}$$

It follows from the expressions (15), (17) that the demagnetization factor is determined both by geometrical parameters and physical properties of the magnet material has the same expression for all the portions of a hysteresis loop.

Let us analyze the expressions (15), (17) that are obtained for the demagnetization factor.

If

$$\chi\Big|\iiint\limits_{\Omega} K_{\varsigma,\xi}\,dV_\xi\Big| \ll 1,$$

then

$$N_{Maver} \approx \frac{1}{\chi}\left(\Omega\Big/\left(\Omega-\Omega\chi\Big|\iiint\limits_{\Omega} K_{p,\xi}\,dV_\xi\Big|\right)-1\right)$$

$$\approx\Big|\iiint\limits_{\Omega} K_{\varsigma,\xi}\,dV_\xi\Big|_{aver}.$$

If

$$\chi\Big|\iiint\limits_{\Omega} K_{\varsigma,\xi}\,dV_\xi\Big| \gg 1,$$

then

$$N_{Maver} \approx \frac{1}{\chi}\left(\Omega\Big/\Omega\Big/\Big|\iiint\limits_{\Omega} K_{\theta,\xi}\,dV_\xi\Big|\right)-1$$

$$\approx\Big|\iiint\limits_{\Omega} K_{\theta,\xi}\,dV_\xi\Big|.$$

Hence, it is concluded:

1) At the initial portion of the demagnetization curve

$$\chi\Big|\iiint\limits_{\Omega} K_{\varsigma,\xi}\,dV_\xi\Big| \ll 1,$$

the demagnetization factor by the average magnetization is determined by the geometrical parameters of the magnet: the form, the size ratio and the magnetization direction,

$$N_{Maver} \approx \Big|\iiint\limits_{\Omega} K_{\varsigma,\xi}\,dV_\xi\Big|_{aver}, \tag{18}$$

and is equal to the average value of the nucleus of the integral expression.

At the final portion of the demagnetization curve

$$\chi\Big|\iiint\limits_{\Omega} K_{\varsigma,\xi}\,dV_\xi\Big| \gg 1$$

the demagnetization factor by the average magnetization is determined by the geometrical parameters of the magnet: the form, the size ratio and the magnetization direction.

$$N_{Maver} \approx \Big|\iiint\limits_{\Omega} K_{\theta,\xi}\,dV_\xi\Big|. \tag{19}$$

The point θ is a certain point that belongs to the magnet region. Strictly speaking, the value of

$$\Big|\iiint\limits_{\Omega} K_{\theta,\xi}\,dV_\xi\Big|$$

does not coincide with

$$\Big|\iiint\limits_{\Omega} K_{\varsigma,\xi}\,dV_\xi\Big|_{aver}.$$

3) At the curvilinear portion of the curve

$$\chi\Big|\iiint\limits_{\Omega} K_{\varsigma,\xi}\,dV_\xi\Big| \approx 1$$

the demagnetization factor by the average magnetization is determined by the physical parameters of the magnetic

$$N_{Maver} \approx \frac{1}{\chi}. \tag{20}$$

It follows from the relationships (18)−(20) that it is possible to tabularize the values of the demagnetization factor for the various forms, size ratios and magnetic parameters of the permanent magnets. For the ring-shaped magnets that are axially or radially magnetized, these values are shown in the figures 2−5 as graphical dependences. The value of the demagnetization factor, as taken from the graph, can be used for determining the average magnetization and subsequently calculating the magnetic fields both within the magnet region and outside it as well.

3. Experimental and computational results

The experiments were carried out with disc-shaped and ring-shaped permanent magnets that were made of ferrite, cast and rare-earth hard magnetic materials.

The magnetic fields were measured by the magnetometer IMI-641 of the class 2.5, i.e. the fields were measured

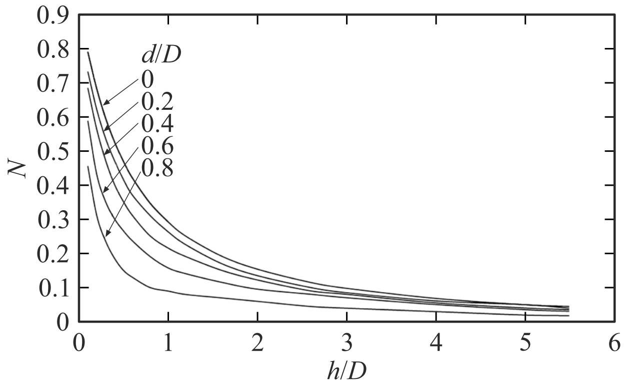

Figure 2. Demagnetization factor of the axially-magnetized ring-shaped magnet provided that $\chi \cdot N \ll 1$ or $\chi \cdot N \gg 1$.

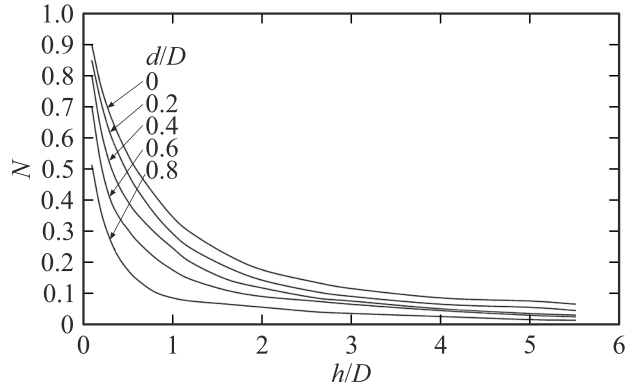

Figure 3. Demagnetization factor of the axially-magnetized ring-shaped magnet provided that $0.8 < \chi \cdot N < 1.2$.

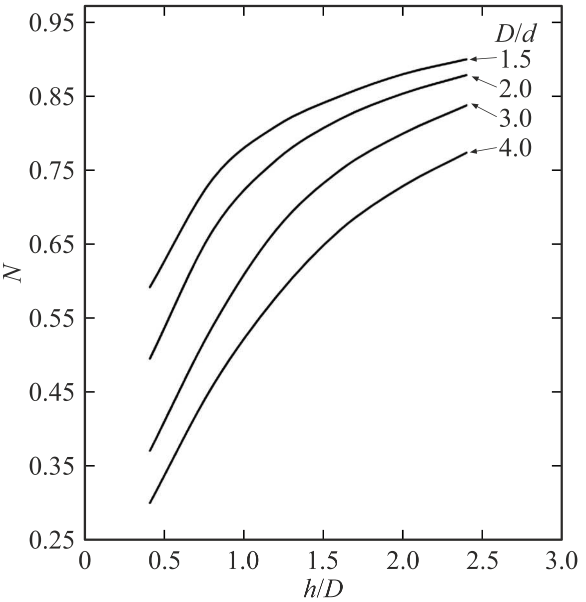

Figure 4. Demagnetization factor of the radially-magnetized ring-shaped magnet provided that $\chi \cdot N \ll 1$ or $\chi \cdot N \gg 1$.

with an error of 2.5% at the end of the scale of each measurement range: 1–200 mT, 200–2000 mT. The instrument passport specifies that it is designed „for measuring ... magnetic induction of constant magnetic fields in gaps and on surfaces of the magnetic systems, ... as well as for measuring residual magnetization of the materials when assembling, installing, operating ... the magnetic systems in mechanical engineering, instrumentation, electrical engineering and power engineering ...".

The magnetic fields of the disc-shaped magnets were measured by a flat probe of the thickness of 1 mm with the Hall sensor. The magnetic fields of the ring-shaped magnets were measured by a round probe of the diameter of 4,mm with the Hall sensor. During the measurements, the probe position was axially fixed by shims, whose total thickness was measured with an error of at most 0.1 mm. Radially, the probe position was determined based on the fact that

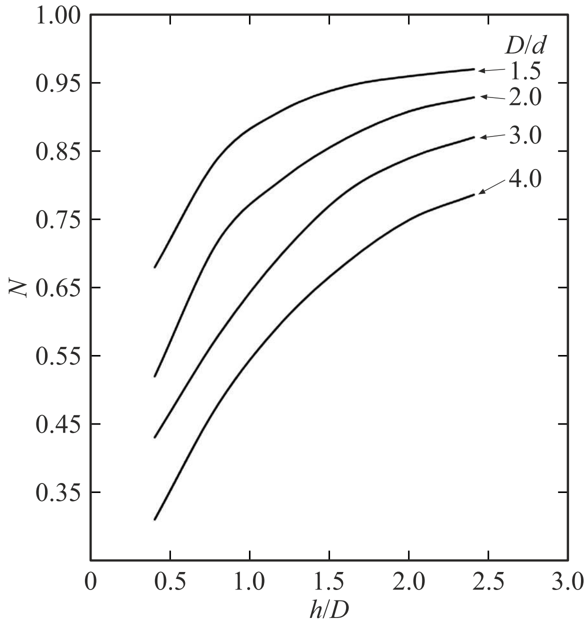

Figure 5. Demagnetization factor of the radially-magnetized ring-shaped magnet provided that $0.8 < \chi \cdot N < 1.2$.

the axial points in the magnets of the surveyed form are points of an extremum of induction of the magnetic field.

The demagnetization factor was determined by the characteristics of the figures 2–5 for each magnet by its form, size ratio, magnetization direction and parameters of the hard magnetic material. A load line was constructed on the material demagnetization curves by a respective demagnetization factor and magnetization of the magnet was determined on them, too. Its value was used when determining the magnetic field by the formulas (1) and (2).

The calculation also included the observation points, in which the constant-sign condition of the nucleus $K(p, q)$ of the integral expression (6) is not fulfilled. These results are incorrect and demonstrate inadmissibility of measurements in such points for evaluating the quality of the parameters of the hard magnetic material by comparing with calculated values.

3.1. Example 1

The axially-magnetized magnet disc with the sizes $D = 30$ mm, $h = 6$ mm. Here, $d/D = 0$, $h/D = 0.2$. Fig. 1 shows results for the values of the demagnetization factor and reverse susceptibility for the considered hard magnetic material (Fig. 6). The magnet is made of the ferrite hard magnetic material. The demagnetization factor of this magnet should be assumed to be equal to $N = 0.73$, as $\chi \cdot N$ is close to unity (Fig. 3) and the magnetization value that follows from a kind of the demagnetization curve (Fig. 6) is $\mu_0 \cdot Maver = 0.36$ T. Fig. 6 shows results of measurement and calculation of induction of the magnetic field at the axis of the considered magnetic element. It has been calculated according to the formula (1).

Table 1. Demagnetization factor, reverse susceptibility and their product in Example 1

Demagnetization factor (from the figures 2, 3)		Reverse susceptibility		Product of the demagnetization factor and the reverse susceptibility	
N_1	N_2	χ_{dif1}	χ_{dif2}	$N_1 \cdot \chi_{dif1}$	$N_2 \cdot \chi_{dif2}$
0.68	0.73	1.45	1.50	0.986	1.095

3.2. Example 2

The axially-magnetized magnet disc with the sizes $D = 22.6$ mm, $h = 30$ mm. Here, $d/D = 0$, $h/D = 1.33$. The magnet is made of the cast hard magnetic material of the ALNICO type with the demagnetization curve shown in Fig. 7. Here, $\chi_{dif} \cdot N = 36$. The demagnetization factor of this magnet is $N = 0.28$ (Fig. 2) and the magnetization value is $\mu_0 \cdot Maver = 0.221$ T. Fig. 7 shows results of measurement and calculation of induction of the magnetic field at the axis of the considered magnetic element. It has been calculated according to the formula (1).

3.3. Example 3

The axially-magnetized magnet disc with the sizes $D = 30$ mm, $h = 10$ mm. Here, $d/D = 0$, $h/D = 0.33$. The magnet is made of the cast hard magnetic material of the Nd-Fe-B type with the demagnetization curve shown in Fig. 8. Here, $\chi_{dif} \cdot N = 0.54$. The demagnetization factor is $N = 0.64$ (Fig. 2), the magnetization value is $\mu_0 \cdot Maver = 1.32$ T. Fig. 8 shows results of measurement and calculation of induction of the magnetic field at the axis of the considered magnetic element. It has been calculated according to the formula (1).

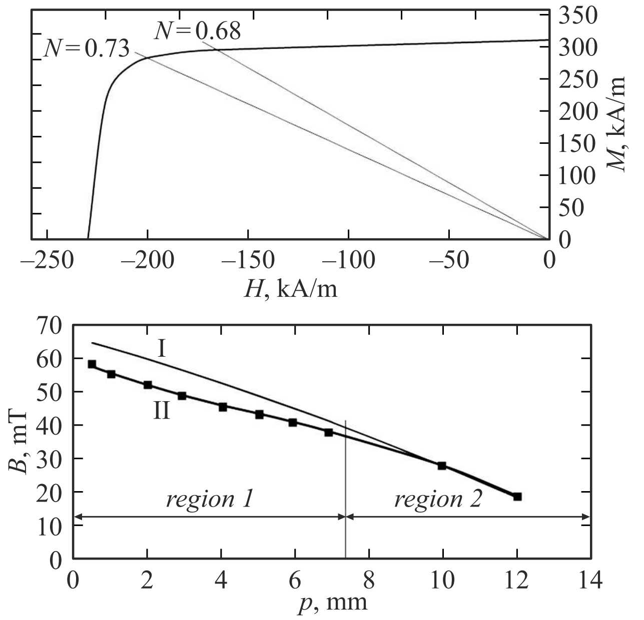

Figure 6. Curve of demagnetization of the ferrite hard magnetic material. The calculated I and measured II values of magnetic induction at an axis of the disc-shaped magnet. The constant-sign condition of the nucleus of the integral relationship is not fulfilled with in the region *1*, but fulfilled in the region *2*.

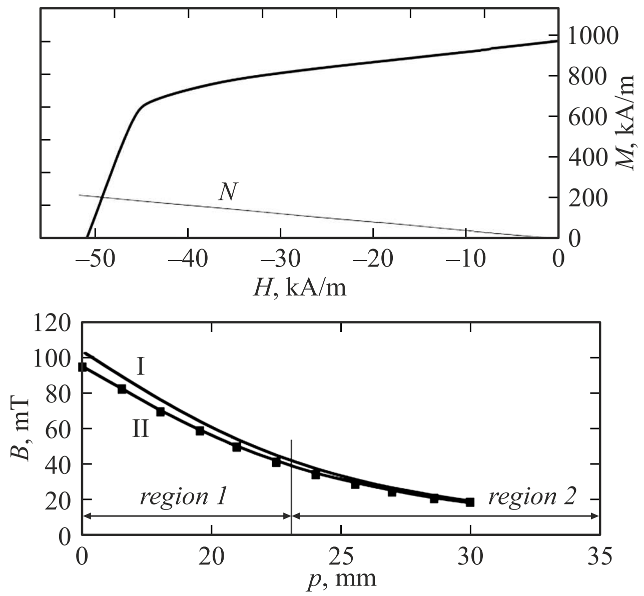

Figure 7. Curve of demagnetization of the cast hard magnetic material of the ALNIKO type. The calculated I and measured II values of magnetic induction at an axis of the disc-shaped magnet. The constant-sign condition of the nucleus of the integral relationship is not fulfilled with in the region *1*, but fulfilled in the region *2*.

3.4. Example 4

The axially-magnetized magnetic ring that has the sizes $D = 30$ mm, $d = 16$ mm, $h = 10$ mm. Here, $d/D = 0.53$, $h/D = 0.33$. The magnet is made of the hard magnetic material of the ALNICO type with the demagnetization curve shown in Fig. 9. Here, $\chi_{dif} \cdot N = 3.2$. The demagnetization factor is $N = 0.4$ (Fig. 2) and the magnetization value is $\mu_0 \cdot Maver = 0.32$ T. Fig. 9 shows results of measurement and calculation of induction of the magnetic field at the axis of the considered magnetic element. It has been calculated according to the formula (2).

In practice it is convenient to use the tables 2, 3 when controlling the parameters of the permanent magnets material.

Table 2. Relative positions of the points of extremums of z/d and respective relative extremum values of field induction $B/(\mu_0 \cdot Maver)$ for the radially-magnetized ring-shaped magnet

h/d	D/d								
	Calculated values	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.05	z/d	0. 300	0.329	0.354	0.366	0.385	0.387	0.395	0.400
	$B/(\mu_0 \cdot Maver)$	0.014	0.021	0.024	0.027	0.028	0.029	0.030	0.030
0.10	z/d	0.302	0.334	0.356	0.371	0.381	0.391	0.040	0.402
	$B/(\mu_0 \cdot Maver)$	0.028	0.042	0.049	0.053	0.056	0.058	0.060	0.061
0.15	z/d	0.305	0.337	0.359	0.375	0.386	0.394	0.400	0.405
	$B/(\mu_0 \cdot Maver)$	0.042	0.062	0.073	0.080	0.084	0.087	0.089	0.091
0.20	z/d	0.310	0.341	0.363	0.379	0.390	0.398	0.404	0.410
	$B/(\mu_0 \cdot Maver)$	0.055	0.082	0.096	0.105	0.111	0.115	0.118	0.120
0.25	z/d	0.316	0.347	0.369	0.384	0.396	0.404	0.410	0.416
	$B/(\mu_0 \cdot Maver)$	0.068	0.101	0.119	0.130	0.138	0.143	0.147	0.149
0.30	z/d	0.323	0.354	0.375	0.393	0.403	0.411	0.417	0.422
	$B/(\mu_0 \cdot Maver)$	0.080	0.119	0.141	0.180	0.164	0.170	0.174	0.178
0.35	z/d	0.331	0.362	0.383	0.399	0.410	0.419	0.426	0.434
	$B/(\mu_0 \cdot Maver)$	0.091	0.137	0.162	0.178	0.189	0.196	0.201	0.205
0.40	z/d	0.340	0.371	0.392	0.408	0.418	0.428	0.434	0.439
	$B/(\mu_0 \cdot Maver)$	0.102	0.153	0.183	0.201	0.213	0.222	0.228	0.232
0.45	z/d	0.351	0.381	0.403	0.418	0.429	0.438	0.445	0.450
	$B/(\mu_0 \cdot Maver)$	0.112	0.169	0.202	0.223	0.237	0.246	0.253	0.258
0.50	z/d	0.363	0.392	0.413	0.429	0.440	0.449	0.456	0.462
	$B/(\mu_0 \cdot Maver)$	0.121	0.184	0.220	0.233	0.259	0.270	0.277	0.283

Note: . D — the external diameter, d — the internal diameter, h — the height of the ring. It has been calculated according to the formula (3).

3.5. Example of using Table 2

The radially-magnetized magnetic ring with the sizes $D = 50$ mm, $d = 25$ mm, $h = 10$ mm. The demagnetization factor $N = 0.5$ (Fig. 4). Let the magnetization value with this demagnetization factor be $\mu_0 \cdot Maver = 1$ T. Then, according to Table 2[2], the maximum of magnetic induction is achieved when $z/d = 0.371$ ($z = 9.275$ mm) and its value in this point is $B = 0.153 \cdot 1\,\mathrm{T} = 0.153$ T.

3.6. Examples of using Table 3

3.6.1. Example 1

The axially-magnetized magnetic ring with the sizes $D = 50$ mm, $d = 25$ mm, $h = 7.5$ mm. The demagnetization factor $N = 0.6$ (Fig. 2). Let the magnetization value be $\mu_0 \cdot Maver = 1$ T at this. Then, according to Table 3, the extremums of magnetic induction are achieved when $z_1 = 0$and its value in this point is $B = 0.139 \cdot 1\,\mathrm{T} = 0.139$ T, and when $z_3/d = 0.895$ ($z_3 = 22.4$ mm), with the value of magnetic induction in the point $B = -0.191 \cdot 0.139\,\mathrm{T} = -0.0265$ T.

3.6.2. Example 2

The axially-magnetized magnetic ring with the sizes $D = 50$ mm, $d = 25$ mm, $h = 50$ mm. The demagnetization factor $N = 0.12$ (Fig. 2). Let the magnetization value be $\mu_0 \cdot Maver = 1$ T at this. Then, according to Table 3, the extremums of magnetic induction are achieved:

• in the point $z_1 = 0$, where the value of magnetic induction is

$$B = 0.187 \cdot 1\ \mathrm{T} = 0.187\,\mathrm{T};$$

• in the point $z_2/d = 0.38$ ($z_2 = 9.5$ mm), where the value of magnetic induction is

$$B = 1.019 \cdot 0.187\,\mathrm{T} = 0.1906\,\mathrm{T};$$

Table 3. Relative extremum value of field induction $B_1/(\mu_0 \cdot Maver)$ in the point $z_1 = 0$, relative positions of the points of other extremums of z_2/d, z_3/d and relative ratios B_2/B_1, B_3/B_1 of field induction in these points to induction in the point $z_1 = 0$ for the axially-magnetized ring-shaped magnet

h/d	D/d						
	Calculated values	1.5	2.0	3.0	4.0	6.0	8.0
0.1	z3/d	0.755	0.880	1.084	1.264	1.580	1.850
	B_3/B_1	−0.196	−0.183	−0.157	−0.136	−0.106	−0.087
	$B_1/(\mu_0 \cdot Maver)$	0.033	0.050	0.066	0.075	0.083	0.087
0.2	z_3/d	0.765	0.880	1.088	1.270	1.584	1.560
	B_3/B_1	−0.200	−0.186	−0.160	−0.138	−0.108	−0.088
	$B_1/(\mu_0 \cdot Maver)$	0.064	0.097	0.130	0.146	0.163	0.171
0.3	z_3/d	0.775	0.895	1.100	1.280	1.592	1.866
	B_3/B_1	−0.207	−0.191	−0.164	−0.142	−0.111	−0.091
	$B_1/(\mu_0 \cdot Maver)$	0.091	0.139	0.188	0.213	0.237	0.250
0.6	z_3/d	0.850	0.960	1.156	1.320	1.620	1.900
	B_3/B_1	−0.241	−0.220	−0.186	−0.160	−0.125	−0.102
	$B_1/(\mu_0 \cdot Maver)$	0.143	0.227	0.318	0.366	0.415	0.440
0.8	z_3/d	0.918	1.016	1.200	1.364	1.656	1.920
	B_3/B_1	−0.277	−0.248	−0.208	−0.178	−0.139	−0.113
	$B_1/(\mu_0 \cdot Maver)$	0.154	0.253	0.367	0.429	0.493	0.525
1.0	z_3/d	0.993	1.100	1.260	1.416	1.700	1.960
	B_3/B_1	−0.322	−0.283	−0.234	−0.200	−0.155	−0.126
	$B_1/(\mu_0 \cdot Maver)$	0.152	0.260	0.391	0.465	0.543	0.583
2.0	z_3/d	1.449	1.520	1.656	1.776	2.000	2.224
	B_3/B_1	−0.693	−0.556	−0.416	−0.342	−0.258	−0.209
	Z_2/d	0.510	0.380				
	B_2/B_1	1.094	1.019				
	$B_1/(\mu_0 \cdot Maver)$	0.094	0.187	0.340	0.447	0.578	0.652
3.0	z_3/d	1.938	2.012	2.120	2.224	2.412	2.600
	B_3/B_1	−1.309	−0.992	−0.682	−0.534	−0.386	−0.308
	Z_2/d	1.050	1.000	0.820	0.636		
	B_2/B_1	1.626	1.338	1.098	1.022		
	$B_1/(\mu_0 \cdot Maver)$	0.054	0.117	0.242	0.349	0.501	0.598
4.0	z_3/d	2.433	2.504	2.612	2.700	2.868	3.028
	B_3/B_1	−2.172	−1.599	−1.039	−0.778	−0.536	−0.419
	Z_2/d	1.560	1.500	1.372	1.256	0.980	
	B_2/B_1	2.459	1.904	1.386	1.174	1.032	
	$B_1/(\mu_0 \cdot Maver)$	0.034	0.076	0.170	0.263	0.415	0.522

Note: D — the external diameter, d — the internal diameter, h — the height of the ring. It has been calculated according to the formula (2).

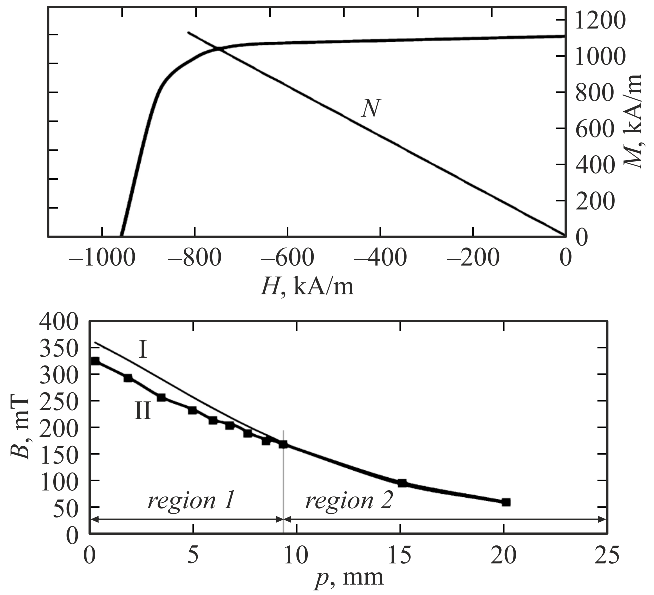

Figure 8. Curve of demagnetization of the rare-earth hard magnetic material of the Nd-Fe-B type. The calculated I and measured II values of magnetic induction at an axis of the disc-shaped magnet. The constant-sign condition of the nucleus of the integral relationship is not fulfilled with in the region *1*, but fulfilled in the region *2*.

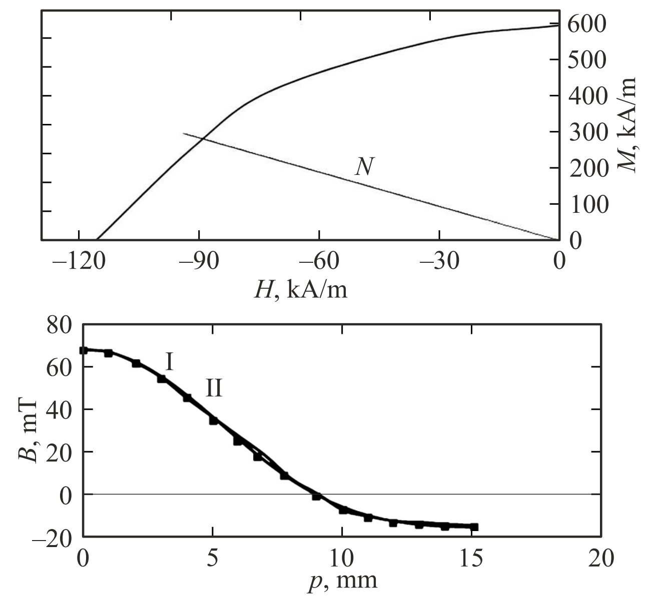

Figure 9. Curve of demagnetization of the cast hard magnetic material of the ALNIKO type. The calculated I and measured II values of magnetic induction at an axis of the ring-shaped magnet.

• in the point $z_3/d = 1.52$ ($z_3 = 38\,\mathrm{mm}$), where the value of magnetic induction

$$B = -0.556 \cdot 0.187\,\mathrm{T} = -0.104\,\mathrm{T}.$$

Conclusion

The examples of calculations and measurements of the field the permanent magnets that are made of the hard magnetic materials of various types (ferrite, cast and rare-earth) demonstrate that the values of the average magnetization for any portion of the demagnetization curve, which are obtained based on the value of the demagnetization factor, allow calculating the magnetic fields in the various observation points.

It includes the graphic dependences for determining the values of the demagnetization factor of the axially- and radially-magnetized ring-shaped magnets for the various size ratios. Besides, for the ring-shaped magnets, the tables of a position of the extremums of the magnetic field and the values of fields in relation magnetization for the extremum points are constructed.

Thus, using the formulas, graphs and tables that are provided in the study, enables successfully solving a practical task of evaluating the magnetic parameters of a specific permanent magnet, thereby essentially simplifying control of compliance of the material to the specified parameters.

Conflict of interest

The author declares that he has no conflict of interest.

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