Optics and Spectroscopy, 2025, Vol. 133, No. 6

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Optical theorem and vacuum dichroism in electromagnetic fields producing pairs

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Received May 08, 2025
Revised May 08, 2025
Accepted May 12, 2025

The relationship between the processes of photon decay into an electron-positron pair and photon emission from vacuum with pair production in an external electromagnetic field is investigated. It is known that in the case when the external field is not capable of producing particles from vacuum within the zeroth order in radiative interaction, the contribution of radiation is also zero, and the probability of photon decay in accordance with the optical theorem can be associated with the imaginary part of the second-order Feynman diagram containing a fermion loop. In this paper, the main attention is paid to the problem with unstable vacuum. It is shown that in this case the statement of the optical theorem is modified since a nonzero probability of emission with pair production must be added to the probability of photon decay. In our numerical calculations, both of these probabilities are obtained nonperturbatively with respect to the interaction with an external alternating electric field for various photon polarizations. The results of calculating the imaginary part of the one-loop diagram turned out to be in complete agreement with the optical theorem. It is also shown that the locally-constant field approximation is inapplicable in the region of low photon energies and can give a significant error in the high-energy region. The paper also analyzes the phenomenon of vacuum dichroism, i.e., the dependence of the above-described contributions on the photon polarization.

Keywords: quantum electrodynamics, strong fields, nonlinear effects, dichroism, polarization tensor, birefringence.

DOI: 10.61011/EOS.2025.06.61664.8144-25

1. Introduction

Description of electromagnetic interactions within quantum field theory includes consideration of processes with non-conserved particle numbers (electrons, positrons, photons, etc.). For example, to first order in the fine structure constant in quantum electrodynamics α there are processes of photon emission by an electron, photon absorption by an electron, photon decay into an electron-positron pair, etc. It is well known that all these elementary reactions are prohibited by the laws of conservation of energy and momentum in the absence of any other interactions [1]. The situation changes qualitatively if we introduce interaction of the quantized electron-positron field with an external classical electromagnetic field. In this case, the probabilities of all processes of the first order in alpha α, are generally non-zero, and the theoretical description of the corresponding effects in the regime of a strong external field represents an extremely non-trivial problem (see, for example, monograph [2], work [3] and recent reviews [4–6]). Interest in this problem is determined not only by its fundamental nature but also by the development of experimental setups in which increasingly strong electromagnetic fields can be created, for example, through laser radiation generation (references in [5,6]) or collisions of heavy nuclei [7–9].

In the present work, we are interested in the process of photon decay into a pair (in the first order of α) and photon emission from vacuum with pair creation in a strong electromagnetic field. These two phenomena are closely related for several reasons. Firstly, the amplitudes of such processes differ only by complex conjugation of the „wave function" of the corresponding photon. Secondly, the difference in probabilities of these two reactions determines the first-order contribution in α to the number of photons in the final state, as shown in work [10] (see also [2]). Thirdly, the sum of such probabilities can be related to the imaginary part of a one-loop diagram with two external photon lines according to the optical theorem. We emphasize that in a theory with an unstable vacuum, the statement of the optical theorem must include the probabilities of both processes, although in standard QED without vacuum pair production, it is sufficient to consider only the contribution from photon decay [1]. In diagram language, this statement is shown in Fig. 1. In the present work, we will discuss how this relation is proven and perform calculations of the two separate contributions on the right-hand side of the equality in Fig. 1. Using the optical theorem, we will test our independent numerical approach for finding the polarization tensor, which determines the left-hand side of the equation. Besides direct verification of our non-perturbative calculation methods, this will allow us to investigate the dependence of photon decay and pair-creation photon emission processes on photon polarization. The fact that photon decay probability depends on polarization is

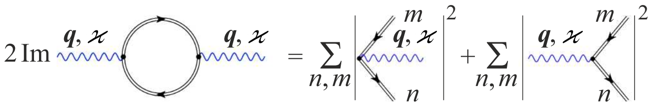

Figure 1. Diagram representation of the optical theorem. Double lines correspond to electron functions in an external field, and wavy lines represent the initial and final photon with momentum $\boldsymbol{q}$ and polarization $\varkappa$. Fermion in-statesn and m are usually characterized by momenta and spin quantum numbers at initial times $t_{\text{in}} \to -\infty$.

called vacuum dichroism [11]. Below, we will investigate the dichroism phenomenon through numerical non-perturbative calculations. Further development of theoretical methods is of great importance for planning experiments to observe corresponding nonlinear QED effects in strong fields.

In the main text of the article, we use units $\hbar = c = 1$ ($\hbar$ is Planck's constant, c is the speed of light), the fine structure constant $\alpha = e^2/(4\pi)$, m and $e < 0$ are the electron mass and charge.

2. Theory with stable vacuum. Constant crossed fields

Consider first a relatively simple case where the external electromagnetic field does not produce pairs from vacuum. Such a situation is realized, for example, in a constant magnetic field or in the field of a plane electromagnetic wave [2]. In this case, the initial vacuum state $|0, \text{in}\rangle$ and final state $|0, \text{out}\rangle$ coincide up to a complex phase factor, which does not affect physical observables. This state can simply be denoted as $|0\rangle$. Let the initial state contain a photon with momentum $\boldsymbol{q}$ and polarization $\varkappa$. Within QED in the interaction representation, the photon number density in the final state has the form [2]

$$n_{\boldsymbol{k}\lambda} = \langle 0|c_{\boldsymbol{q}\varkappa} S^\dagger c^\dagger_{\boldsymbol{k}\lambda} c_{\boldsymbol{k}\lambda} S c^\dagger_{\boldsymbol{q}\varkappa}|0\rangle, \tag{1}$$

where $\boldsymbol{k}$ and λ specify the momentum and polarization of the final photon, $c^\dagger_{\boldsymbol{k}\lambda}$ is the photon creation operator, and S is the scattering operator in the interaction representation:

$$S = \mathscr{T} \exp\left[-i \int d^4x \, \mathscr{H}_{\text{int}}(x)\right]. \tag{2}$$

Here $\mathscr{T}$ is the time-ordering operation, $\mathscr{H}_{\text{int}}(x) = j^\mu(x) A_\mu(x)$ is the interaction operator of quantized fields, and $j^\mu(x) = (e/2)[\bar{\psi}(x)\gamma^\mu, \psi(x)]$ is the current density operator of the electron-positron field in the presence of an external classical field $\mathscr{A}_\mu(x)$. As shown in [10], to first order in the fine-structure constant α, quantity (1) includes the difference between the first and second terms on the right-hand side of Fig. 1, of which only the negative contribution remains in the theory with stable vacuum. Thus, the first-order contribution in α is minus the squared magnitude of the photon decay diagram, summed over final fermion states. The presence of non-trivial dependence of this quantity on photon polarization means that vacuum dichroism occurs — photons of different polarizations decay with different probabilities.

To second order in α density (1) is determined by the squared magnitude of the one-loop diagram shown on the left side of Fig. 1, but with the final photon state $\boldsymbol{k}, \lambda$ [11]. The transition amplitude for this diagram is related to the polarization tensor by the relation

$$T(q, k) = \frac{1}{\sqrt{4q_0 k_0}} \varepsilon_\mu(q) \Pi^{\mu\nu}(q, k) \varepsilon^*_\nu(k), \tag{3}$$

where $\varepsilon_\mu(q)$ and $\varepsilon_\nu(k)$ are 4-vectors of polarization of the initial and final photons, respectively. The problem of finding this amplitude reduces to calculating the polarization tensor $\Pi^{\mu\nu}(q, k)$ in a given external field. Consider as an example a specific configuration of the external field that does not produce pairs from vacuum. Assume that a constant electric field $\boldsymbol{E}$ of magnitude E_0 is directed along the x axis, and an equal magnitude magnetic field $\boldsymbol{H}$ is directed along the y axis. Since in such a field the relativistic invariants $\boldsymbol{E}^2 - \boldsymbol{H}^2$ and $\boldsymbol{E} \cdot \boldsymbol{H}$ are exactly zero, it is well known that the vacuum is stable [1–3]. The polarization operator for this case was calculated analytically in works [12–15]. Assuming that the photon propagates along the z axis, we can write the result in the following form:

$$\begin{pmatrix} \Pi^{11}(q, k) \\ \Pi^{22}(q, k) \end{pmatrix} = -\frac{16\pi^3\alpha}{3} m^2 \delta(k - q) \chi^{2/3} \begin{pmatrix} A - B \\ A + 2B \end{pmatrix}, \tag{4}$$

where the quantum nonlinearity parameter is

$$\chi = \frac{2|eE_0|q^0}{m^3}, \tag{5}$$

and the following notations are introduced:

$$A = \int\limits_{-1}^{1} dv w^{1/3} f'(u), \qquad B = \int\limits_{-1}^{1} dv w^{-2/3} f'(u), \tag{6}$$

$$w = \frac{4}{1 - v^2}, \qquad u = \left(\frac{w}{\chi}\right)^{2/3}, \tag{7}$$

$$f(u) = i \int\limits_0^\infty d\tau \, \mathrm{e}^{-i(u\tau + \tau^3/3)} = \pi \mathrm{Gi}(u) + i\pi \mathrm{Ai}(u). \tag{8}$$

Here $\mathrm{Gi}(u)$ and $\mathrm{Ai}(u)$ are real-valued Scorer [16] and Airy functions, respectively.

Now let's calculate twice the imaginary part of amplitude (3) at $k = q$ per unit time and unit volume, for example, for a photon polarized along the x axis ($\mu = \nu = 1$):

$$\frac{2 \operatorname{Im} T^{(1)}(q, q)}{VT} = -\frac{\alpha m^2 \chi^{2/3}}{3\pi q^0} \operatorname{Im}(A - B). \tag{9}$$

Due to the evenness of the integrand functions in (6), we can write

$$\mathrm{Im}\,(A-B) = 2\pi \int_0^1 dv\, \frac{w-1}{w^{2/3}}\,\mathrm{Ai}'(u)$$

$$= 4\pi \int_4^\infty dw\, \frac{w-1}{w^{5/3}\sqrt{w(w-4)}}\,\mathrm{Ai}'(u). \quad (10)$$

If we now switch to the integral over variable u, we obtain

$$\frac{2\,\mathrm{Im}\,T^{(1)}(q,q)}{VT} = -\frac{2\alpha m^2\chi}{q^0}\int_{u_0}^\infty \frac{du}{\sqrt{u}}\frac{w-1}{w\sqrt{w(w-4)}}\,\mathrm{Ai}'(u)$$

$$= -\frac{\alpha m^2\chi}{8q^0}\int_{u_0}^\infty \frac{du}{\sqrt{u}}\frac{4\mathfrak{w}-1}{\mathfrak{w}\sqrt{\mathfrak{w}(\mathfrak{w}-1)}}\,\mathrm{Ai}'(u), \quad (11)$$

where $u_0 = (4/\chi)^{2/3}$ and $\mathfrak{w} = (\chi/4)u^{3/2}$. Expression (11) completely coincides with the probability of decay of a polarized photon with pair creation per unit time from Ritus's work [3] (item 5.23). For the second possible polarization, it is also not difficult to verify consistency of the results, which confirms the validity of the optical theorem in its simplest formulation: twice the imaginary part of the diagram with a closed fermion loop equals the total probability of decay of the initial photon into an electron-positron pair. If the external field violates vacuum stability, then to the decay probability one needs to add the probability of pair production with photon emission. The remaining part of this paper is devoted to the analysis of this more general case.

At the end of this section, we note that closed-form expressions (4) are often used for approximate description of radiation-induced processes in inhomogeneous fields. For this purpose, χ is taken as the local value of parameter (5), and then contributions (4) are integrated over time and spatial coordinates [11,17,18]. This approach is called the locally constant field approximation (LCFA).

3. Theory with unstable vacuum

If the external field produces pairs from vacuum to zeroth order in α, then the initial and final Heisenberg vacuum states $|0,\mathrm{in}\rangle$ and $|0,\mathrm{out}\rangle$ differ not only by a phase in this case $|\langle 0,\mathrm{out}|0,\mathrm{in}\rangle| < 1$, and the initial vacuum transitions with non-zero probability into states with real particles [19]. Expression (1) is now written as

$$n_{\boldsymbol{k}\lambda} = \langle 0,\mathrm{in}|c_{\boldsymbol{q}\varkappa}S^\dagger c^\dagger_{\boldsymbol{k}\lambda}c_{\boldsymbol{k}\lambda}Sc^\dagger_{\boldsymbol{q}\varkappa}|0,\mathrm{in}\rangle. \quad (12)$$

To first order in α the contribution to photon number density has the form [10]

$$n^{(1)}_{\boldsymbol{k},\lambda} = e^2\delta(\boldsymbol{k}-\boldsymbol{q})\delta_{\lambda\varkappa}\sum_{n,m}\left|\int d^4x\, {}_+\bar{\varphi}_n(x)\gamma^\mu f^*_{\boldsymbol{q},\varkappa,\mu}(x)\, {}_-\varphi_m(x)\right|^2$$

$$- e^2\delta(\boldsymbol{k}-\boldsymbol{q})\delta_{\lambda\varkappa}\sum_{n,m}\left|\int d^4x\, {}_+\bar{\varphi}_n(x)\gamma^\mu f_{\boldsymbol{q},\varkappa,\mu}(x)\, {}_-\varphi_m(x)\right|^2, \quad (13)$$

where $f_{\boldsymbol{q},\varkappa,\mu}(x)$ is the "wave function" of a photon with momentum $\boldsymbol{q}$ and polarization $\varkappa$, and ${}_+\varphi_n(x)$ and ${}_-\varphi_m(x)$ are in-solutions of the Dirac equation in an external field, specified in the asymptotic past by quantum numbers n and m (these numbers usually include momentum and spin quantum number). At times $x^0 \to +\infty$ in-solutions become superpositions of solutions of the free Dirac equation with different energy signs. The presence of a solution with the opposite energy sign precisely means vacuum pair production [2]. The contribution to the total number of photons is obtained by integrating (13) over $\boldsymbol{k}$ and multiplying by factor $(2\pi)^3/V$, where V is the system volume. If one needs to consider a spatially localized external field, the initial single-photon states in (12) should be chosen in the form of wave packets. The two terms in formula (13) correspond to diagrams on the right-hand side of Fig. 1. The minus sign before the second sum is related to the fact that as a result of the possible photon decay, the number of final quanta decreases. The first term is positive and can be interpreted as stimulated emission from vacuum. Note that quantity $n^{(1)}_{\boldsymbol{k},\lambda}$ also contains contributions from vacuum emission in the absence of an initial photon [2,10,20–26], but we do not account for these terms since they do not contain $\delta(\boldsymbol{k}-\boldsymbol{q})$. When detecting photons propagating along direction $\boldsymbol{q}$ in a small angular vicinity, vacuum contributions will be negligibly small.

Now let's turn to the general expression for the polarization tensor in an external field, taking into account vacuum instability:

$$\Pi^{\mu\nu}(q,k) = ie^2\int d^4x\int d^4y\, \mathrm{e}^{-iqx}\mathrm{e}^{iky}\,\mathrm{Tr}\,[\gamma^\mu$$

$$\times S_{\mathrm{in}}(x,y)\gamma^\nu S_{\mathrm{in}}(y,x)] - \{\text{zero-field contribution}\}. \quad (14)$$

Here $S_{\mathrm{in}}(x,y) = i\langle 0,\mathrm{in}|\mathscr{T}[\psi(x)\bar{\psi}(y)]|0,\mathrm{in}\rangle$ is the electron Green's function in an external field with respect to the in-vacuum. We will consider an external time-dependent electric field directed along the x axis, and the photon momentum will again be directed along the z axis. Photon polarizations along x and along y will be denoted by numbers 1 and 2, respectively. Let $P^{(1,2)}_{\mathrm{d}}$ denote the photon decay probability, and $P^{(1,2)}_{\mathrm{e}}$ denote the total probability of emission of such a photon with pair creation. According to the optical theorem formulated above, we have

$$P^{(1)}_{\mathrm{d}} + P^{(1)}_{\mathrm{e}} = \frac{1}{V}\frac{1}{q^0}\,\mathrm{Im}\,\Pi^{11}(q,q). \quad (15)$$

A similar relation holds for polarization 2. On the right side, volume V cancels out due to the presence of delta function $\delta(\boldsymbol{k}-\boldsymbol{q})$ in $\Pi^{\mu\nu}(q,k)$. In Section 2, we verified this statement in the case when $P_{\mathrm{e}}^{(1,2)}=0$. To prove the optical theorem, it is sufficient to consider integration in formula (14) over regions $x^0>y^0$ and $x^0<y^0$. For each term, one can use spectral decomposition of propagators $S_{\mathrm{in}}(x,y)$. When calculating the imaginary part, the integral over each half-plane can easily be rewritten through an integral over all x^0 and y^0 and the result is written as the squared magnitude of the first-order diagram.

Calculation of the polarization tensor in our chosen external field is performed by using the spectral decomposition of Green's functions. Solutions of the Dirac equation are constructed numerically. It is important to emphasize that the external field is accounted for fully nonperturbatively. Similarly, though technically significantly simpler, we find the individual probabilities $P_{\mathrm{d}}^{(1,2)}$ and $P_{\mathrm{e}}^{(1,2)}$. The numerical method in this case is described in detail in works [10,25]. In the next section, we will compare the results of numerical calculations, verify compliance with relations (15), and investigate properties of vacuum dichroism. Finally, comparison with LCFA method will be carried out, within which expressions (4) are integrated over time taking into account local dependence of parameter χ through the time dependence of the external field.

Note that to second order in α quantity (12) is determined by the squared magnitude of the diagram with a closed fermion loop. In particular, the corresponding contribution describes the phenomenon of vacuum birefringence [11,27–39].

4. Numerical results and discussion

We will consider a time-dependent electric field directed along the x axis and defined using the following x-projection of the vector potential:

$$\mathscr{A}_x(t)=\frac{E_0}{\omega}\,\mathrm{e}^{-t^2/\tau^2}\sin\omega t, \tag{16}$$

where E_0 is the pulse amplitude, τ is the characteristic duration, and ω is the carrier frequency. The projection of the electric field on the x axis is $E_x(t)=-\mathscr{A}_x'(t)$. In our chosen system of units, frequency, energy, and momentum have dimension m; the critical (Schwinger) value of field strength is $E_{\mathrm{c}}=m^2/|e|\approx 1.3\times 10^{18}$ V/m.

First, calculations of individual contributions on the right-hand side of Fig. 1 were performed, i.e., photon decay probabilities $P_{\mathrm{d}}^{(1,2)}$ and photon emission with pair creation $P_{\mathrm{e}}^{(1,2)}$ were obtained for two different polarizations (1 - along x, 2 — along y). Note that due to the finite duration of the external electric pulse, we are dealing with total (dimensionless) probabilities. Figure 2 shows dependencies of these quantities on photon energy for the following set of external field parameters: $E_0=0.2E_{\mathrm{c}}$, $\tau=2m^{-1}$, $\omega=0.2m$. We observe several characteristic features of the

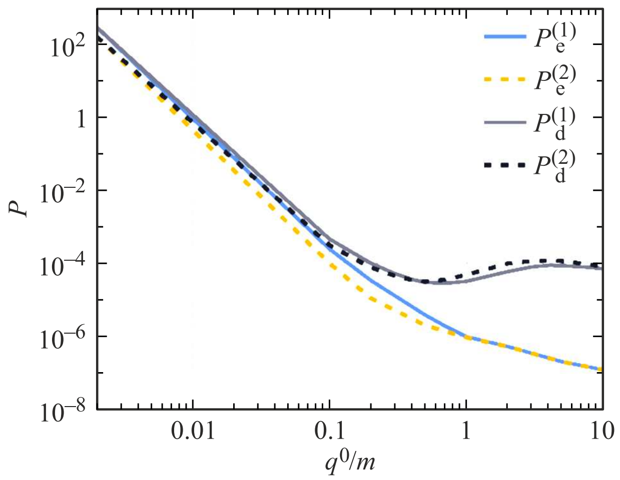

Figure 2. Dependence of photon decay probabilities $P_{\mathrm{d}}^{(1,2)}$ and photon emission with pair creation $P_{\mathrm{e}}^{(1,2)}$ on photon energy q^0. Indices 1 and 2 denote polarization along the external electric field x) axis) and perpendicular to the field y axis). The photon propagates along the z; axis; the external field is given by expression (16). The following parameters were chosen: $E_0=0.2E_{\mathrm{c}}$, $\tau=2m^{-1}$, $\omega=0.2m$ (E_{c} is the Schwinger field strength).

obtained graphs. First, at low photon energies, probabilities grow proportionally to $(1/q_0)^3$. This property was described in detail in works [10,23,25], where the corresponding asymptotics was established analytically. An important circumstance is that such behavior is possible only in fields producing pairs; otherwise, probability rapidly tends to zero as $q^0\to 0$. In terms of photon number (12), the observed growth means, for example, that when pairs are produced, a very large number of soft photons can also be emitted. It should be noted that at $P\gtrsim 1$ quantity P obviously cannot be interpreted as probability anymore, and contributions of higher orders need to be considered. Second, at small q^0 we have $P_{\mathrm{d}}^{(1,2)}\approx P_{\mathrm{e}}^{(1,2)}$ (solid and dashed lines on Fig. 2 coincide pairwise). This fact automatically follows from the fact that, as noted in the Introduction, diagrams on the right-hand side of Fig. 1 differ only by complex conjugation of the photon function, so when expanding in powers of $1/q^0$ odd contributions will coincide, and even ones will differ in sign. Further, as photon energy increases, quantities P decrease, but after a certain energy, photon decay probability $P_{\mathrm{d}}^{(1,2)}$ begins to grow. This growth conventionally occurs in problems with stable vacuum immediately as $q^0\geqslant 0$. For example, in crossed fields, probability (11) behaves as $(\chi/q^0)\exp(-8/3\chi)$, at small energies, where $\chi\sim q^0$ for fixed field magnitude. It should be noted that at high energies this same probability is proportional to $\chi^{2/3}/q^0\sim(q^0)^{-1/3}$, which qualitatively explains the fact that curves $P_{\mathrm{d}}^{(1,2)}$ in Fig. 2 increase only over a fairly limited interval of photon energies, when parameter $\chi=|eE_0|q^0/m^3$ changes approximately from 0.1 to 1 (in crossed fields parameter χ in (5) contained factor

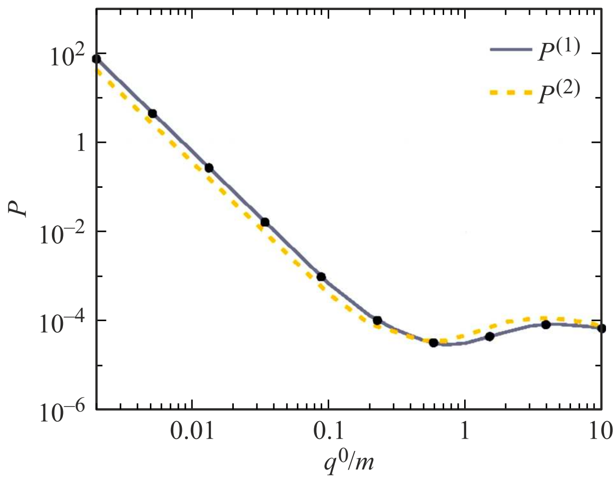

Figure 3. Dependence of sum of probabilities $P^{(1,2)} = P_e^{(1,2)} + P_d^{(1,2)}$ on photon energy. Points mark data obtained by independent calculation of the imaginary part of polarization tensor Π^{11} using the optical theorem. External field parameters are chosen the same as in Fig. 2.

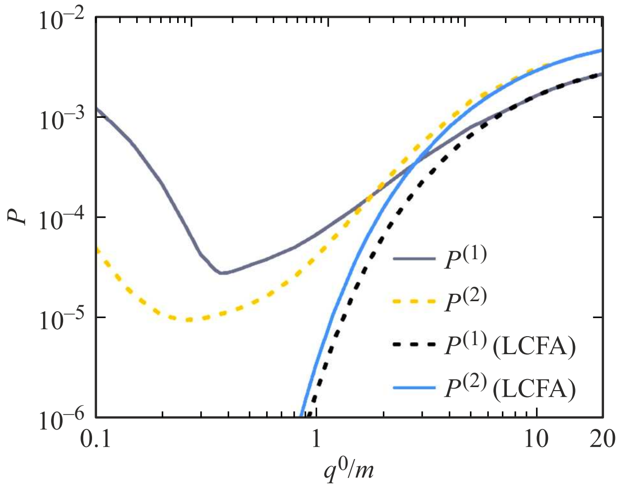

Figure 4. Dependence of sum of probabilities $P^{(1,2)} = P_e^{(1,2)} + P_d^{(1,2)}$ on photon energy similar to Fig. 3, but for another set of parameters: $E_0 = 0.4E_c$, $\tau = 10m^{-1}$, $\omega = 0.1m$. Besides results of direct numerical calculations, the graph also shows curves obtained within the LCFA approximation.

2 due to the presence of magnetic component). We emphasize that comparison with the crossed fields case for the chosen parameters is possible only qualitatively, since duration τ does not allow considering the external field as locally constant (see below). Finally, note that at sufficiently large photon energies, photon decay probability exceeds pair-creation photon emission probability by several orders of magnitude, so $P_e^{(1,2)} + P_d^{(1,2)} \approx P_d^{(1,2)}$ and in the optical theorem, the emission channel can be neglected. The discrepancy between curves $P_d^{(1)}$ and $P_d^{(2)}$ indicates vacuum dichroism. We see that the difference between these two quantities changes sign as q^0 varies, indicating non-trivial behavior of the dichroism signal and must be considered when searching for the most favorable scenarios for experimental observation of the effect.

Figure 3 shows sums of probabilities $P^{(1,2)} = P_e^{(1,2)} + P_d^{(1,2)}$ for two polarizations (solid and dashed curves). Values of these sums can be directly compared with results of calculating the imaginary part of the polarization tensor in the chosen external field. Through independent calculation of the loop diagram and considering the optical theorem, we obtained values for $P^{(1)}$ shown in Fig. 3 as dots. The two different approaches turned out to be in complete agreement, which indicates a high degree of reliability of the numerical methods used. As noted above, we observe growth of probabilities when moving to low energies, which occurs in fields that produce particles. At high energies, total probabilities are determined primarily by the photon decay channel.

In the example above, pulse duration and „frequency" are related by $\omega\tau = 0.4$, which means that no slow envelope can be distinguished in such a pulse. In fact, the external field barely changes with further decrease of ω, so the characteristic frequency is determined by parameter τ and has magnitude of order m. Then the ratio of field amplitude to frequency in units of $m/|e|$ equals $\xi \sim 0.2$. For LCFA applicability, the field should change slowly, i.e., condition $\xi \gg 1$ [3] is required (or at least $\xi \gtrsim 1$). Now consider a field with larger amplitude and lower frequency. Figure 4 shows similar dependencies as in Fig. 3, but for the following parameters: $E_0 = 0.4E_c$, $\tau = 10m^{-1}$, $\omega = 0.1m$. The electric pulse has qualitatively the same shape as before, but now its frequency is of order $\omega = 0.1m$, giving $\xi = |e|E_0/(m\omega) = 4$. Figure 4 also shows approximate curves obtained within the LCFA. We see that at sufficiently high photon energies, LCFA agrees with results of exact calculations. Of course, agreement is impossible in the region of small energies since LCFA completely ignores the vacuum instability effect. In terms of χ the condition for LCFA applicability at high energies has the form $\chi^2 \gg E_0/E_c$ [3]. We observe high accuracy at energies of order $5m$–$10m$, which corresponds to $\chi = 2{-}4$ and agrees with this criterion. It is worth noting separately that when deviating from LCFA predictions, exact curves exhibit quite non-trivial behavior, including crossing of curves as in the previous example, i.e., change of the sign of the vacuum dichroism signal.

5. Conclusion

In this work, probabilities of photon decay and photon emission with pair creation in a time-dependent external electric field were calculated. In particular, the dependence of probabilities on photon energy and polarization was studied. The sum of probabilities was also obtained by independent calculation of the imaginary part of the polarization tensor in the given external field. It was shown that the above processes depend on photon polarization, which is a manifestation of vacuum dichroic properties. Within the locally constant field approximation, the imaginary part of the polarization tensor was calculated, and it was found that this approximate approach does not account for growth at low photon energies, which is associated with vacuum instability. In the domain of strong slowly varying fields and high photon energies, agreement between the approximate method and exact numerical approach was found.

Funding

This research was financially supported by the Russian Science Foundation (RSF) under project No. 23-72-01068.

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by J.Savelyeva