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Polarization dependence of cesium vapor absorption on the D1 line in low magnetic fields: isotropy of transition $F = 3 \rightarrow F' = 4$

© G.G. Kozlov

Spin Optics Laboratory, Saint Petersburg State University, St. Petersburg, Russia

e-mail: gkozlov@photonics.phys.spbu.ru

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The paper studies the polarization dependence of cesium vapor absorption in the spectral region of the T line in weak magnetic fields, when the Zeeman structure of the spectrum is not resolved. The developed theory shows that the experimentally observed dependence of absorption on the relative orientation of the azimuth of the probe beam polarization and the magnetic field is a nonlinear effect and can be interpreted using the expansion of the energy absorbed by the atom over the time T of its flight through the beam to terms $\sim T^2$. Within the framework of the proposed theory, it was also possible to explain the absence of an orientation dependence of absorption at the transition $F = 3 \rightarrow F' = 4$ (isotropic transition) and to quantitatively describe this dependence at other transitions of the D1 cesium line.

Keywords: alkali metal vapor spectroscopy, D1 cesium line, nonlinear absorption, unresolved Zeeman splitting.

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1. Introduction

Alkali metal vapors are a classic subject of atomic spectroscopy and have been studied for many decades. Their nonlinear magnetic properties attract special attention from researchers [1]. Nevertheless, even now, experiments with atomic vapors often reveal effects whose interpretation is not always obvious and can be of interest both for gathering information about these systems and for applications. One such effect is discussed in this work.

The effect was observed in an experiment on resonant absorption of cesium vapors at the D1 line ($\lambda = 894.593$ nm). The diagram of the setup is shown in Fig. 1. A cuvette with cesium vapors 5 was placed in a solenoid 4 creating a sinusoidally time-varying magnetic field $\mathbf{B}_s$ with amplitude $\sim 2-4$ Gs and frequency ~ 50 Hz. A linearly polarized probe beam with a diameter $2r \sim 4$ mm, obtained by a tunable diode laser 1, was attenuated by an attenuator 2 to power $P \sim 5-50\,\mu$W, passed through the cuvette 5 with cesium vapors, and then detected by photodetector 6. A half-wave wafer 3 allowed changing the polarization azimuth mutual orientation of the probe beam polarization and the magnetic field of the solenoid, which was directed along the laboratory coordinate system axis x. The cuvette length l was 50 mm. Observations were usually conducted in Voigt geometry (the solenoid magnetic field perpendicular to the probe beam direction), but the effect was also observed in Faraday geometry (the solenoid magnetic field parallel to the probe beam). The experiments were performed at a temperature of 22°C.

The cesium D1 line corresponds to transitions from two ground-state multiplets with total momenta $F = 4$, and $F = 3$ to two excited multiplets with momenta $F' = 3$, and $F' = 4$ (Fig. 1). Accordingly, the absorption spectrum of cesium vapors in the spectral region near the D1 line consists of four components ($F = 4 \rightarrow F' = 3$, $F = 4 \rightarrow F' = 4$, $F = 3 \rightarrow F' = 3$, $F = 3 \rightarrow F' = 4$). The frequency of the probe optical beam was tuned in resonance with one of these, after which the dependence of the transmitted beam intensity P' on the solenoid magnetic field was observed.

The discussed effect consisted of a narrow feature in the transmitted beam intensity when the solenoid $\mathbf{B}_s$ magnetic field passed through zero and changed sign (Fig. 2, *a, c*). The character of this feature changed with the azimuthal angle of the probe laser linear polarization (Fig. 2, *c*). In our experiments, such a feature appeared on all D1 line components except component $F = 3 \rightarrow F' = 4$ (Fig. 2, *a*). Similar effects are called magneto-optical resonances and have been observed in Rb vapors [2,3] (in longitudinal magnetic fields) and in Na and Rb vapors [4,5] (in fluorescence intensity in transverse magnetic fields).

A simplest qualitative interpretation connects this feature with the presence of a static Earth (laboratory) magnetic field $\mathbf{B}_e$ (Fig. 1), since no magnetic shielding was used in our experiments. The atomic system absorption starts to change significantly when the solenoid field becomes comparable to the Earth's field and the direction of the total magnetic field $\mathbf{B}$ (Fig. 1) experiences a substantial change[1]. This direction can be associated with the quantization axis of the atomic system, whose orientation relative to the probe beam linear polarization direction determines the dipole moment operator matrix elements defining the probe beam absorption. Therefore, at these times the field dependence of atomic absorption exhibits a feature.

[1] In our experiments, the Earth field had both components parallel and perpendicular to the solenoid field.

When the solenoid magnetic field significantly exceeds the Earth's field, the dipole moment matrix elements do not depend on the magnetic field, as its direction (and thus the quantization axis direction) is almost unchanged. This directional dependence is decisive here since the Zeeman splitting (~ 1 MHz) in the applied magnetic fields (~ 1 Gs) is much smaller than both the Doppler ($\Delta \sim 400$ MHz), and homogeneous ($\delta \sim 10$ MHz) widths of the optical atomic transitions. The Earth (laboratory) field can be largely compensated using an additional electromagnet, suppressing the aforementioned feature. Such experiments were performed, and the calibration of the compensating field allowed estimation of the Earth's field, which in our case was about ~ 0.5 Gs.

Let us note here that similar experiments in ultra-low fields of about $\sim 10^{-4}$ Gs are described in [6], although the nature of the observed effects and the methods of their analysis differ from those presented below.

The first difficulty of the given interpretation is the fact that, as will be shown later, the described effect should not exist in linear theory. This is because the quantization axis direction is conveniently chosen along the magnetic field, but in principle, this direction can be arbitrary. The second difficulty is the need to explain the absence of the effect on the component $F = 3 \rightarrow F' = 4$. Finally, the dependence of the effect on the polarization azimuth mutual orientation and the magnetic field also requires interpretation.

Special experiments have shown that despite the low intensity of the probe beam (in typical experiments $\sim 5\ \mu$W), the described effect is indeed nonlinear — further decreasing the intensity leads to a reduction of the effect. Theoretical analysis of magneto-optical resonance effects is usually based on the Maxwell-Bloch equations [3–5,7] and is used under conditions when the nonlinearity of atomic dynamics is not assumed to be small. A consistent treatment of this type (taking into account renormalization of atomic states by the probe field, transit effects and related spatial dispersion, nonlinear electrodynamics construction, etc.) generally leads to a system of nonlinear integro-differential equations, whose solution (even numerically) is known to be challenging. Below, a simple theory of absorption of the considered atomic system in the presence of weak nonlinearity will be constructed, demonstrating all features of the described effect and its orientational dependence. In calculating absorption, we do not use the stationary density matrix of the atomic system (as is often done [6]), assuming the probe beam pumping is so weak that atoms do not have time to significantly change their state while passing through the laser beam. The small parameter in our consideration is the transit time T. Our theory uses only the known characteristics of the considered atomic system and in this sense contains no fitting parameters.

2. Nonlinear Theory of Cesium Vapor Absorption

In this section, we present the theory of absorption in atomic systems with certain simplifications and additions that take into account the specifics of our experiments. For a qualitative interpretation of the described effects, it is sufficient to consider the case of weak absorption, where propagation effects can be neglected and the optical field can be assumed to be the same for all atoms in the beam. For a quantitative interpretation of our experiments, in which the absorption was approximately $\sim$ 50%this approximation may prove insufficient. A generalization of the developed theory is provided in Section 4. In the calculations presented below, we also do not take into account interatomic collisions, since at the cesium vapor pressures relevant to our experiments ($\sim 10^{-6}$ Torr) the atoms pass through the probe beam without collisions. The calculations are based on a model whose main assumptions are as follows.

i) Let us assume that cesium atoms enter the probing beam in a state corresponding to an equal population distribution among the sublevels of both lowest multiplets. Upon entering the probing beam, the atoms begin making transitions to the states of the resonant excited multiplet, from which fast radiative disintegration occurs into both main multiplets. At the same time, the non-resonant ground multiplet is effectively populated, from which the probing beam does not induce transitions — this is the phenomenon of hyperfine pumping [8], which inevitably takes place and must be taken into account. According to the conditions of our experiments, we will assume that the rate of radiative disintegration is significantly greater than the rate of induced transitions.

ii) In calculating the dynamics of the atomic state in the optical beam, we neglect relaxation processes causing transitions between the main multiplets (collisions with the cell walls, interatomic collisions), assuming the corresponding relaxation times to be much longer than the atomic flight time through the probing beam.

iii) We assume that at the intensity of the probing beam relevant to our experiments, the energy of an atom does not change significantly during its flight time Tthrough the beam. The corresponding condition is $\omega_R^2/\delta < T^{-1}$, where T is the characteristic atomic flight time through the beam, ω_R is the Rabi frequency of the probing beam (see below), and, δ is the homogeneous linewidth of the relevant optical transitions. In our experiments, the parameters appearing here are estimated as $T \sim 10^{-5}$ s, $\omega_R \sim 5 \cdot 10^5$ s^{-1}, $\delta \sim 2\pi \cdot 10^7$ s^{-1}. Thus, the above condition holds. Our consideration is based on the expansion of the atomic energy change in powers of the flight time T. The first, linear in T, contribution corresponds to linear absorption. This contribution does not describe the effect under consideration and it is necessary to include the$\sim T^2$contribution. The corresponding calculations are given below.

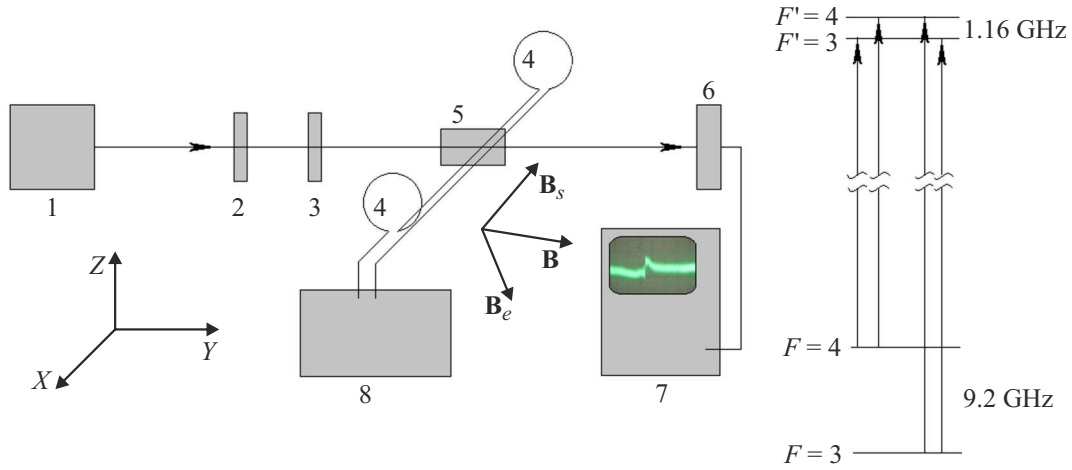

Figure 1. Diagram of the experimental setup. 1 — tunable laser, 2 — attenuator, 3 — half-wave wafer used to rotate the azimuth of the probe beam linear polarization, 4 — solenoid, 5 — cuvette with Cs vapors, 6 — photodetector, 7 — oscilloscope, 8 — sinusoidal current generator. The laboratory coordinate system is shown in the lower-left corner. On the right is the scheme of transitions of the cesium D1 line. $\mathbf{B}_s$ — solenoid field, $\mathbf{B}_e$ — Earth field, $\mathbf{B}$ — total field.

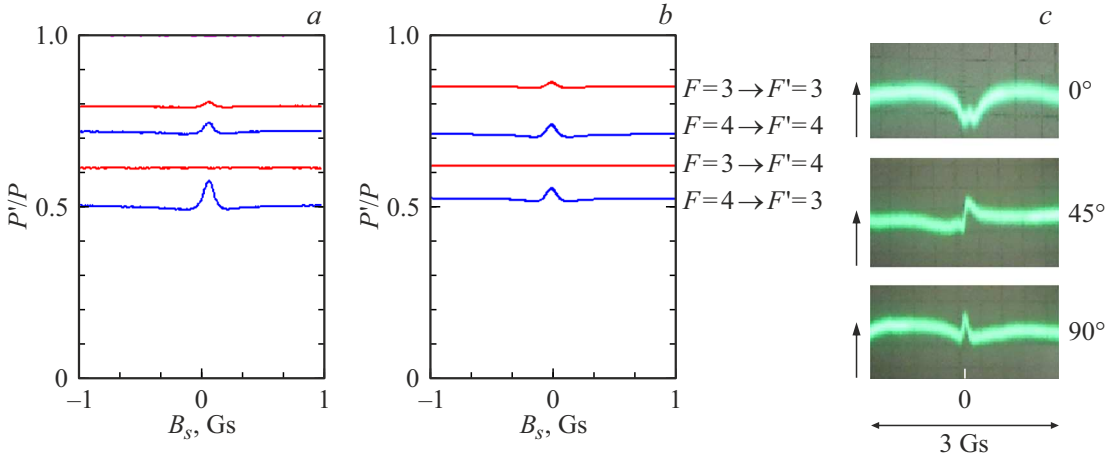

Figure 2. Absorption dependence on magnetic field in Voigt geometry for various transitions of the cesium D1 line: *a* — experiment, *b* — theory. Dependencies *a* and *b* are obtained at an angle between the probe beam linear polarization direction and the solenoid magnetic field B. 90°. (*c*) Behavior of absorption P'/P on transition $F = 4 \to F' = 3$ at solenoid field passing through zero for various angles (angles shown to the right) between the solenoid magnetic field and the probe beam linear polarization direction.

2.1. Absorption of a given velocity group

Consider a velocity group of atoms entering the probing beam at time $t = 0$ with velocity $\mathbf{v}$ and leaving it at time $t = T$. Denote by $dN$the number of atoms in this group. When analyzing the population dynamics of the states for this group, we consider, besides the resonant main and excited multiplets associated with the probing beam, also the non-resonant ground multiplet, to which radiative disintegration occurs from the resonant excited multiplet (hyperfine pumping [8,9]). The non-resonant excited multiplet will not be considered. Denote the total angular momenta of the resonant ground and excited multiplets by F and F', respectively, and the total angular momentum of the non-resonant ground multiplet by J (Fig. 3, *a*). We assume that at $t = 0$ the states of both the main multiplets F and J are equally populated, and therefore the population of each state in these multiplets equals $dN/[2J + 2F + 2]$. Below it will be convenient to normalize all populations to this value $dN/[2J + 2F + 2]$. For populations normalized in this manner, we introduce the following notations:

$n_M, M = -F, \ldots, F$ — populations of the states $|F, M\rangle$ of the resonant ground multiplet (capital Latin letters denote the projection of the angular momentum),

$p_m, m = -F', \ldots, F'$ — populations of the states $|F', m\rangle$ of the resonant excited multiplet (lowercase Latin letters denote the projection of the angular momentum),

$N_\alpha, \alpha = -J, \ldots, J$ — populations of the states $|J, \alpha\rangle$ of the non-resonant ground multiplet (Greek letters denote the projection of the angular momentum).

Here, quantum numbers M, m and α correspond to the projection of the total angular momentum along the direction of the total magnetic field **B**, acting on the atomic system, which, according to the remarks presented in the Introduction, will be considered the quantization axis. Recall that this direction does not coincide with the z axis of the laboratory coordinate system (Fig. 1). Thus, the atomic system under consideration loses isotropy and acquires a distinguished direction determined by the magnetic field **B** [2].

The dynamics of the normalized populations introduced above at $t > 0$ will be determined by the following kinetic equations and initial conditions:

$$\dot{n}_M = -n_M \sum_{m=-F'}^{F'} A_{M\to m} + \sum_{m=-F'}^{F} \gamma_{m\to M} p_m,$$

$$\dot{p}_m = -p_m \left\{ \sum_{M=-F}^{F} \gamma_{m\to M} + \sum_{\alpha=-J}^{J} \gamma_{m\to\alpha} \right\} + \sum_{M=-F}^{F} A_{M\to m} n_M, \dot{N}_\alpha = \sum_{m=-F'}^{F'} \gamma_{m\to\alpha} p_m, \quad (1)$$

$$n_M(0) = N_\alpha(0) = 1, \quad p_m = 0. \quad (2)$$

Here: $A_{M\to m}$ is the transition rate from the state $|F, M\rangle$ of the ground multiplet to the state $|F', m\rangle$ of the excited multiplet due to the probing beam $\gamma_{m\to M}$ ($\gamma_{m\to\alpha}$) is the rate of radiative decay from the state $|F', m\rangle$ of the excited multiplet into the state $|F, M\rangle$ ($|J, \alpha\rangle$) of the resonant (non-resonant) ground multiplet.

We will assume that the energy dE, absorbed by the considered velocity group dN of atoms is determined by the number of transitions from the multiplet F to the multiplet F' during the flight time T, multiplied by the energy of the atomic transition$\hbar\Omega$. Let us denote the specified number of transitions (normalized by the factor $dN/[2J + 2F + 2]$, as was done above for the populations) by q. From equation (1), we obtain the following expression for q:

$$q = \int_0^T dt \sum_{M=-F}^{F} n_M \sum_{m=-F'}^{F'} A_{M\to m} = \sum_{M=-F}^{F} \int_0^T dt \sum_{m=-F'}^{F'} \gamma_{m\to M} p_m - \sum_{M=-F}^{F} \int_0^T \dot{n}_M dt. \quad (3)$$

Then,

$$\frac{dE}{dN} = \frac{1}{2} \frac{\hbar\Omega q}{J + F + 1}. \quad (4)$$

[2] The magnetic field in our experiments is of the order of the Earth's field; nevertheless, its perturbation on the atomic system must exceed that caused by the probing beam, since only in this case does the nomenclature of atomic states having definite angular momentum projections along the magnetic field direction and between which the probing optical field induces transitions, make sense.

Thus, the calculation of the contribution to the absorption of the considered velocity group of atoms reduces to calculating the quantity q (3). For the calculations, we use the fact that the sum of coefficients $\gamma_{m\to M}$ over the projections of momentum M of the states of the ground multiplet does not depend on the projection m of the momentum of the excited multiplet (see Appendix 1 or [10,11]). Denote these sums as follows:

$$\sum_{M=-F}^{F} \gamma_{m\to M} \equiv \Gamma_F^{F'}, \qquad \sum_{\alpha=-J}^{J} \gamma_{m\to\alpha} \equiv \Gamma_J^{F'} \quad (5)$$

(explicit expressions for $\Gamma_F^{F'}$ are given in Appendix 1). Taking this into account and performing the integration in the last term of (3), we obtain the following expression for the quantity of interest q:

$$q = \Gamma_F^{F'} \int_0^T dt \sum_{m=-F'}^{F'} p_m - \sum_{M=-F}^{F} \Big[n_M(T) - 1 \Big]. \quad (6)$$

Next, let us note that in our experiments the rates of induced transitions are much smaller than the rates of radiative disintegration: $A_{M\to m} \ll \gamma_{m\to M}$. This allows neglecting the derivative $\dot{p}_m$in the second equation of system (1). Considering the expressions (5), one can write the following expression for p_m:

$$p_m = \frac{\sum_{M=-F}^{F} A_{M\to m} n_M}{\Gamma_F^{F'} + \Gamma_J^{F'}}. \quad (7)$$

Substituting this expression into (6) and recalling the definition of the quantity q given in (3), we obtain the following expression:

$$q = \frac{\Gamma_F^{F'} + \Gamma_J^{F'}}{\Gamma_J^{F'}} \sum_{M=-F}^{F} \Big[1 - n_M(T) \Big]. \quad (8)$$

Solving the equations (1) for the given velocity group, one can find the values $n_M(T)$. Then formulas (4) and (8) allow computing the energy dE, absorbed from the probing beam by the velocity group consisting ofdN atoms flying through the beam during time T:

$$\frac{dE}{dN} = \frac{\hbar\Omega}{F + J + 1} \frac{\Gamma_F^{F'} + \Gamma_J^{F'}}{2\Gamma_J^{F'}} \sum_{M=-F}^{F} \Big[1 - n_M(T) \Big]. \quad (9)$$

2.2. Averaging over velocity groups

Let us calculate the energy absorbed by atoms of the velocity group $\mathbf{v} = (v_x, v_y, v_z)$ passing through the beam per unit time and perform averaging over velocity groups. We assume the beam to be cylindrical with radius r and axis coinciding with the y axis of the laboratory coordinate system (Fig. 1). On the beam cross-section along the level y designate an element of area „beam surface“ $d\mathbf{S}$ (Fig. 3, b). Through the area element $d\mathbf{S}$ during the time

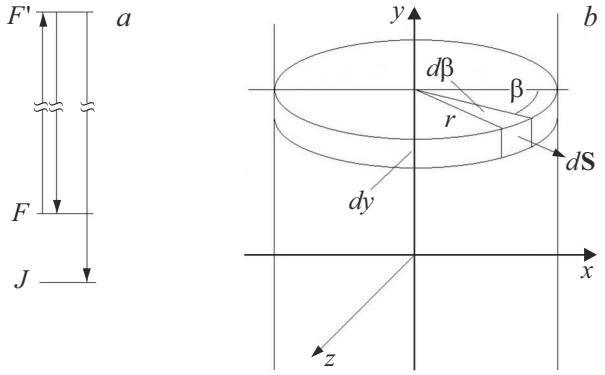

Figure 3. *a*) Resonant multiplets F (ground) and F' (excited). Downward arrows indicate radiative disintegration and hyperfine pumping of the non-resonant multiplet J. *b*) To calculation of energy absorbed by velocity groups from the probing beam.

interval dt passes $dn = (\mathbf{v}, d\mathbf{S})c\mathscr{M}_3(\mathbf{v})d\mathbf{v}dt$ atoms of the velocity group $\mathbf{v}$ (c where is the atomic concentration, $\mathscr{M}_3(\mathbf{v}) = \mathscr{M}(v_x)\mathscr{M}(v_y)\mathscr{M}(v_z)$ is the Maxwell distribution (10), $d\mathbf{v}$ is the „volume" of the considered velocity group in velocity space). All these atoms will pass the entire beam and exit it, absorbing energy $\frac{dE}{dN}dn$, which they take from the optical beam, thus causing the absorption we are interested in. The total energy taken from the beam per unit time by all velocity groups (i.e., the absorbed power, denote it $W \equiv \frac{dE}{dN}\frac{dn}{dt}$), is determined by the integral:

$$W = -c\int \Theta\Big(-(\mathbf{v}, d\mathbf{S})\Big)(\mathbf{v}, d\mathbf{S})\frac{dE}{dN}\mathscr{M}_3(\mathbf{v})d\mathbf{v}, \qquad (10)$$

$$\mathscr{M}_3(\mathbf{v}) = \mathscr{M}(v_x)\mathscr{M}(v_y)\mathscr{M}(v_z),$$

$$\mathscr{M}(v) \equiv \exp\left(-\frac{v^2}{v_T^2}\right)\frac{1}{\sqrt{\pi}v_T}, \qquad v_T \equiv \sqrt{\frac{2k_BT}{m_A}}.$$

Here m_A is the atomic mass, and, $\Theta$is a function accounting for the fact that only incoming atoms contribute to absorption; for these atoms$(\mathbf{v}, d\mathbf{S}) < 0$. The position of the area element $d\mathbf{S}$ is conveniently described in polar coordinates: $x = r\cos\beta, z = r\sin\beta$ (Fig. 3, *b*). Then $(\mathbf{v}, d\mathbf{S}) = [v_x\cos\beta + v_z\sin\beta]rd\beta dy$. Appendix 2 shows that the flight time of atoms of the velocity group $\mathbf{v}$ through the beam depends only on v_x and v_z and is given by the formula:

$$T(v_x, v_z, \beta) = -2r\,\frac{v_x\cos\beta + v_z\sin\beta}{v_x^2 + v_z^2}. \qquad (11)$$

For a given flight time T the populations of the ground resonant multiplet $n_M(T)$ depend only onv_y, since only this velocity component determines the Doppler shift, which in turn affects transition probabilities $A_{M\to m}$ (see below (23)). Thus, the dependence of these populations on velocity components has the form $n_M = n_M(v_y, T(v_x, v_z))$. Now use the fact that the integrand in (10) does not depend on the coordinate y, and integrating over it simply yields the length of the cell l along the probing beam axis.

Finally, recall again that only atoms flying into the beam contribute to absorption. For these atoms, the flight time (11) is positive (see Appendix 2). Taking all this into account, as well as expression (9) for dE/dN, from formula (10) we get the expression for the power absorbed by the atomic system:

$$W = \frac{\Gamma_F^{F'} + \Gamma_J^{F'}}{2\Gamma_J^{F'}}\frac{clr\hbar\Omega}{F+J+1}\int d\mathbf{v}d\beta\,\mathscr{M}_3(\mathbf{v})\Theta\Big(T(v_x, v_z, \beta)\Big)$$

$$\times\Big[v_x\cos\beta + v_z\sin\beta\Big]\sum_{M=-F}^{F}\Big[n_M(v_y, T(v_x, v_z, \beta)) - 1\Big]. \qquad (12)$$

This formula shows that the energy absorbed by the atoms is effectively proportional to the decrease in their number in the ground resonant multiplet, which is an expected result.

2.3. Expansion in powers of flight time T

As stated above, we consider the case of not too strong absorption when the populations n_M of states of the ground resonant multiplet do not have time to change significantly during the flight time T of atoms through the beam. Based on this, expand the quantities in the square brackets of the last multiplier in (12) in powers of T and keep only the first two terms of the expansion:

$$n_M(T) - 1 \equiv n_{1M}T + n_{2M}T^2 + O(T^3). \qquad (13)$$

Note that the flight time T (11) depends only on the v_x, $v_z$component of the atomic velocity, and the quantities n_{1M} and n_{2M} depend only on v_y (see the note after (11)). Substituting (13) into (12) allows writing the absorption W as:

$$W = \frac{\Gamma_F^{F'} + \Gamma_J^{F'}}{2\Gamma_J^{F'}}\frac{clr\hbar\Omega}{F+J+1}\Big[S_1\Sigma_1 + S_2\Sigma_2\Big] + O(T^3), \quad (14)$$

where

$$S_p \equiv \int dv_x dv_z d\beta\,\mathscr{M}(v_x)\mathscr{M}(v_z)\Theta\Big(T(v_x, v_z, \beta)\Big)$$

$$\times\Big[v_x\cos\beta + v_z\sin\beta\Big]T^p(v_x, v_z, \beta), \qquad (15)$$

$$S_1 = -\pi r, \qquad S_2 = -\frac{2\sqrt{\pi}r^2}{3v_T}, \qquad (16)$$

$$\Sigma_p \equiv \int dv_y\mathscr{M}(v_y)\sum_{M=-F}^{F} n_{pM}, \qquad p = 1, 2.$$

Calculations of the integrals (15) are given in Appendix 3. Substitution of (7) into the first of the equations (1) allows obtaining a closed equation for the populations $n_M, M = -F, \ldots, F$, which can be represented in matrix form:

$$\dot{n}_M = \sum_{M'=-F}^{F} B_{MM'}n_{M'}, \qquad (17)$$

where

$$B_{MM'} \equiv \sum_{m=-F'}^{F'} \frac{A_{M'\to m}\gamma_{m\to M}}{\Gamma_F^{F'}+\Gamma_J^{F'}} - \delta_{MM'} \sum_{m=-F'}^{F'} A_{M\to m}.$$

The solution of equation (17) has the form $n_M(T) = \sum_{M'=-F}^{F}[e^{BT}]_{MM'} n_{M'}(0)$. Expanding the exponent into a series up to T^2 and considering the initial condition (2), we get:

$$n_M(T) - 1 = T \sum_{M'=-F}^{F} B_{MM'} + \frac{T^2}{2} \sum_{M'=-F}^{F} \sum_{M''=-F}^{F} B_{MM'}B_{M'M''} + O(T^3). \quad (18)$$

Comparing this expression with (13), and using (16), we obtain:

$$\Sigma_1 = \int dv_y \mathscr{M}(v_y)\sigma_1, \quad \sigma_1 \equiv \sum_{M=-F}^{F} \sum_{M'=-F}^{F} B_{MM'}$$

$$= -\frac{\Gamma_J^{F'}}{\Gamma_F^{F'}+\Gamma_J^{F'}} \sum_{M=-F}^{F} \sum_{m=-F'}^{F'} A_{M\to m}, \quad (19)$$

$$\Sigma_2 = \int dv_y \mathscr{M}(v_y)\sigma_2, \quad (20)$$

$$\sigma_2 \equiv \frac{1}{2} \sum_{M=-F}^{F} \sum_{M'=-F}^{F} \sum_{M''=-F}^{F} B_{MM'}B_{M'M''}$$

$$= -\frac{\Gamma_J^{F'}}{\Gamma_F^{F'}+\Gamma_J^{F'}} \sum_{M=-F}^{F} \sum_{m=-F'}^{F'} A_{M\to m} \sum_{M'=-F}^{F} \frac{B_{MM'}}{2}.$$

Here, we used the fact that the matrix elements $A_{M\to m}$ and $B_{MM'}$ depend only on the v_y-component of the atomic velocity, which determines the Doppler shift. We will now proceed to calculating these matrix elements.

2.4. Probabilities of induced transitions $A_{M\to m}$

In our experiments, the dependence of resonant absorption of cesium vapor on the solenoid field is recorded (Fig. 1), which is directed along the axis x of the laboratory coordinate system $\mathbf{B}_s = (B_s, 0, 0)$. As was said, the states between which the probing beam induces transitions are referred to the quantization axis parallel to the total magnetic field $\mathbf{B} = \mathbf{B}_s + \mathbf{B}_e$ acting on the atomic system (Fig. 1). We assume that in the laboratory coordinate system the field $\mathbf{B}$ has components defined by angles ϕ and η, and the electric field $\mathbf{E}$ of the probing beam in this system is characterized by the azimuth of the polarization plane θ. Then

$$\mathbf{B} \equiv B \begin{pmatrix} \sin\phi\cos\eta \\ \sin\eta \\ \cos\phi\cos\eta \end{pmatrix}, \quad \mathbf{E} = E \begin{pmatrix} \sin\theta \\ 0 \\ \cos\theta \end{pmatrix} \cos\omega t, \quad (21)$$

besides

$$\phi = \operatorname{arctg}\frac{B_s + B_{ex}}{B_{ez}}, \qquad \eta = \operatorname{arctg}\left\{\frac{B_{ey}}{B_s + B_{ex}}\sin\phi\right\},$$

$$B = \sqrt{[B_s + B_{ex}]^2 + B_{ey}^2 + B_{ez}^2}.$$

The matrix D of the operator of interaction of the atomic system with the optical field of the probing beam in the wave function representation with definite projection along the direction of the magnetic field has the form $D = a_F^{F'}(E'_x S_x + E'_y S_y + E'_z S_z)$ (we omit the factor $\cos\omega t$), where $a_F^{F'}$ is the reduced matrix element of the dipole moment of the transition between the ground and excited atomic multiplets, $E'_{x,y,z}$ are components of the electric field of the probing beam, which differ from (21), in the coordinate system with the quantization axis z', directed along the magnetic field $\mathbf{B}$, and $S_{x,y,z}$ are standard vector operator matrices in the wave function representation with definite projection of momentum along the quantization axis z' [12]. Direct calculation of the components $E'_{x,y,z}$ leads to the following expression for the matrix D:

$$D \equiv E a_F^{F'} \left[\overbrace{\sin[\theta-\phi]}^{X} S_x - \overbrace{\sin\eta\cos[\theta-\phi]}^{Y} S_y + \overbrace{\cos\eta\cos[\theta-\phi]}^{Z} S_z \right]. \quad (22)$$

Denoting the detuning between the frequency of the probing beam ω and the transition frequency between the ground and excited resonant multiplets Ω by $\nu \equiv \Omega - \omega$ and introducing the standard matrix $S_+ \equiv S_x + \imath S_y$ [12], the expression for the transition rate $A_{M\to m} = 2\pi|\langle F, M|D|F', m\rangle|^2 \mathscr{L}([E_m^{F'} - E_M^F]/\hbar)/\hbar^2$ can be written as follows [12]:

$$A_{M\to m} = \frac{2\pi[a_F^{F'}E]^2}{\hbar^2}\mathscr{L}\Big(\nu + kv_y + m\omega_{L1} - M\omega_{L2}\Big)a_{M\to m}, \quad (23)$$

$$a_{M\to m} \equiv H^2\Big\{\delta_{m,M+1}|\langle F'M+1|S_+|FM\rangle|^2 + \delta_{m,M-1}|\langle FM|S_+|F'M-1\rangle|^2\Big\} + Z^2\delta_{m,M}|\langle F'M|S_z|FM\rangle|^2,$$

$$\mathscr{L}(x) \equiv \frac{1}{\pi}\frac{\delta}{\delta^2+x^2},$$

where $H^2 \equiv [X^2+Y^2]/4 = \frac{1}{4}[\cos^2[\theta-\phi]\sin^2\eta + \sin^2[\theta-\phi]]$, $Z^2 = \cos^2[\theta-\phi]\cos^2\eta$, $k \equiv \omega/c_0$ (c_0 – speed of light, and $4H^2 + Z^2 = 1$), δ – homogeneous linewidth, ω_{L2} and $E_M^F = \omega_{L2}M$ (ω_{L1} and $E_m^{F'} = \Omega + \omega_{L1}m$) are the Larmor frequencies and energies of the ground (excited) multiplet states respectively. Formula (23) also defines the coefficients $a_{M\to m}$, which we will use later. The matrix elements S_+ entering (23) differ from zero only when

$F' = F \pm 1$ or $F' = F$ and are given by the expressions [12]

$$\langle F, M+1|S_+|F-1, M\rangle = \sqrt{\frac{(F+M)(F+M+1)}{2}},$$
$$\langle F-1, M+1|S_+|F, M\rangle = -\sqrt{\frac{(F-M)(F-M-1)}{2}}, \quad (24)$$
$$\langle F, M|S_z|F-1, M\rangle = \langle F-1, M|S_z|F, M\rangle = -\sqrt{\frac{F^2-M^2}{2}},$$
$$\langle F, M+1|S_+|F, M\rangle = \sqrt{F(F+1)-(M+1)M}, \quad (25)$$
$$\langle F, M|S_z|F, M\rangle = M.$$

As seen from (23), the transition rates depend only on the component of the atomic velocity determining the Doppler shift kv_y, and when calculating the quantities Σ_1 (19) and Σ_2 (20) integrals of the following type will appear

$$I_1(x) \equiv \int \mathscr{M}(v)\mathscr{L}(kv+x)dv, \quad (26)$$
$$I_2(x,y) \equiv \int \mathscr{M}(v)\mathscr{L}(kv+x)\mathscr{L}(kv+y)dv.$$

The following circumstance allows simplifying formulas (19), (20). Since we are interested in the absorption behavior of the atomic system in magnetic fields so small that the Zeeman splittings are significantly less than the homogeneous linewidth $\omega_{L1,2} \ll \delta$, one can assume that the integrals in (26) depend only on the optical detuning ν. For example, $I_1(\nu + m\omega_{L1} - M\omega_{L2}) \approx I_1(\nu) \approx k^{-1}\mathscr{M}(\nu/k)$ and $I_2(\nu + m\omega_{L1} - M\omega_{L2}, \nu + m'\omega_{L1} - M'\omega_{L2}) \approx I_2(\nu, \nu) \approx k^{-1}\mathscr{M}(\nu/k)/[2\pi\delta]$. At this stage of calculations, the dependence on the magnitude of the magnetic field, whose direction sets the quantization axis, disappears.

For further calculations, we construct from the matrix B which does not depend on optical detuning, a matrix b, in which the transition rates $A_{M\to m} \to a_{M\to m}$ (17)

$$b_{MM'} \equiv \sum_{m=-F'}^{F'} \frac{a_{M'\to m}\gamma_{m\to M}}{\Gamma_F^{F'} + \Gamma_J^{F'}} - \delta_{MM'} \sum_{m=-F'}^{F'} a_{M\to m}. \quad (27)$$

are Taking into account the remarks made above, the quantities $\Sigma_{1,2}$ (19), (20) can now be written as:

$$\Sigma_1 = -\mathscr{M}(\nu/k)\frac{2\pi[a_F^{F'}E]^2}{k\hbar^2}\frac{\Gamma_J^{F'}}{\Gamma_F^{F'} + \Gamma_J^{F'}} \sum_{m=-F'}^{F'}\sum_{M=-F}^{F} a_{M\to m}, \quad (28)$$

$$\Sigma_2 = -\frac{k^{-1}\mathscr{M}(\nu/k)}{4\pi\delta}\left[\frac{2\pi[a_F^{F'}E]^2}{\hbar^2}\right]^2 \frac{\Gamma_J^{F'}}{\Gamma_F^{F'} + \Gamma_J^{F'}} \times \sum_{m=-F'}^{F'}\sum_{M=-F}^{F}\sum_{M'=-F}^{F} a_{M\to m}b_{MM'}. \quad (29)$$

In obtaining these formulas, wherever possible, summation over M was performed using the relations (5). Substituting the obtained expressions for $\Sigma_{1,2}$ into formula (14), the expression for the power W absorbed by the atomic system can be brought to the form:

$$W = \sqrt{\pi}N\frac{e^{-\nu^2/\Delta^2}}{\Delta}\frac{\omega_R^2\hbar\Omega}{F+J+1}\Bigg[\overbrace{\sum_{m=-F'}^{F'}\sum_{M=-F}^{F} a_{M\to m}}^{\mathscr{Y}} + \omega_R^2 T_1 T_2 \overbrace{\sum_{m=-F'}^{F'}\sum_{M=-F}^{F}\sum_{M'=-F}^{F} a_{M\to m}b_{MM'}}^{\mathscr{A}}\Bigg], \quad (30)$$

$$N = c\pi r^2 l, \quad \Delta = kv_T, \quad \omega_R \equiv \frac{a_F^{F'}E}{\hbar},$$
$$T_1 \equiv \frac{1}{3\sqrt{\pi}}\frac{r}{v_T}, \quad T_2 \equiv \frac{1}{\delta}.$$

The quantities N and Δ represent, respectively, the number of atoms in the beam and the Doppler width of the atomic absorption line, while ω_R is the Rabi frequency determined by the intensity of the probing beam. In the case of weak absorption, the power P' of the probing beam at the output of the cell is related to the input power $P$by the relation $P' = P - W$. Note here that the nomenclature of the atomic eigenstates used, relying on the quantum numbers M and m projections of the total angular momentum — is justified when the optical excitation of the atomic system is not too strong and $\omega_R \ll \omega_{L1,2}$.

The first sum $\mathscr{Y}$ in the square brackets describes the linear absorption independent of the probing beam intensity. The linear contribution $\sim \mathscr{Y}$ is the standard expression for absorbed power by an atomic system under unresolved multiplet structure conditions and, being the dominant term, can be used for the experimental evaluation of the reduced matrix elements $a_F^{F'}$. Using expressions (24), (25) for the matrix elements entering this sum, explicit formulas for $\mathscr{Y}$ can be derived and it can be shown (Appendix 4, expressions (56)–(58)), that this quantity does not depend on the angles θ, ϕ, η. Thus, as noted above, linear theory does not yield the considered effect of absorption dependence on the mutual orientation of the magnetic field and the linear polarization of the probing beam. This dependence is described by the second term $\sim \mathscr{A}$ in the square brackets (30), whose contribution is proportional to the probing beam intensity $\sim E^2$. The factor appearing before this term in (30) has the form of a standard saturation factor $\omega_R^2 T_1 T_2$, which includes the Rabi frequency ω_R, the phase relaxation time $T_2 \equiv 1/\delta$ and the effective „population relaxation time“ T_1, determined by the transit time $\sim r/v_T$of atoms through the probing beam. The nonlinear contribution to absorption $\sim \mathscr{A}$ is key to the effect discussed in this work. This contribution is anisotropic, i.e., it depends on the angle between the magnetic field acting on the atomic system and the direction of linear polarization of the probing beam. This dependence is fully contained in the factor $\mathscr{A}$, and will be described in the next section.

3. Nonlinear anisotropic contribution to the absorption of an atomic system. Isotropy of the $F = 3 \to F' = 4$transition

Substituting (27) into (30), one can obtain the following expression for the quantity $\mathscr{A}$:

$$\mathscr{A} = \sum_{m,m'=-F'}^{F'} \sum_{M=-F}^{F} \sum_{M'=-F}^{F} \frac{a_{M'\to m'}\gamma_{m'\to M}a_{M\to m}}{\Gamma_F^{F'} + \Gamma_J^{F'}} - \sum_{m,m'=-F'}^{F'} \sum_{M=-F}^{F} a_{M\to -m}a_{M\to m'}. \quad (31)$$

Let us introduce the quantities $\Phi_M^{F'F}$ and $U_M^{F'F}$ using the relations:

$$\sum_{m=-F'}^{F'} a_{M\to m} = \overbrace{= H^2 \left\{ |\langle F'M+1|S_+|FM\rangle|^2 + |\langle FM|S_+|F'M-1\rangle|^2 \right\}}^{\Phi_M^{F'F}} + Z^2 \overbrace{|\langle F'M|S_z|FM\rangle|^2}^{U_M^{F'F}} \equiv H^2\Phi_M^{F'F} + Z^2U_M^{F'F}. \quad (32)$$

Using (24), it can be verified that

$$\sum_{M=-F}^{F} a_{M\to m} = H^2\Phi_m^{FF'} + Z^2U_m^{FF'}. \quad (33)$$

In terms of the introduced quantities $\Phi_M^{F'F}$ and $U_M^{F'F}$ the anisotropic contribution to the absorption $\mathscr{A}$ can be written as:

$$\mathscr{A} = \sum_{m=-F'}^{F'} \sum_{M=-F}^{F} \frac{[H^2\Phi_M^{F'F} + Z^2U_M^{F'F}]\gamma_{m\to M}[H^2\Phi_m^{FF'} + Z^2U_m^{FF'}]}{\Gamma_F^{F'} + \Gamma_J^{F'}} - \sum_{M=-F}^{F} [H^2\Phi_M^{F'F} + Z^2U_M^{F'F}]^2. \quad (34)$$

From the given expression, it follows that the general form of the dependence of $\mathscr{A}$ on the angular factors H and Z is:

$$\mathscr{A} = \alpha H^4 + \beta Z^4 + \gamma H^2Z^2, \quad (35)$$

where

$$\alpha \equiv \sum_{m=-F'}^{F'} \sum_{M=-F}^{F} \frac{\Phi_M^{F'F}\gamma_{m\to M}\Phi_m^{FF'}}{\Gamma_F^{F'} + \Gamma_J^{F'}} - \sum_{M=-F}^{F} [\Phi_M^{F'F}]^2,$$

$$\beta \equiv \sum_{m=-F'}^{F'} \sum_{M=-F}^{F} \frac{U_M^{F'F}\gamma_{m\to M}U_m^{FF'}}{\Gamma_F^{F'} + \Gamma_J^{F'}} - \sum_{M=-F}^{F} [U_M^{F'F}]^2,$$

$$\gamma \equiv \sum_{m=-F'}^{F'} \sum_{M=-F}^{F} \frac{\Phi_M^{F'F}\gamma_{m\to M}U_m^{FF'} + U_M^{F'F}\gamma_{m\to M}\Phi_m^{FF'}}{\Gamma_F^{F'} + \Gamma_J^{F'}} - 2\sum_{M=-F}^{F} U_M^{F'F}\Phi_M^{F'F}.$$

From definition (23) for the quantities H and Z it is clear that they can be considered respectively as half the sine and the cosine of the angle ξbetween the polarization direction of the probe beam and the magnetic field:

$$H \equiv \frac{1}{2}\sin\xi, \qquad Z = \cos\xi. \quad (36)$$

For arbitrary F and $F' = F, F \pm 1$ using relations (24) and (25, the following explicit expressions for the quantities $\Phi_M^{F'F}$ and $U_M^{F'F}$can be obtained:

$$\Phi_M^{F'F} = (F+1)(F+2) + M^2, \quad U_M^{F'F} = [(F+1)^2 - M^2]/2, \quad F' = F+1,$$
$$\Phi_M^{F'F} = F(F-1) + M^2, \quad U_M^{F'F} = [F^2 - M^2]/2, \quad F' = F-1,$$
$$\Phi_M^{F'F} = 2[F(F+1) - M^2], \quad U_M^{F'F} = M^2, \quad F' = F. \quad (37)$$

Since the expression (35) for the anisotropic contribution $\mathscr{A}$ includes ratios of the form $\gamma_{m\to M}/[\Gamma_F^{F'} + \Gamma_J^{F'}]$ (see (35), only relative values of the reduced transition matrix elements $[a_F^{F'}]^2$, that enter expressions (44), (45) for $\gamma_{m\to M}, \Gamma_F^{F'}, \Gamma_F^{F'}$ are needed for the calculation of this contribution. For the D1 line of cesium, these relative values are known [10]:

$$\begin{aligned} &[a_3^4]^2 = 7/12\ \ d^2, && F = 3 \to F' = 4,\\ &[a_4^3]^2 = 3/4\ \ d^2, && F = 4 \to F' = 3,\\ &[a_4^4]^2 = 5/12\ \ d^2, && F = 4 \to F' = 4,\\ &[a_3^3]^2 = 1/4\ \ d^2, && F = 3 \to F' = 3, \end{aligned} \quad (38)$$

where the overall dipole moment d^2 of the cesium D1 line can be determined experimentally and compared with its tabulated value [10].

The angular dependence of the anisotropic contribution $\mathscr{A}$ (35) for all four transitions of the cesium D1 line, calculated using these data, is shown in Fig. 4. As seen from this figure, this contribution is always negative, and for the $F = 3 \to F' = 4$ transition, the angular dependence is practically absent, which fully agrees with the experiment. In our experiments, the power P' of the beam transmitted through the cell with cesium vapor was recorded. If the absorption of the atomic system were small [3], this power

[3] i.e., when propagation effects can be neglected and the single scattering approximation is applicable

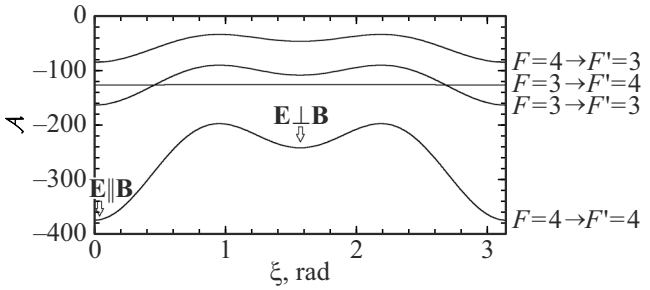

Figure 4. Dependence of the nonlinear contribution $\mathscr{A}$ (35) to the transmission on the angle ξ between the magnetic field and the direction of linear polarization of the probe beam (36) for all transitions of the cesium D1 line. It can be seen that, in agreement with the experiment, the developed theory shows essentially complete isotropy of the $F = 3 \to F' = 4$transition.

would be related to the power P of the incident laser beam by the relation $P' = P - W$, where the quantity W is defined by expression (30). However, in our experiments, absorption could be $\sim 50\%$ or greater, so it seems natural to take propagation effects into account, relating the calculated quantity W to the optical density of the vapor in the cell. This will be done in the next section.

4. Accounting for Propagation Effects

The formula (30) derived above for the absorbed power is strictly applicable when the intensity of the probing beam changes little while passing through the cell containing alkali metal vapor — the Rabi frequency ω_R entering this formula is considered independent of position inside the cell and determined by the input laser beam power. In a real experiment, the transmission coefficient can be ~ 0.5 or less, which can noticeably affect the accuracy of estimates made using formula (30). In this regard, it is appropriate to indicate a straightforward method to account for propagation effects and to provide expressions for the resonant optical density of atomic vapors. For this, one may consider that the above calculation of absorbed power applies not to the entire cell length l, but to an element dy of the probing beam inside the cell. The power p of the probing beam at the input of this element and its power $p + dp$ at the output are related by$p + dp = p - W$, where in expression (30) for W one should replace $l \to dy$ and express the Rabi frequencyω_R through the „current“ power p of the probing beam using the known relation [10]:

$$\omega_R^2 = \left[\frac{a_F^{F'}}{\hbar}\right]^2 E^2 = \left[\frac{a_F^{F'}}{\hbar}\right]^2 \frac{2\mu_0 c_0}{\pi r^2}\ p. \qquad (39)$$

This leads to the following equation for the probing beam power p, which becomes a function of the coordinate y:

$$\frac{dp}{dy} = -\varkappa p - \varepsilon p^2, \qquad (40)$$

where

$$\varkappa = \frac{e^{-\nu^2/\Delta^2}}{\Delta}\ frac2\sqrt{\pi}\mu_0 c_0 c\ \hbar\Omega F + J + 1\left[\frac{a_F^{F'}}{\hbar}\right]^2 \mathscr{I},$$

$$\varepsilon = \frac{e^{-\nu^2/\Delta^2}}{\Delta}\ \frac{4c\ \hbar\Omega}{F + J + 1}\left[\frac{a_F^{F'}}{\hbar}\right]^4 \frac{\mu_0^2 c_0^2}{3\pi r v_T \delta}\ \mathscr{A}.$$

The solution of equation (40) with the initial condition $p(0) \equiv P$ (where P is the laser beam power at the cell entrance) has the form:

$$P' \equiv p(l) = \frac{\varkappa}{\varkappa + P[1 - e^{-\varkappa l}]\varepsilon}\ Pe^{-\varkappa l}. \qquad (41)$$

As is seen from this expression, $\varkappa$can be interpreted as the optical density of the atomic vapor inside the cell, and formula (41) is applicable not only for small absorption. Note that at very high beam power, formula (41) becomes inapplicable. Section 3 stated that the quantity $\varepsilon \sim \mathscr{A}$, describing the nonlinear contribution to absorption, is negative $\varepsilon < 0$. Therefore, the relations presented here only make sense when the beam power does not exceed the power

$$P_c \equiv -\varkappa/\varepsilon = -\frac{3}{2}\ \frac{\pi^{3/2} r v_T \hbar^2 \delta}{\mu_0 c_0 [a_F^{F'}]^2}\ \frac{\mathscr{I}}{\mathscr{A}}, \qquad (42)$$

at which the denominator in (41) can become zero, and they remain valid for beam power $P < P_c$.

Results of calculations using formulas (40) and (41) are shown in Fig. $2\,d$ and Fig. 5. For clarity of comparison with experiment, the probing beam power was chosen to be sufficiently large ($\sim 50\ \mu W$) — in this case, the relative magnitude of the absorption feature's dependence on magnetic field was $\sim 10\%$ (Figs. 2b, d and 5) and was clearly visible. Although the applicability condition $\omega_R \ll \omega_{L1,2}$ of our theory might have been somewhat violated in this case, the experimental data presented in Fig. $2, b$ and Fig. 5 could be interpreted using relations (30),(40) and (41), with the values of the reduced matrix elements (38) $a_F^{F'} \sim d \sim 1.6 \cdot 10^{-30}$ C · m used corresponding to the cesium vapor absorption cross-section known from other sources [10] within an average error $\sim 30\%$.

Note that in our experiments we used the Earth's magnetic field $\mathbf{B}_e$ to observe the dependence of atomic system absorption on the mutual orientation of the probing beam polarization and the total magnetic field $\mathbf{B}$. This dependence can be described by a single angle ξ and is effectively presented in Fig. 4. The experiments in Figs. $2, b,c$ and 5 show absorption dependence on the solenoid field $\mathbf{B}_s$, with the mutual orientation of the total magnetic field and the probing beam polarization direction determined by formulas (21) and (22). The required components of the Earth's field were determined by fitting and corresponded to known values $B_{ex}, B_{ey}, B_{ez} \sim 0.5$ Gs.

Since the described effect essentially represents dependence of absorption on the azimuth of linear polarization (linear dichroism), beam propagation in the atomic medium

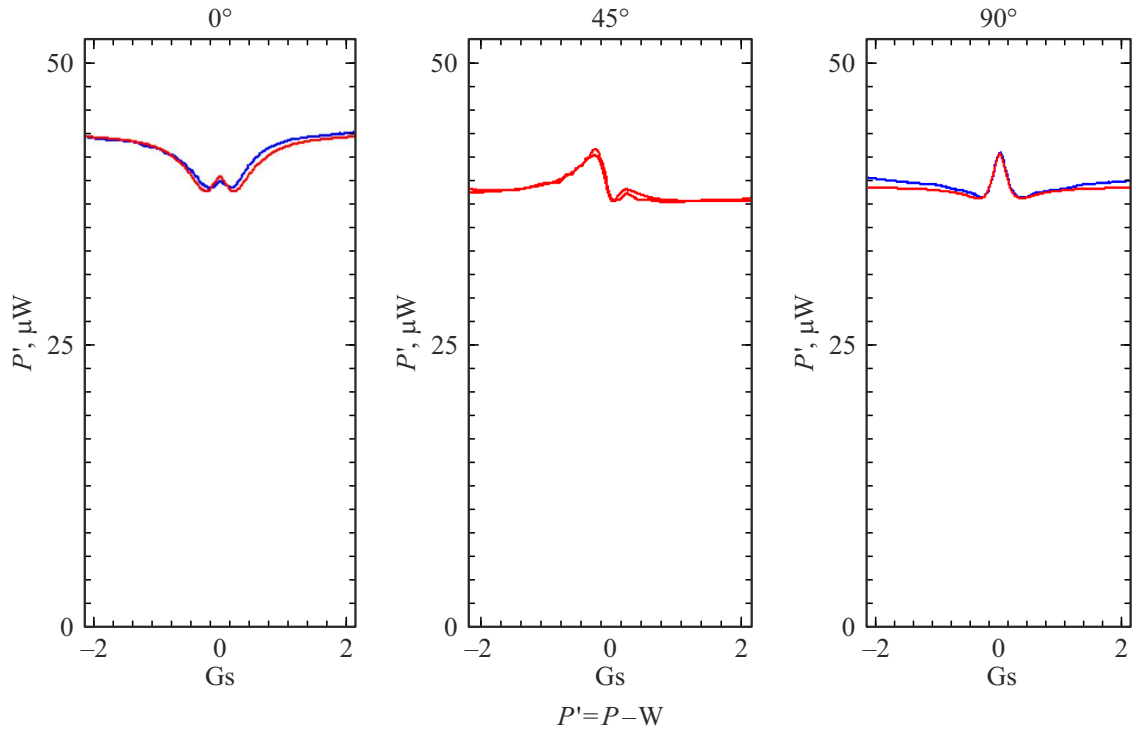

Figure 5. Resonant probing on the$F = 4 \rightarrow F' = 3$ transition of the cesium D1 line. Calculated (smooth) and experimental (noisy) dependencies of the beam power P' after passing through the cell on the solenoid magnetic field, in the Voigt geometry, for various mutual orientations of the azimuth of the probing beam polarization and the solenoid magnetic field (compare with Fig. 2, *c*).

may be accompanied by changes in its polarization. Such an effect is described in [13–15].

5. Conclusion

This work investigates the dependence of cesium atomic vapor absorption near the D1 line on a small (close to Earth's) magnetic field. It is shown that even under unresolved Zeeman structure conditions, the nonlinear absorption significantly depends on the mutual orientation of the magnetic field and the azimuth of linear polarization of the probing beam at all transitions except $F = 3 \rightarrow F' = 4$, where this dependence is suppressed by at least two orders of magnitude. A theory of nonlinear absorption of the atomic system is constructed, explaining these properties of cesium vapor atomic systems.

Note that in the above consideration, the described effect does not explicitly depend on the magnitude of the magnetic field. Nevertheless, anisotropy of the atomic system is caused in the calculation by the magnetic field, which must be sufficiently strong such that the Rabi frequency of the linearly polarized probing optical beam satisfies the inequality $\omega_R \ll \omega_{L1,2}$. Only in this case will the eigenstate nomenclature be determined by the projection of total angular momentum along the magnetic field direction, and equations (3) (essentially, only the diagonal elements of the atomic density matrix), containing transition rates between these states, will have meaning.

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Conflict of interest

The author declares that he has no conflict of interest.

Appendix 1 (Radiative Sums)

The radiative disintegration rate of the excited atomic state is determined by the interaction of the atom with the quantized photon field, which may be in the vacuum state. In this case, atomic excitation decay occurs due to

vacuum fluctuations of the photon field accompanied by photon emission. The disintegration rate from the excited multiplet state $|m, F'\rangle$ with momentum projection m to the ground multiplet state $|M, F\rangle$ can be expressed through the matrix elements of operators S_+ and S_z and the reduced inter-multiplet matrix element $a_F^{F'}$ as follows [11]:

$$\gamma_{m\to M} = \frac{1}{\tau_F^{F'}} \sum_{i=x,y,z} |\langle F', m|S_i|F, M\rangle|^2$$

$$= \frac{1}{\tau_F^{F'}} \left\{ \frac{\delta_{m,M+1}}{2} \left| \langle F', M+1|S_+|F, M\rangle \right|^2 \right.$$

$$+ \frac{\delta_{m,M-1}}{2} \left| \langle F, M|S_+|F', M-1\rangle \right|^2$$

$$\left. + \delta_{m,M} \left| \langle F', M|S_z|F, M\rangle \right|^2 \right\},$$

$$\frac{1}{\tau_F^{F'}} \equiv \frac{4[a_F^{F'}]^2\Omega^3}{3\hbar c_0^3}. \tag{43}$$

Using the expressions (25) for the matrix elements entering this formula, one obtains expressions for the sums (5) of radiative decay rates over the ground multiplet states $\Gamma_F^{F'} \equiv \sum_{M=-F}^{F} \gamma_{m\to M}$:

$$\begin{aligned} \Gamma_F^{F'} &= F(F+1)/\tau_F^{F'}, & F' &= F, \\ \Gamma_F^{F'} &= (F+1)(2F+1)/2\tau_F^{F'}, & F' &= F+1, \\ \Gamma_F^{F'} &= F(2F+1)/2\tau_F^{F'}, & F' &= F-1, \end{aligned} \tag{44}$$

and confirms that these sums do not depend on the index m of the excited state.

Appendix 2 (Transit Time)

Write the equations of motion of an atom entering the beam at time $t = 0$ at an arbitrary point on the beam surface, characterized in the plane xz by the angle β (Fig. 3, *b*):

$$\begin{aligned} x(t) &= r\cos\beta + v_x t, \\ z(t) &= r\sin\beta + v_z t. \end{aligned} \tag{45}$$

After the flight time T the atom will again be on the beam surface; hence, $x^2(T) + z^2(T) = r^2$. Substituting relations (45) into this condition and solving for T, yields relation (11).

Appendix 3 (Calculation of integrals S_p (15))

Substituting the Maxwell distribution (10) and the expression (11) for the flight time T into integral (15), it is transformed into:

$$S_p = \frac{(-2r)^p}{\pi} \int \frac{dv_x dv_z}{v_T^2} d\beta \exp\left[-\frac{v_x^2 + v_z^2}{v_T^2} \right]$$

$$\times \Theta\Big(-v_x\cos\beta - v_z\sin\beta - \Big)$$

$$\times \frac{[v_x\cos\beta + v_z\sin\beta]^{p+1}}{[v_x^2 + v_z^2]^p}.$$

In this integral, we perform the variable change $x = v_x/v_T, z = v_z/v_T$, introduce polar coordinates $x = \rho\cos\phi, z = \rho\sin\phi$ and a new variable $\xi = \beta - \phi$. This yields for the quantity S_p an integral of the form:

$$S_p = 2(-2r)^p v_T^{1-p} \int_0^\infty d\rho e^{-\rho^2} \rho^{2-p} \int_{\pi/2}^{3\pi/2} d\xi \ \cos^{p+1}\xi,$$

calculation of which is straightforward and leads to relations (16).

Appendix 4 (Isotropy of Linear Absorption)

Consider the first term $\mathcal{Y}$ in the square brackets (30), describing the linear absorption of the atomic system. Calculate $\mathcal{Y} \equiv \sum_{M=-F}^{F} \sum_{m=-F'}^{F'} a_{M\to m}$ (24) for the case $F = F'$. Using the matrix elements (25), verify that the first term in the curly braces (24) yields the following contribution[4]:

$$\frac{1}{4}\sum_M \left\{ |\langle F, M+1|S_+|FM\rangle|^2 + |\langle FM|S_+|F, M-1\rangle|^2 \right\}$$

$$= \frac{1}{4}\sum_M \Big\{ F(F+1) - M(M+1) + F(F+1) - M(M-1) \Big\}$$

$$= \frac{1}{2}\Big\{ (2F+1)F(F+1) - \sum_M M^2 \Big\} = \frac{1}{3}F(F+1)(2F+1). \tag{46}$$

Now consider the sum with the factor Z^2

$$\sum_M |\langle F, M|S_z|FM\rangle|^2 = \sum_M M^2 = \frac{1}{3}F(F+1)(2F+1). \tag{47}$$

Since $4H^2 + Z^2 = 1$ always holds, for $F' = F$ we get

$$\mathcal{Y} = \sum_{M=-F}^{F} \sum_{m=-F'}^{F'} a_{M\to m} = \frac{F(F+1)(2F+1)}{3}. \tag{48}$$

Similarly, for $F' = F + 1$

$$\mathcal{Y} = \sum_{M=-F}^{F} \sum_{m=-F'}^{F'} a_{M\to m} = \frac{(2F+3)(2F+1)(F+1)}{6} \tag{49}$$

[4] In the calculations, the relation $\sum_{M=-F}^{F} M^2 = \frac{1}{3}F(F+1)(2F+1)$ is used.

and for $F' = F - 1$

$$\mathcal{Y} = \sum_{M=-F}^{F} \sum_{m=-F'}^{F'} a_{M\to m} = \frac{(2F+1)(2F-1)F}{6}. \quad (50)$$

From these expressions, it follows that the linear absorption of the atomic system in small magnetic fields, when the Zeeman structure is unresolved (i.e., $\omega_{L1}, \omega_{L2} < kv_T$), does not depend on the mutual orientation of the magnetic field and the azimuth of linear polarization of the probing beam.

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