15

Numerical simulation of supercontinuum generation based on similaritons in femtosecond fiber lasers

© U.S. Averkieva¹, F. Yan², A. Ismaeel^{1,3}, I.O. Orekhov¹, S.G. Sazonkin¹, D.A. Dvoretskiy¹, V.E. Karasik¹, L.K. Denisov¹

Received December 26, 2024 Revised February 14, 2025 Accepted February 17, 2025

In this paper, we conducted numerical simulations of the supercontinuum generation characteristics using a femtosecond fiber laser source. The research focused on analyzing the output characteristics of the supercontinuum generated in different types of optical fibers. The primary objective of this work was to optimize the output pulse parameters of the laser and the characteristics of highly nonlinear fibers to achieve a supercontinuum with the broadest spectral bandwidth and high coherence. To accomplish this, we developed and employed a mathematical model that describes the evolution and propagation of ultrashort pulses in optical fibers. The study examines key processes contributing to spectral broadening, including self-phase modulation, stimulated Raman scattering, four-wave mixing, and group velocity dispersion. Our findings demonstrate that hybrid-structure fibers with tailored dispersion profiles enable significant spectral broadening of the supercontinuum while requiring lower peak input pulse power and shorter fiber lengths. Additionally, we investigated the effects of critical factors on supercontinuum formation, which allowed us to propose strategies for improving the performance of such light sources in various applications, including spectroscopy, medicine, metrology, and telecommunications.

Keywords: supercontinuum, femtosecond fiber laser, highly nonlinear optical fiber, numerical simulation of supercontinuum generation.

DOI: 10.61011/EOS.2025.04.61416.7501-24

Introduction

A broad spectral range and a variety of applications in fundamental and applied research fields make supercontinuum (SC) generation a key direction in the development of modern optical technology [1]. Supercontinua are used widely to detect industrial toxic gases [2], in medicine for optical coherence tomography [3], in metrology [4], and in multiplexed data transmission systems [5]. The unique properties of a supercontinuum, such as its broad spectrum and high spatial coherence, make it indispensable in applications requiring precise spectral analysis [6,7] or in the field of high-precision optical measurements [8,9].

The use of femtosecond fiber lasers is one of the most efficient ways to generate a supercontinuum [10–12]. These lasers provide an opportunity to generate ultrashort pulses with high intensity, which establishes the conditions for excitation of nonlinear effects in highly nonlinear fibers. The high intensity of pulses promotes such nonlinear effects as self-phase modulation (SPM), stimulated Raman scattering (SRS), and four-wave mixing (FWM), which leads to significant broadening of the emission spectrum.

Highly nonlinear fiber (HNLF), which is used to form a broad spectrum due to nonlinear effects, is an important element in supercontinuum generation. Highly nonlinear fibers with different dispersions allow one to form a broader spectrum with lower energy expenditure, which makes them an efficient solution for the design of high-power stable supercontinuum sources [13].

Numerical modeling plays an important part in the optimization of parameters of such systems. It provides an opportunity to predict the behavior of nonlinear processes in the fiber and adjust the laser and fiber parameters to achieve the best supercontinuum characteristics. Simulations serve as a substitute for complex and expensive experiments, revealing efficient ways to improve the performance of optical systems.

In the present study, numerical simulations of supercontinuum generation were performed for a femtosecond fiber laser and highly nonlinear fiber. The main objective was to examine the system parameters and output SC characteristics.

Method of numerical simulation of supercontinuum generation

The numerical simulation of supercontinuum generation characterized the generation of output pulses obtained with the use of an all-fiber femtosecond laser with passive mode locking (ML) based on the effect of nonlinear polarization evolution and highly nonlinear fiber. The structural diagram

¹ Bauman Moscow State Technical University, Moscow, Russia

² Harbin Polytechnic University, Harbin, China

³ Moscow Institute of Physics and Technology MIPT, Dolgoprudny, Moscow region, Russia e-mail: USL99@mail.ru

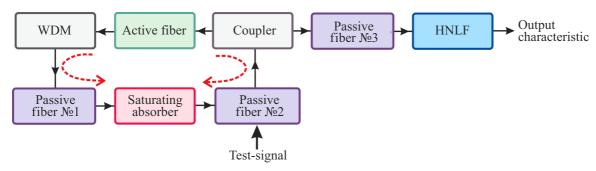


Figure 1. Diagram of the mathematical model of supercontinuum generation.

of the mathematical model of the SC generator is shown in Fig. 1. The characteristics of passive fibers Nos. 1, 2, and 3 in the mathematical model of the SC generator corresponded to those of standard telecommunication fiber The test signal was modeled as spontaneous SMF-28. emission that initiated lasing. The saturating absorber combined the functions of polarization controllers and a polarizer isolator. Germanosilicate fiber HNLF Ge-225 was used for supercontinuum generation. The active fiber corresponded to the laboratory equivalent of erbium fiber The coupler and the fiber multiplexer (WDM) had the parameters and characteristics of devices from the experimental setup discussed in [14].

The method for modeling the generation of the output optical SC spectrum presented in this paper is based on the results reported in [15], where the efficiency of nonlinearity and dispersion control in generation of ultrashort pulses and ultrawide output optical SC spectra with highly nonlinear germanosilicate fibers was demonstrated experimentally.

The solution of the Ginzburg-Landau equation, which characterizes the balance between nonlinear effects, medium dispersion, losses, and gain in fiber cavities with passive ML was used for numerical simulations. This equation allowed us to model the dynamics of pulse propagation with account for the key features of interactions of various nonlinear effects inside the laser system cavity. It was solved using the split-step Fourier method (SSFM), which made it possible to model linear and nonlinear interactions efficiently. The convergence condition was enforced by monitoring the relative change in pulse amplitude at each Nth cavity pass, which confirmed the stability of calculations and the accuracy of the proposed model [16].

One feature of the mathematical model was the potential for parameter optimization, which made it possible to establish efficient control over the generation process and form an ultrawide output optical spectrum within the wavelength range from 1200 to 2400 nm.

Simulation results

At the first stage of numerical modeling, stable generation in the form of similaritons was obtained. This is consistent with the earlier data reported in [17,18]. Specifically, the investigation of conditions of self-similar propagation and amplification of parabolic pulses in optical fibers in [17] laid the foundation for the analysis of similariton dynamics. The studies conducted in [18] demonstrate the evolution of similariton pulses in erbium fiber lasers with hybrid ML, verifying the applicability of this approach to generation of stable pulses under the considered conditions.

The obtained single pulse energy was as high as 411 pJ, and the average output power was 10.57 mW. These pulses with a duration of 73 fs were introduced into a Ge-225 HNLF with nonlinearity coefficient $\gamma=0.009\,\mathrm{W/m}$ and second-order dispersion $\beta_2=-2.1\,\mathrm{ps^2/km}$ at a wavelength of 1560 nm. Figure 2 shows the output optical SC spectrum plots corresponding to the highly nonlinear fiber 3 m in length. The obtained output optical SC spectrum was extended from 1228 to 2001 nm with a width of 773 nm at the level of $-20\,\mathrm{dB}$. This spectral range confirms the efficiency of application of highly nonlinear fibers with high nonlinearity and small core diameter.

Figure 3 presents the evolution of the output optical spectrum and the pulse in the course of propagation within a 3-m-long Ge-225 HNLF. At the initial stage of pulse propagation (within the first 0.15 m), the similariton pulse was subjected to temporary compression, inducing symmetric broadening of the optical spectrum. Under the influence of SRS and higher-order dispersion, the pulse then started to split, which led to asymmetric broadening of the optical spectrum. The split pulse shifted to the long-wavelength region under the influence of SRS, gradually moving away from the residual signal pulse due to differences in group velocity.

The primary spectral broadening was largely confined to the first 0.5 m; further broadening of the optical spectrum was associated with the red shift of the newly formed pulse, while the short-wavelength SC region remained virtually unchanged.

The peak power of pulses introduced into the highly nonlinear fiber also had a direct effect on the width and structure of the output optical SC spectrum. It can be seen from Fig. 4 that the influence of such nonlinear effects as SPM and SRS increases significantly with an increase in peak power of the input pulses from 2.79 to 6.70 kW, which contributes to significant broadening of the SC spectrum. Numerical simulation data revealed that the output optical

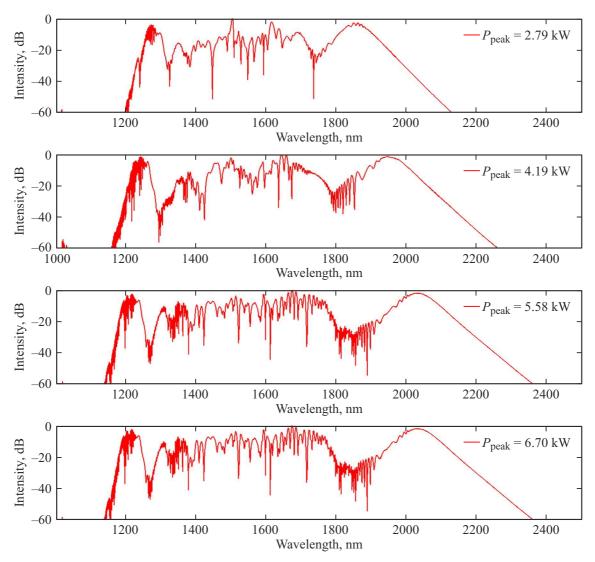


Figure 2. Output optical SC spectrum obtained in numerical simulations.

spectrum may be expanded to $1126 \,\mathrm{nm}$ at the $-20 \,\mathrm{dB}$ level at a pulse power of $6.70 \,\mathrm{kW}$.

For further analysis, the spatial coherence of the output optical SC spectrum was examined at different values of second-order dispersion. The supercontinuum coherence was estimated using the ensemble averaging method, which made it possible to determine quantitatively the degree of first-order coherence in the output optical spectrum and take the influence of modulation instability on the formation of the output signal into account. Figure 5 shows the coherence plots for the output optical SC spectrum with HNLFs with positive and negative dispersion.

The simulation results revealed the presence of chaotic processes, which are caused by modulation instability and splitting of similaritons, in the case of application of highly nonlinear fibers with negative dispersion $\beta_2 = -2.1 \,\mathrm{ps^2/km}$. These processes had a negative effect on coherence of the output optical spectrum. The generation of sub-pulses induced by nonlinear effects made the process of broadening

of the output optical spectrum sensitive to external noise and changes in initial conditions.

In contrast, highly nonlinear fibers with positive dispersion $\beta_2 = 2.1 \, \mathrm{ps^2/km}$ demonstrated a more stable SC generation process. The SPM effect provided symmetrical spectrum broadening and minimum influence of external noise on spatial coherence. The simulation results confirmed that positive-dispersion fibers ensure high spatial coherence of the output optical spectrum and uniformity of phase characteristics, which makes them preferable in applications requiring stable SC generation.

However, despite the high spatial coherence, the spectral broadening in positive-dispersion fibers was limited compared to negative-dispersion ones. This made it necessary to combine the advantages of both dispersion types in one structure. At the final stage of simulation, the influence of the hybrid fiber structure [19] with alternating dispersion on the output supercontinuum characteristics was investigated. An iterative method for adjusting the values of quadratic

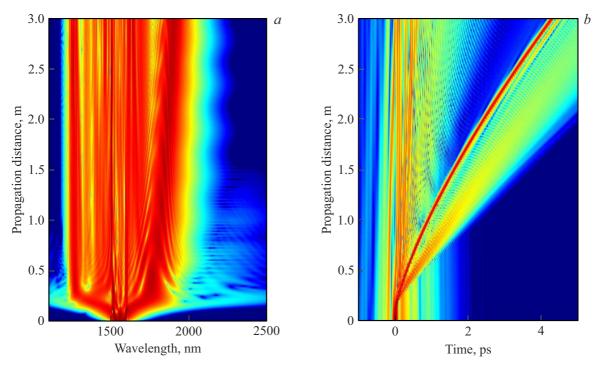


Figure 3. Pulse propagation in highly nonlinear fiber: (a) evolution of the output optical spectrum; (b) pulse evolution in the time domain.

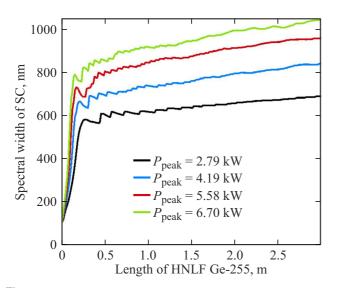


Figure 4. Variation of width of the output optical SC spectrum with peak power of a pulse introduced into highly nonlinear fiber.

dispersion was used to choose the optimum parameters of the hybrid HNLF structure. The dispersion parameters of fiber segments were varied in the course of numerical simulation in order to maximize spectrum broadening and maintain high coherence. The values of $\beta_2=2.1\,\mathrm{ps^2/km}$ and $-2.1\,\mathrm{ps^2/km}$ were determined by searching through the possible combinations and provided the best results in terms of spectral broadening and supercontinuum generation stability.

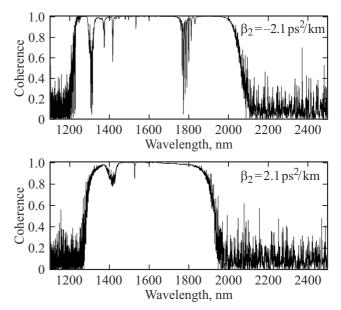


Figure 5. Spatial coherence of the output optical SC spectrum: (a) HNLF with negative dispersion; (b) HNLF with positive dispersion.

In this structure, segments with negative dispersion $\beta_2 = -2.1 \,\mathrm{ps^2/km}$ alternated with segments with positive dispersion $\beta_2 = 2.1 \,\mathrm{ps^2/km}$, which made it possible to combine the advantages of both types of fibers: in segments with negative dispersion, a pulse was compressed and its peak power was increased; in segments with positive

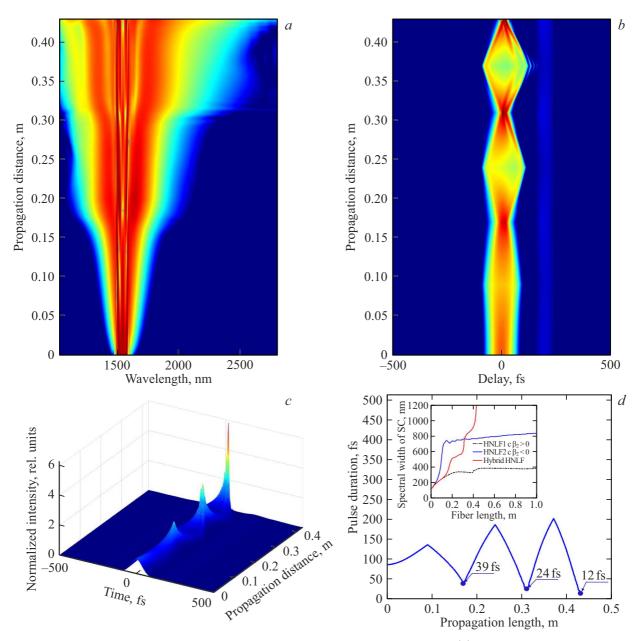


Figure 6. Output characteristics in propagation of a pulse through the hybrid HNLF structure: (a) evolution of broadening of the output optical spectrum in the frequency domain; (b) pulse shape evolution in the time domain; (c) normalized average peak power of pulses after propagation through each HNLF segment; (d) pulse duration evolution. Inset: comparison of widths of the output optical spectra in HNLF.

dispersion, the pulse was expanded, which contributed to stability and coherence of output emission. This approach is consistent with the data obtained in [20], where the use of a hybrid HNLF consisting of several segments with alternating dispersion was proposed. The results of the study demonstrated that such a structure allows one to form an output optical SC spectrum within the wavelength range from 900 to 2200 nm, which is significantly better than the parameters achieved in fibers with uniform dispersion.

At the same time, the concept of highly nonlinear fibers with a hybrid structure (fibers with alternating dispersion) featuring alternating segments with positive and negative

dispersion was presented in [21]. It was found that the correct choice of fiber parameters and length allows for an exponential increase in width of the SC spectrum, confirming that hybrid fibers are highly efficient in generation of ultrawide optical spectra.

Figure 6, a shows the evolution of spectral broadening of the supercontinuum in the frequency domain, while Fig. 6, b illustrates the changes in shape of a pulse in the time domain occurring as it propagates through a 43-cm-long hybrid HNLF structure. In the course of pulse propagation, the output optical SC spectrum broadened under the influence of SPM in the section with positive dispersion, while a

compression of the pulse in the time domain, which led to an increase in peak power and further broadening of the SC spectrum, was observed in the section with negative dispersion.

It can be seen from Fig. 6, c that an increase in peak power and a reduction in pulse duration are observed after the passage through each segment with negative dispersion. This enhanced the action of nonlinear effects and contributed to effective broadening of the output optical spectrum in the process of propagation through the subsequent segment with positive dispersion. The peak pulse power increased by a factor of 6 relative to the initial level as a result of passing through the 43-cm-long hybrid HNLF structure.

In addition, Fig. 6, d makes it clear that the hybrid structure prevents pulse splitting, allowing one to obtain an ultrashort output pulse with a duration of 12 fs and a wide spectral coverage of 1204 nm at the $-20\,\mathrm{dB}$ level. The inset in Fig. 6, d presents a width comparison of the output optical spectra obtained using the hybrid highly nonlinear fiber structure. When fibers with positive and negative dispersion were used for SC generation, the rate of broadening of the output optical spectrum plateaued at a level of 400 and 800 nm, respectively. At the same time, the hybrid HNLF structure 43 cm in length ensured further broadening of the optical spectrum. The output optical SC spectrum spanned from 1087 nm to 2291 nm, exceeding an octave in width (1204 nm) at the $-20\,\mathrm{dB}$ level.

Thus, the hybrid HNLF structure combined the advantages of fibers with positive and negative dispersion, making it possible to reduce significantly the length of the highly nonlinear fiber used and increase the peak power of the output pulse to 2kW without additional amplifiers. The simulated SC source featured not only a wide spectral coverage, but also pulse shape stability in the time domain.

Conclusion

Numerical modeling of supercontinuum generation with a femtosecond fiber laser with passive ML based on nonlinear polarization evolution and highly nonlinear fibers with different dispersion was performed. The main objective of the study was to optimize the parameters of the laser system and highly nonlinear fiber to obtain a wide spectral coverage and a stable pulse shape in the time domain.

The parameters of SC generation in fibers with positive, negative, and hybrid dispersion were examined. Simulations revealed that fibers with negative dispersion provide a significant broadening of the output optical spectrum due to the development of nonlinear effects, such as SPM and SRS. However, the processes of modulation instability and pulse splitting caused a reduction in coherence of the output optical spectrum. In contrast, positive-dispersion fibers demonstrated high spatial coherence and optical spectrum stability, but the spectral width was limited.

A hybrid highly nonlinear fiber structure with alternating segments with positive and negative dispersion was pro-

posed and investigated as a design capable of overcoming the identified limitations. This structure provided an opportunity to combine the advantages of both types of fibers. In segments with negative dispersion, the pulse was compressed, which enhanced its peak power and intensified the influence of nonlinear effects; in segments with positive dispersion, the pulse was expanded, which ensured stabilization of the shape and coherence of the optical spectrum.

The results of numerical simulation demonstrated that the 43-m-long hybrid HNLF structure provided an optical spectrum width of 1204 nm (1087–2291 nm) at the $-20\,\mathrm{dB}$ level. The peak pulse power did also increase by a factor of 6 to 2 kW, while the pulse shape and duration (12 fs) remained stable. In addition, the hybrid HNLF structure prevented pulse splitting, which made it the optimum choice for generating stable ultrawide supercontinuum spectra. This approach is promising for various scientific and engineering SC applications.

Funding

This study was supported by the Ministry of Science and Higher Education of the Russian Federation (state assignment project No. FSFN-2024-0018).

Conflict of interest

The authors declare that they have no conflict of interest.

References

- [1] A.M. Heidt, A. Hartung, H. Bartelt. *The Supercontinuum Laser Source*, ed. by R.R. Alfano (Springer, N.Y., 2016), 3rd ed., p. 247–280. DOI: 10.1007/978-1-4939-3326-6-6
- [2] A. Adamu, M. Dasa, O. Bang, C. Markos. IEEE Sens. J., 20 (18), 10591-10597 (2020).
 DOI: 10.1109/JSEN.2020.2993549
- [3] S.R.D.S. Rao, M. Jensen, L. Grüner-Nielsen, J.T. Olsen, P. Heiduschka, B. Kemper, J. Schnekenburger, M. Glud, M. Mogensen, N.M. Israelsen, O. Bang. Light Sci. Appl., 10, 133 (2021). DOI: 10.1038/s41377-021-00574-x
- [4] Q. Hao, T. Liu, H. Zeng. In: High Energy and Short Pulse Lasers, ed. by R. Viskup (IntechOpen, Austria, 2016). DOI: 10.5772/64209
- [5] S.V. Smirnov, J.D. Ania-Castañón, S. Kobtsev, S.K. Turitsyn. In: The Supercontinuum Laser Source: The Ultimate White Light, ed. by R.R. Alfano. 4th ed. (Springer International Publishing AG, 2023), p. 397—432. DOI: 10.1007/978-3-031-06197-4_9
- [6] A. Reyes-Reyes, Z. Hou, E. van Mastrigt, R.C. Horsten, J.C. de Jongste, M.W. Pijnenburg, H.P. Urbach, N. Bhattacharya. Opt. Express, 22 (15), 18299 (2014). DOI: 10.1364/OE.22.018299
- [7] K. Eslami Jahromi, M. Nematollahi, R. Krebbers, M.A. Abbas, A. Khodabakhsh, F.J.M. Harren. Opt. Express, 29 (8), 12381 (2021). DOI: 10.1364/OE.418072
- [8] I. Zorin, P. Gattinger, A. Ebner, M. Brandstetter. Opt. Express, 30 (4), 5222 (2022). DOI: 10.1364/OE.447269

- [9] S. Schilt, T. Südmeyer. Appl. Sci., 5 (4), 787 (2015).DOI: 10.3390/app5040787
- [10] S.-S. Lin, S.-K. Hwang, J.-M. Liu. Opt. Express, 22 (4), 4152 (2014). DOI: 10.1364/OE.22.004152
- [11] E. Genier, P. Bowen, T. Sylvestre, J.M. Dudley, P.M. Moselund, O. Bang. J. Opt. Soc. Am. B., 36 (2), A161 (2019). DOI: 10.1364/JOSAB.36.00A161
- [12] D.H. Martínez-Suá rez, M.C.S. Araujo, D. Steinberg, L.A.M. Saito, E.A. Thoroh de Souza, J.D. Zapata. Opt. Laser Technol., 174, 110588 (2024). DOI: 10.1016/j.optlastec.2024.110588
- [13] M.H.M. Shamim, L. Brilland, R. Chahal, J. Troles, M. Rochette. J. Phys. Photonics, 6 (4), 045018 (2024). DOI: 10.1088/2515-7647/ad819e
- [14] U.S. Averkieva, Y. Feifei, A. Ismaeel, I.O. Orekhov, S.G. Sazonkin, D.A. Dvoretskii, V.E. Karasik, L.K. Denisov. In: Lazery v nauke, tekhnike, meditsine, Ed. by V.A. Petrov (Mosk. NTO Radiotekh., Elektron. Svyazi im. A.S. Popova, 2025) (in Russian).
- [15] A. Ismaeel, A. Krylov. Opt. Lett., 49 (20), 5977 (2024). DOI: 10.1364/OL.540588
- [16] F. Yan, A. Ismaeel, I. Orekhov, S. Sazonkin, D. Dvoretskiy, A. Krylov, L.K. Denisov, V.E. Karasik. In: Proc. of the 2024 International Conference on Laser Optics (ICLO), edited by IEEE (IEEE, 2024), p. 59. DOI: 10.1109/ICLO59702.2024.10623989
- [17] M.E. Fermann, V.I. Kruglov, B.C. Thomsen, J.M. Dudley, J.D. Harvey. Phys. Rev. Lett., 84 (26), 6010 (2000). DOI: 10.1103/PhysRevLett.84.6010
- [18] A.Y. Fedorenko, A. Ismaeel, I.O. Orekhov, D.A. Dvoretskiy, S.G. Sazonkin, L.K. Denisov, V.E. Karasik. Photonics, 11 (4), 387 (2024). DOI: 10.3390/photonics11040387
- [19] S. Zhang, H. Qiao, T. Zhang, M. Zhou, X. Xu. Front. Phys., 10, 915266 (2022). DOI: 10.3389/fphy.2022.915266
- [20] I.I. Korel, B.N. Nyushkov, V.S. Pivtsov, S.N. Bagaev. Laser Phys., 24 (7), 074012 (2014).
 DOI: 10.1088/1054-660X/24/7/074012
- [21] H. Zia, N.M. Lüpken, T. Hellwig, C. Fallnich, K.-J. Boller. Laser Photon. Rev., 14, 2000031 (2020). DOI: 10.1002/lpor.202000031

Translated by D.Safin