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Electric dipole moment of the W- boson at the quark-gluon level

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The electric dipole moment (EDM) of the W- boson in a three-loop approximation at the quark-gluon level is investigated. The violation of combined $\mathcal{CP}$- symmetry is introduced through a complex phase in the Cabibbo–Kobayashi–Maskawa matrix, which characterizes the flavor structure of the quark loop. For the first time, an estimate for the EDM of the W- boson is obtained taking into account the Glashow–Iliopoulos–Maiani mechanism: $d_W \sim 10^{-41}\,e$cm. It is known that at the quark-gluon level, the EDM of the W- boson can induce both the EDM of the electron and the EDM of the neutron. Thus, from experimental constraints on the EDM of these fermions, obtained through various spectroscopic methods, one can derive a constraint on the EDM of the W- boson. It is shown that this constraint exceeds the predictions of the Standard Model for the EDM of the W- boson obtained in this work by many orders of magnitude.

Keywords: electric dipole moment, parity nonconservation, W- boson, Cabibbo–Kobayashi–Maskawa matrix, Glashow–Iliopoulos–Maiani mechanism.

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1. Introduction

The nature of interactions non-invariant with respect to time reversal ($\mathcal{T}$) still remains a mystery. One of the most widely discussed manifestations of such effects is the presence of a non-zero electric dipole moment (EDM) of particles that are not truly neutral (i.e., do not transform into themselves when charge conjugation operation C is applied). Indeed, according to the Wigner–Eckart theorem, the EDM of a particle must be directed along its spin. This proportionality automatically ensures the violation of both $\mathcal{T}$- and $\mathcal{P}$-symmetries ($\mathcal{P}$ is spatial parity). According to the fundamental $\mathcal{CPT}$-theorem, the violation of $\mathcal{T}$-symmetry is equivalent to the violation of combined $\mathcal{CP}$-parity. The first $\mathcal{CP}$-odd effects have been discovered in decays of neutral kaons [1] and, later, other exotic mesons [2–5]. The first direct observation of $\mathcal{T}$-invariance violation has also been performed in meson physics [6]. However, the existence of EDM of particles may be indicative of the universal nature of interactions that violate $\mathcal{T}$-symmetry.

The search for $\mathcal{T}$-odd effects has been initiated approximately 75 years ago in [7], where the possibility of observing the neutron EDM (nEDM) by magnetic resonance was discussed. The tightest constraint on the value of nEDM (d_n) has been reported recently in [8]: $d_n < 1.8 \cdot 10^{-26}\,e$cm. Here, $e$cm is elementary charge e multiplied by cm. As for the search for the EDM of other particles, the most advanced experiments were those focused on the EDM of an electron (eEDM). The search for eEDM began with the publication of [9]. The author of this study suggested that the observation of a non-zero EDM of a paramagnetic atom (i.e., associated with an unpaired electron spin) may be regarded as a manifestation of eEDM. Several different experimental designs employing various atomic and molecular systems have been proposed since then [10–16]. The tightest constraint on eEDM: $d_e < 4.1 \cdot 10^{-30}\,e$cm [12].

Theoretical predictions of the EDM of various particles within the Standard Model (SM) are quite uncertain and lie far from current experimental constraints. For example, the estimate for nEDM is $d_n \sim 10^{-32}\,e$cm [17]. The estimates for eEDM at the quark-gluon level are $d_e \sim (10^{-44}–10^{-50})\,e$cm [18–20]. At the hadron level, the SM yields an estimate of $d_e \sim 10^{-39}\,e$cm [19]. Note also that both the eEDM effect and the effect of $\mathcal{T}$, $\mathcal{P}$-odd interaction of an electron with a nucleus are present in experiments on the search for $\mathcal{T}$, $\mathcal{P}$-odd effects in paramagnetic atomic and molecular systems [21,22]. Generally speaking, one may distinguish these two effects by performing several experiments with different atomic and/or molecular systems, since these interactions have different dependences on nucleus charge Z [23]. The latter interaction is usually expressed in terms of equivalent eEDM d_e^{eqv}. Several mechanisms of $\mathcal{T}$, $\mathcal{P}$-odd electron–nucleus interaction have been proposed [18,24,25]; the largest predicted value was reported in [25], $d_e^{\text{eqv}} \sim 10^{-35}\,e$cm. At the same time, the interest in searching for „new physics“ beyond the SM, where larger EDM values may be predicted [26], remains high.

Not only fermious, but also bosons, can possess an electric dipole moment. It is theoretically possible to measure the EDM of a W- boson (WEDM) directly in, e.g., high-energy scattering experiments [27]. However, as far as we know, no such experiments focused on the WEDM have been performed. The study of mechanisms of this effect is not of purely methodological interest. It turns out that the WEDM may induce both the eEDM and the nEDM within the SM at the quark-gluon level. Thus, a constraint on the WEDM may be derived from experimental constraints on the EDM of the mentioned fermions. However, it should be noted that the restrictions obtained this way are model-dependent (i.e., they depend on the specific mechanism within the SM) and, consequently, may be nondominant. The possibility of existence of W-EDM has been first discussed in [28–30]. Since a phase in the Cabibbo–Kobayashi–Maskawa (CKM) matrix is the source of $\mathcal{CP}(\mathcal{T})$-invariance violation in the SM, a quark loop is needed. To extract this phase, four changes of quark flavors from up-type to down-type and vice versa are needed; accordingly, four W- boson-quark vertices in the loop are required. Therefore, the WEDM could potentially emerge in a two-loop approximation. An estimate with the Glashow–Iliopoulos–Maiani (GIM) mechanism taken into account in the two-loop approximation was made in [31]; however, it was demonstrated soon afterward in [32] that the WEDM does not arise in the two-loop approximation. The author of [33] has made an attempt to obtain an estimate in the three-loop approximation with the addition of gluon exchange in the quark loop. The estimate was derived by analyzing the infrared region of integration in the three-loop integral. However, various estimates of similar loops for the eEDM have been presented in recent decades, and it has been demonstrated that the main contribution to the integral is produced by the ultraviolet region of integration (with account for cancelations associated with the use of the GIM mechanism) [18–20]. In view of this, the present study is focused on estimating the WEDM at the quark-gluon level in the three-loop approximation with account for the GIM mechanism. We also compare our SM predictions for this quantity with constraints that may be derived from spectroscopic experiments on the search for eEDM and nEDM.

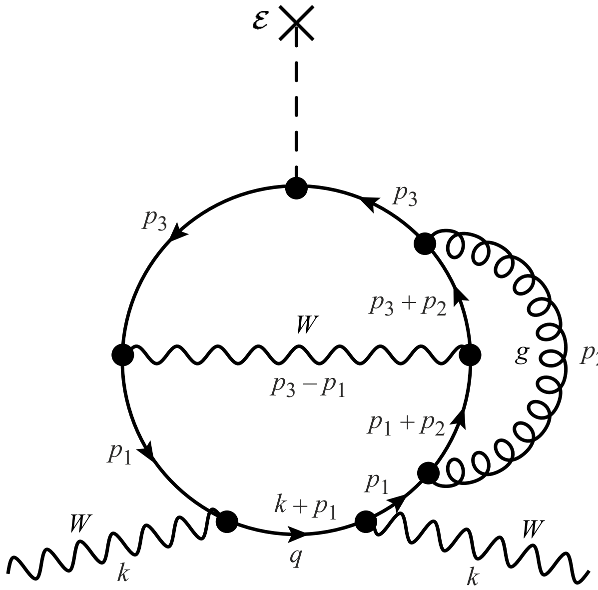
The following units are used in the text: $\hbar = c = 1$ ($\hbar$ is the Planck constant and c is the speed of light) and fine structure constant $\alpha = e^2/(4\pi)$.

2. Estimation of the three-loop integral for WEDM

The interaction of the WEDM with an external electric field violating $\mathcal{T}$, $\mathcal{P}$-parities may be presented in the form of the following effective Lagrangian:

$$\mathcal{L}_{\mathcal{W}} = i2m_W d_W \widetilde{F}^{\alpha\beta} W_\alpha^\dagger W_\beta, \tag{1}$$

where m_W and d_W are the mass and the EDM of a W-boson, respectively; W_α is the field of a W- boson; and $\widetilde{F}^{\alpha\beta} = (1/2)\varepsilon^{\gamma\delta\alpha\beta}F_{\gamma\delta}$ is a tensor dual to electromagnetic tensor $F_{\gamma\delta}$. Owing to its relative smallness, the contribution



Three-loop Feynman diagram characterizing the interaction of the WEDM with constant uniform electric field $\mathcal{E}$. The 4-momenta of particles are also indicated. Solid lines correspond to different quarks q. Wavy lines correspond to W- bosons. The curved line corresponds to gluon g, and the dashed line with a cross denotes the interaction with the external field.

from derivatives with respect to the W- boson field was neglected in Eq. (1).

As was noted in the Introduction, the first non-zero contribution to the WEDM arises in the three-loop approximation at the quark-gluon level (see the figure). In the figure, uniform constant electric field $\mathcal{E}$ must be coupled either to one of the quark propagators or to the internal W- boson line. The W- boson energy shift in a uniform constant (arbitrarily weak) electric field is determined by the amplitude of the process illustrated in the figure and is given by

$$\Delta E = -\boldsymbol{d}_W \boldsymbol{\mathcal{E}}. \tag{2}$$

Notably, the amplitude is considered for a process without a change in energy of a particle at rest (according to the definition of EDM). The expression for amplitude may be written in accordance with standard Feynman rules. We do not present it here, since only the region of large 4-momenta circulating in the loops is relevant to the purpose of this study (namely, to estimating the multiloop integral with account for the GIM mechanism). Let us note certain features characteristic of the discussed problem. The figure shows the 4-momenta of all particles. We formally introduce the classical 4-potential of a constant external electric field into the Feynman diagram technique as $\mathcal{A}^\mu(t) = (0, -\boldsymbol{\mathcal{E}}t)^t$. Integration over the time variable at the „quark–external field" vertex then leads to the emergence of a delta function derivative. In other words, one of the quark propagators $S^q(p)$ should be substituted with its energy derivative $\partial S^q(p)/\partial p_0$. The following obvious identity may be used for the quark propagator derivative:

$$\frac{\partial S^q(p)}{\partial p_0} = -S^q(p)\gamma_0 S^q(p). \quad (3)$$

Since asymptotically

$$\lim_{p\to\infty} S^q(p) = \frac{1}{p}, \quad (4)$$

we have

$$\lim_{p\to\infty} \frac{\partial S^q(p)}{\partial p_0} = -\frac{1}{p^2}. \quad (5)$$

The second characteristic feature is related to the W-boson propagator:

$$D^W_{\mu\nu} = \frac{-1}{p^2 - m_W^2 + i0}\left[g_{\mu\nu} + \frac{(\xi-1)p_\mu p_\nu}{p^2 - \xi m_W^2}\right], \quad (6)$$

where ξ is a gauge parameter. The Feynman and Landau gauges correspond to $\xi = 1$ and $\xi = 0$, respectively, and $\xi = \infty$ in the unitary gauge [34]. We choose the unitary gauge, since it obviates the need to include the interaction with Faddeev–Popov scalar ghosts. In the unitary gauge, the ghost propagators turn to zero [34]. The W- boson in the unitary gauge is then written in the following form:

$$D^W_{\mu\nu,\mathrm{unit}} = \frac{-1}{p^2 - m_W^2 + i0}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_W^2}\right). \quad (7)$$

Note that asymptotically in the unitary gauge,

$$\lim_{p\to\infty} D^W_{\mathrm{unit}}(p) = \frac{1}{m_W^2}, \quad (8)$$

which is why individual amplitude terms may diverge more strongly in loop integrals. However, this is not a problem, since this divergence normally becomes less significant due to cancellations induced by the GIM mechanism (Section 3).

Being essentially the Green's function of a massless boson, the gluon propagator has the standard asymptotics:

$$\lim_{p\to\infty} D^g(p) = \frac{1}{p^2}. \quad (9)$$

Taking all of the above into account and switching from $\int d^4k$ to $\int k^3 dk$, we may rewrite the dimensionless three-loop integral corresponding to the figure in the asymptotic regime as follows:

$$I \sim \frac{1}{m_W^2}\int \frac{dp_1 dp_2 dp_3 p_2}{(p_3+p_2)(p_1+p_2)}. \quad (10)$$

This expression contains ultraviolet divergences. This indicates that the typical momenta circulating in the loop are quite large (specifically, $p \sim m_W(m_t)$, where m_t is the mass of a heavy t- quark). If we introduce cutoff energy Λ, the integral will have the following estimate:

$$I \sim \frac{\Lambda^2}{m_W^2}. \quad (11)$$

Note also that constant $g = e/\sin\theta_W$, where θ_W is the Weinberg angle (free SM parameter), corresponds to each vertex of the W–q interaction, and $\sin^2\theta_W \approx 0.22$. Each of the two W–q loops then acquires factor $\alpha/(\pi\sin^2\theta_W)$, where π is the standard factor assigned to the loop. The estimate used here for the quark-gluon loop is $(\alpha_s/\pi)C_F \sim \alpha_s/\pi$, where α_s is the coupling constant in quantum chromodynamics, which is approximately equal to 1 for large momenta $p \sim m_t$ circulating in the loop, and factor $C_F = 4/3$ arises as a result of summation over quark colors. Finally, making use of the fact that, according to Eq. (1), the effective operator is „confined" between two wave functions of a W- boson at rest, we find that $W^\dagger_\alpha W_\beta \sim 1/m_W$.

Collecting all the factors, we obtain the following estimate of WEDM:

$$d_W \sim e\frac{\Lambda^2}{m_W^3}\frac{\alpha^2\alpha_s}{\pi^3\sin^4\theta_W}R_{\mathrm{GIM}}, \quad (12)$$

where coefficient R_{GIM} includes suppression both due to the GIM mechanism and due to the $\mathcal{CP}$-odd factor originating from the CKM matrix. This coefficient is estimated in the next section.

3. GIM mechanism for the quark loop

The complex phase in the CKM matrix is the source of $\mathcal{CP}(\mathcal{T})$-symmetry violation in the SM. This phase becomes non-trivial (i.e., impossible to eliminate by any quark rotations) if one takes into account three generations of quarks. The $\mathcal{CP}$-odd flavor structure of the quark loop may presented in the following form [32]:

$$\begin{aligned} U \sim 2iJ[&t(dus - sud + bud - dub - bus + sub) \\ &+ u(dcs - scd + scb - bcs + bcd - dcb) \\ &+ c(dts - std + stb - bts + btd - dtb)]. \end{aligned} \quad (13)$$

The standard parameterization for Jarlskog invariant J [35], which is specified by the product of four elements of the CKM matrix V_{ij} [36], is used here:

$$J = \mathrm{Im}\,(V_{us}V_{td}V^*_{ud}V^*_{ts}) \approx 3.2\cdot 10^{-5}. \quad (14)$$

Letters u, d, c, s, t, b in expression (13) denote the propagators of the corresponding quarks. Only the imaginary part of product V_{ij} in expression (14) contributes to $\mathcal{CP}$-odd effects. Each product of quark propagators in Eq. (13) admits cyclic permutations such as $tdus = dust = ustd = stdu$.

Let us start by positioning, e.g., an s- quark at the top of the figure (i.e., coupling the external field to it). Distributing the remaining quarks along the loop in all possible configurations and using the CKM matrix, we obtain the following amplitude flavor structure:

$$\begin{aligned} U_s \sim 2iJ(sAss)(&uudc - ccdu + ccbu - uubc + ccdt \\ &- ttdc + ttbc - ccbt + ttdu - uudt + uubt - ttbu). \end{aligned} \quad (15)$$

Here, sAs denotes the interaction with the external electric field. Without loss of generality in light of relation (3), one may assume that $sAs \sim Q_s s^3$, where Q_s is the charge of an s- quark and $s^3 \equiv sss$. Next, we introduce cutoff energy Λ once again for all momenta in the loop and rewrite expression (15) as follows:

$$U_s \sim 2iJ(Q_s s^4)(d-b)(uuc - ccu + cct - ttc + ttu - uut). \quad (16)$$

All terms here differ only in quark masses m_q in the propagators. If one assumes the quark masses to be negligible compared to the quark momenta in the propagators, the result is exactly zero. A nonzero result is obtained only if the propagators are expanded in $(m_q/\Lambda)^2$ and corrections are taken into account. In addition, the mass terms in numerators of the propagators may be neglected in the GIM estimate (since $m_q \ll \Lambda$). Note that certain mass terms in numerators of the propagators provide zero contribution, since they are „confined" between two W–q vertices.

Expanding the denominators in Eq. (16) in terms of small parameter m_q^2/Λ^2 (assuming that $\Lambda \gg m_t$ and keeping only the leading-order terms in view of $m_t \gg m_c \gg m_u$ and $m_b \gg m_d$), we then obtain the following dimensionless GIM factor:

$$R^s_{\text{GIM}} \sim J\frac{m_c^2 m_b^2 m_t^4}{\Lambda^8}. \quad (17)$$

After that, expression (16) needs to be summed over all down-type quarks (which corresponds to coupling of the field to d and b- quarks). Let us also take into account that $Q_s = Q_d = Q_b = -1/3$. Denoting

$$\mathscr{F}(u, c, t) = uuc - ccu + cct - ttc + ttu - uut$$

in Eq. (16), we find the following flavor structure for the WEDM:

$$U^{W\text{EDM}} \sim iJQ_s[s^4(d-b) + b^4(s-d) + d^4(b-s)]\mathscr{F}(u, c, t). \quad (18)$$

The obvious expansion of expression (18) yields the following dimensionless GIM factor:

$$R_{\text{GIM}} \sim J\frac{m_c^2 m_s^2 m_b^4 m_t^4}{\Lambda^{12}}. \quad (19)$$

Generally speaking, one should also consider the coupling of the external electric field to up-type quarks (with a different coefficient $Q_t = 2/3$). However, the result will evidently be the same as in expression (19). This is attributable to symmetry inherent in Eq. (19): the masses of the two heaviest up-type and down-type quarks (t and b) are raised to the fourth power, while the masses of the other two quarks (c and s) are squared.

Using Eqs. (12) and (19) and setting $\Lambda \to m_t$, we obtain the following parametric estimate for the WEDM in the three-loop approximation at the quark-gluon level:

$$d_W \sim eJ\frac{m_s^2 m_c^2 m_b^4}{m_W^3 m_t^6}\frac{\alpha^2 \alpha_s}{\pi^3 \sin^4\theta_W} \sim 10^{-41}\, e\text{cm}. \quad (20)$$

Additional numerical factors and the possible logarithmic amplification, which may scale the result up by a couple orders of magnitude, are neglected in this estimate. Note also that the quark loop shown in the figure is essentially identical to the one that contributes to the eEDM and was estimated earlier. That said, our GIM estimate of the quark loop differs from those reported earlier in [18] and [19]. Compared to [18], it includes additional small factor $m_b^4/m_t^4 \sim 3 \cdot 10^{-7}$; compared to [19], it features additional factor $m_b^2/m_t^2 \sim 6 \cdot 10^{-4}$.

As was noted in the Introduction, the WEDM may induce the eEDM and the nEDM at the quark-gluon level. Let us derive a constraint on the WEDM from the experimental constraints on the EDM of an electron and a neutron. To do that, we use the relations between these EDMs and the WEDM obtained in [30,37]:

$$d_e \sim \frac{\alpha}{\pi \sin^2\theta_W}\frac{m_e}{m_W}d_W \quad (21)$$

and

$$d_n \sim \frac{\alpha}{\pi \sin^2\theta_W}\frac{m_n}{m_W}d_W. \quad (22)$$

The following constraint is then obtained from [8] and Eq. (22):

$$d_W < 2 \cdot 10^{-22}\, e\text{cm}. \quad (23)$$

The data from [12] and Eq. (21) yield the constraint

$$d_W < 6 \cdot 10^{-23}\ e\text{cm}. \quad (24)$$

Comparing Eqs. (20) and (24), we conclude that the current best experimental constraints on the WEDM value are very far from the SM predictions at the quark-gluon level.

4. Conclusion

An estimate of the EDM of a W- boson has been obtained for the first time in the three-loop approximation at the quark-gluon level with account for the GIM mechanism. The resulting value $d_W \sim 10^{-41}\, e$cm is many orders of magnitude smaller than the current experimental constraints that may be extracted from experiments on the search for the EDM of an electron and a neutron. The examination of mechanisms contributing to the EDM of a W- boson at the hadron level is an important direction for future research. There are reasons to believe that certain mechanisms produce a greater contribution than the estimate obtained here at the quark-gluon level.

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Conflict of interest

The authors declare that they have no conflict of interest.

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