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Special far-field intensity distribution formed by corner cube retroreflector with spiral phase plate

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This article discusses metal-coated corner cube retroreflectors equipped with spiral phase plates. Far−field diffraction patterns with annular distributions are modeled for retroreflectors with spiral phase plates of various topological charges. These diffraction patterns are then compared with those obtained from a corner cube without a spiral phase plate.

Keywords: corner cube retroreflector, spiral phase plate, vortex beam, retroreflection indicatrix.

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Corner cube retroreflector (CCR) systems are used, for example, to measure the distance to small meteorological research spacecraft launched by universities around the world. The energy efficiency of ranging is affected to a considerable extent by the effect of velocity aberration [1–3]. Being a manifestation of the Doppler effect, it induces a shift of the retroreflected beam, which leads to large energy losses. The standard way to compensate for this effect is to widen the retroreflection indicatrix to achieve acceptable levels of retroreflected radiation energy [3]. However, this method still leads to significant energy losses, since the maximum of the indicatrix is not directed at the receiving optical system. The method of forming an indicatrix with an annular distribution may be considered more energy efficient. The maximum of such an indicatrix is located approximately in the center of the ring, and the radius corresponding to the indicatrix maximum is chosen so as to be equal to the shift caused by velocity aberration (i.e., so that the indicatrix maximum is aligned with the receiving optical system). The simplest way to form such an indicatrix is the method that uses two retroreflectors rotated by 30° relative to each other with the same errors introduced into all three dihedral angles [3]. A more advanced method is to use retroreflectors with an error introduced into one dihedral angle. This provides an opportunity to form a pattern from two spots with the angular distance from the center of the spot to the center of the diffraction pattern being equal to the shift caused by velocity aberration. A set of such retroreflectors rotated by 15° relative to each other allows one to obtain an annular distribution [1,2]. Approaches involving modification of the CCR front face through the application of coatings [4] or the introduction of additional optical elements (e.g., spiral phase plates) are also known. A structure with an annular distribution integrating a spiral phase plate and a retroreflector has been described and experimentally tested for the first time in [5]. A plane wave diffracted by a spiral phase plate naturally produces a vortex beam with an annular intensity profile.

The aim of the present study is to determine the equivalent cross section in the center of the first ring of the retroreflection indicatrix for corner cube retroreflectors with spiral plates with metallized faces and various integer topological charges and to analyze the parameters of the resulting indicatrices.

The thickness of a spiral phase plate increases with azimuth angle, thereby shaping a spiral profile and providing the corresponding phase shift. The far-field diffraction distribution of a plane wave by a spiral phase plate is characterized by an analytical expression that is written here in angular measure [6,7]:

$$E_{pl}(\rho,\gamma,n)=\frac{(-i)^{n+1}\exp(in\gamma)}{(n+2)n!}\left(k\rho_{pl}^{2}\right)^{n}\left(\frac{k\rho_{pl}\rho}{2}\right)$$
$$\times\,{}_1\mathrm{F}_2\left(\frac{n+2}{2},\frac{n+4}{2},n+1,-\left(\frac{k\rho_{pl}\rho}{2}\right)^{2}\right),\qquad(1)$$

where ρ and γ are polar coordinates in the analysis plane, ${}_1\mathrm{F}_2$ is the hypergeometric function, $k=2\pi/\lambda$ is the wave vector, n is the topological charge of a spiral plate, and ρ_{pl} is the plate radius.

This expression is valid only for diffraction by a plate with an integer topological charge; the diffraction distribution for plates with a fractional topological charge may be obtained numerically [8]. The simulated diffraction patterns for $n=1$ (a) and 0.5 (b) are shown in Fig. 1.

The approximate expression for estimating the outer radius of the first ring, which is of interest to us, is

$$R_{1,n}=\frac{\gamma_{1,2n}\lambda}{2\pi\rho_{pl}},\qquad(2)$$

where $\gamma_{1,2n}$ is the first zero of Bessel function J_{2n}.

When one estimates the radius by formula (2) instead of determining it using analytical expression (1), the calculation error is 1−12 % for $n=1−4$. In the case of

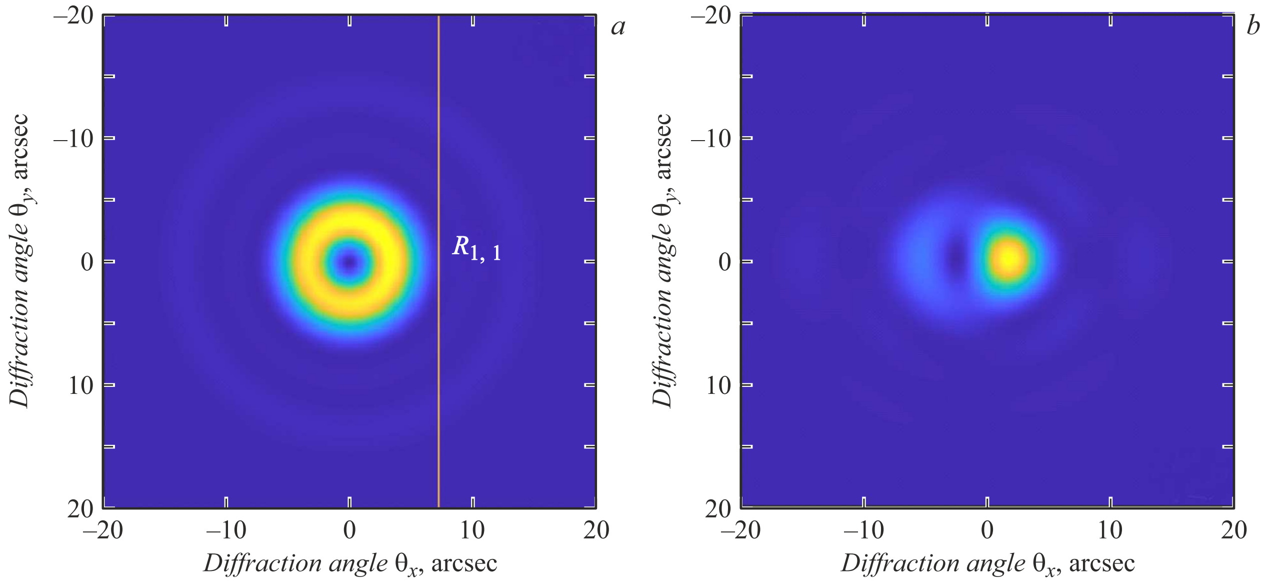

Figure 1. Diffraction pattern from a plate with $n = 1$ (a) and 0.5 (b) for $\lambda = 532$ nm, $\rho_{pl} = 12.7$ mm.

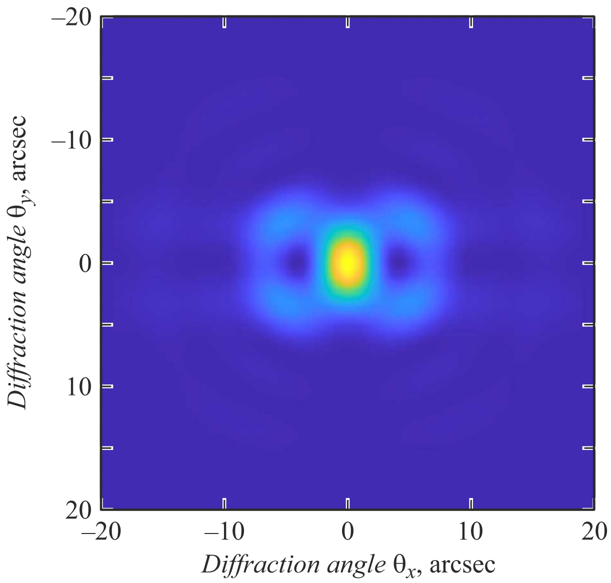

Figure 2. Diffraction pattern from a CCR with a plate with $n = 0.5$ for $\lambda = 532$ nm, $\rho_{pl} = 12.7$ mm.

half-integer topological charges, a vortex does not form in this formulation of the problem.

In geometric optics, a light beam may be presented as a set of rays. When such a beam propagates inside a retroreflector, the ray path geometry is such that rays are reflected symmetrically relative to the principal ray. Only the CCRs with metallized faces are considered here. The influence of the phase shift during reflection is also neglected [1]. If a phase plate is mounted on the front face of the CCR, each beam ray (apart from the principal one) passing through one part of the plate will interact with the other part of the plate (i.e., is subject to a different phase shift) on its backward pass.

Let us simplify the hexagonal shape of the retroreflector aperture to a circle and take a phase plate with a diameter equal to the diameter of this circle. The phase transmittance of the entire plate–retroreflector system is then written as

$$\tau_\varphi(r, \varphi, n) = \exp(2i\varphi n)F(\varphi, n), \qquad (3)$$

where

$$F(\varphi, n) = \{\exp(i\varphi n), \varphi \in [0, \pi);$$

$$\exp(-i\varphi n), \varphi \in [\pi, 2\pi)\}$$

— CCR influence function and r, φ — polar coordinates in the plane of the spiral phase plate.

Notably, although a double pass through a plate with a half-integer topological charge „transforms" it into a plate with an integer charge, the phase shift due to a change in position of rays does not allow one to obtain an annular far-field intensity distribution (Fig. 2).

When a plate with an integer topological charge n is used, the change in position of rays introduces a phase shift divisible by πn, which affects only the polarization state of the beam as a whole, but not the far-field intensity distribution. The influence of the CCR and a double pass through the plate with an integer charge n may then be replaced by a single pass through the plate with charge $l = 2n$.

The parameters of the first ring of far-field diffraction distributions are presented in the table and shown in Fig. 3.

One of the key parameters characterizing the energy efficiency of a retroreflector system is the equivalent cross-section (ECS). The ECS on the axis of a corner cube retroreflector without a plate is written as [9]

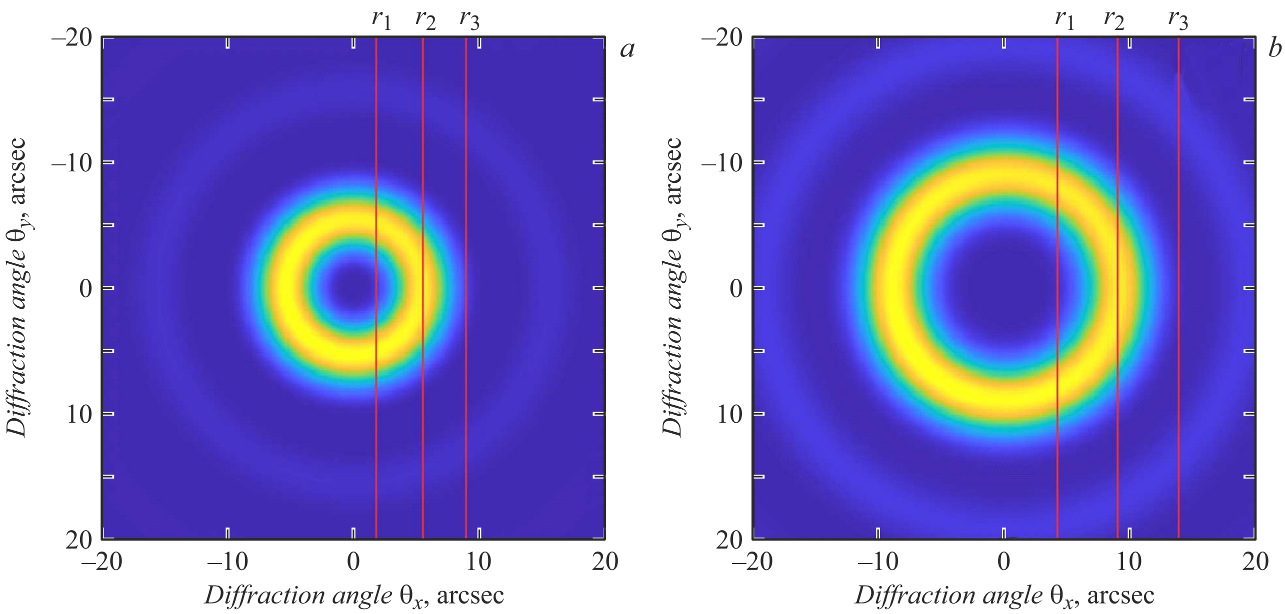

Figure 3. Diffraction pattern from a CCR with a plate with $n = 1$ (a) and 2 (b).

Parameters of the first ring of the diffraction pattern

n	Inner boundary r_1	Ring center r_2	Outer boundary r_3	Ring area S_{Ring}
1	0	Θ	2Θ	$4S_{Airy}$
2	0.5Θ	1.7Θ	2.8Θ	$5.3S_{Airy}$
3	0.8Θ	2.3Θ	3.5Θ	$7.29S_{Airy}$

Note. $\Theta = \frac{3.83\lambda}{2\pi 2\rho_{pl}}$ is the size of the Airy disk for diffraction by an aperture with a radius equal to the phase plate radius and S_{Airy} is the Airy disk area in the analysis plane.

$$\sigma_0^{Airy} = 4\pi \frac{J_r^{Airy}}{J_0} = 4\pi \frac{S^2}{\lambda^2}, \quad (4)$$

where J_r^{Airy} is the intensity of retroreflected radiation in a solid angle corresponding to the angular size of the Airy disk, J_0 is the radiant intensity of an equivalent isotropic source of equal power, and S is the CCR effective aperture area.

It follows from expression (4) that the ECS on the axis is proportional to the retroreflected radiant intensity. The ECS for a retroreflector forming an annular distribution may then be expressed as follows:

$$\sigma_0^{Ring} = \frac{J_r^{Ring}}{J_r^{Airy}} \sigma_0^{Airy}, \quad (5)$$

where J_r^{Ring} is the retroreflected radiant intensity in a solid angle corresponding to the angular size of the first ring.

The ratio of radiant intensities allowing one to estimate the ECS at the peak of the first ring may be derived from the equality of the retroreflected radiation fluxes for retroreflectors with and without a plate. It is needed here to take into account the fractions of total energy of the retroreflected beam contained in the first ring and the Airy disk. Specifically, $\Delta E_{Airy} = 83.7\,\%$ is the fraction of total retroreflected energy contained in the Airy disk (center lobe of the retroreflection indicatrix) and ΔE_{Ring} is the fraction of energy contained in the ring. It varies with topological charge: at $n = 1$ $\Delta E_{Ring} = 87.3\,\%$, at $n = 2$ $\Delta E_{Ring} = 81.4\,\%$, and at $n = 3$ $\Delta E_{Ring} = 76\,\%$. Assuming that the curve of the center lobe in the case of diffraction by a CCR without a plate (Airy disk) has the same shape as the curve for the first ring, we obtain an expression for the ratio of radiant intensities from (5):

$$\frac{J_r^{Ring}}{J_r^{Airy}} = \frac{\Delta E_{Ring}}{\Delta E_{Airy}} \frac{S_{Airy}}{S_{Ring}}. \quad (6)$$

Expressions (5) and (6) characterize the ECS value at the center (peak) of the first ring of the diffraction distribution. At $n = 1$ $J_r^{Rig}/J_r^{Airy} = 0.26$, at $n = 2$ $J_r^{Ring}/J_r^{Airy} = 0.18$, and at $n = 3$ $J_r^{Ring}/J_r^{Airy} = 0.12$.

This method allows one to obtain an annular intensity distribution in the diffraction pattern with the use of a single CCR with metallized faces and a pre-installed spiral phase plate with an integral topological charge and to determine the ECS in the center of the first ring of the diffraction pattern. As the topological charge increases, the first ring increases in radius, enabling the use of corner cube retroreflectors with a larger diameter (compared to experiments with a retroreflector without a plate). Since a larger diameter allows for a higher ECS value, such a system has the potential to be more energy efficient. However, the ECS value at the peak of the first ring (σ_0^{Ring}) for a retroreflector with a plate is 4–10 times smaller (for $n = 1{-}3$) than the value at the peak of the distribution in the form of an Airy disk (σ_0^{Airy}) for a retroreflector without a plate. This may neutralize the effect of a diameter increase. A spiral phase plate–retroreflector system may also be used to double the topological charge. As was demonstrated above, a double pass through a plate due to the retroreflection effect may be presented as a pass through another plate with a topological charge twice as large.

Conflict of interest

The authors declare that they have no conflict of interest.

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