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Determination of the optimal cut of an InP photorefractive crystal at contra-directional four-wave mixing

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Dependence of the reflection coefficient under contra-directional four-wave mixing in the InP crystal on its cut plane orientation in the crystallographic coordinate system has been analyzed. Indices of the crystal cut for which the phase-conjugated wave intensity reaches the absolute maximum under optimal conditions have been found. Diffraction efficiencies during wavefront phase conjugation by combined gratings in different-cut InP crystals have been compared.

Keywords: Four-wave mixing, wavefront conjugation, photorefractive crystal, combined grating, reflection coefficient.

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Regularities of phase conjugation under four-wave mixing (FWM) described by Belarusian researchers in pioneering work [1] underlie dynamic holography and are currently used in a number of optical applications for the spatial-temporal wavefront transformation. Up-to-date techniques for the nonlinear media diagnostics and frequency transformation of images by dynamic holography are described in [2]. The possibility of turbulence auto-suppression in atmospheric optical-communication lines by using the wavefront shape correction under degenerate FWM in a photorefractive crystal was demonstrated in [3]. Deceleration of optical pulses due to nonlinear dispersion under frequency-nondegenerate FWM in the CdTe crystal was studied in [4].

Photorefractive semiconductor InP is one of the nonlinear media most promising for dynamic holography, since such crystals are characterized by a high holographic recording speed close to the theoretical sensitivity limit and also by the ability to transition to the infrared spectrum range [5]. Observation of the photorefractive effect in the InP crystal under FWM was for the first time reported in the previous paper [6]. Results of the study reported in [7–9] show that regularities of the wavefront conjugation should be analyzed taking into account specific features of the light wave diffraction on the holographic grating (hereinafter grating), which are inherent to the semiconductor. Under mixing according to the contra-directional FWM scheme, up to six gratings [7] with the phase-amplitude (combined) structure [8] may be recorded in the InP crystal, since, when electric field of spatially separated charges is formed, additional modulation of the medium absorption coefficient occurs besides the refractive index modulation. The efficiency of diffraction under FWM in the InP crystal significantly depends on the choice of the crystalline sample orientation angle as well as of initial azimuths of light wave polarization. In adjusting the holographic setup by optimally selecting the mentioned parameters, reflection coefficient may be significantly increased [9]. When the theoretical model is developed taking into account data given in [7–9], it is possible to achieve satisfactory agreement between theoretical calculations and experimental data obtained in studying the reflection coefficient dependence on orientation in the case of contra-directional FWM in the InP crystal [10].

In studying specific features of light wave diffraction in the InP crystal, samples with working faces cut parallel to the $\{100\}$, $\{110\}$, $\{111\}$ or $\{112\}$ planes are conventionally considered (see, e.g. [6,9]). By now, it remains unclear whether the absolute reflection coefficient maximum is achievable under optimal experimental conditions in the InP crystals cut as mentioned above. If not, it is reasonable to determine optimal orientation of the crystal cut plane in the crystallographic coordinate system (hereinafter referred to as optimal cut) at which the reflection coefficient reaches its absolute maximum, and also to compare its value with the highest reflection coefficients reachable on samples with the $\{100\}$, $\{110\}$, $\{111\}$ or $\{112\}$ cut planes. Solving this problem will make it possible to utilize the optimal cut selection so as to make the InP crystal employment in optical applications more efficient, which is just the main goal of this study.

Consider the case when a conjugated light wave arises under the contra-directional frequency-degenerate FWM [7] on combined gratings recorded in the InP crystal. The mixing geometry, light wave characteristics, as well as the coupled wave equations used to find the reflection coefficient and a list of initial conditions for numerical solution of the problem, are described in detail in [11]. We use in calculation the following InP semiconductor material parameters corresponding to wavelength $\lambda = 1064 \cdot 10^{-9}$ m: unperturbed crystal refractive index $n_0 = 3.29$ [12]; linear absorption coefficient $\alpha = 30\,\mathrm{m}^{-1}$ [12]; electro-optic coefficient $r_{41} = 1.45 \cdot 10^{-12}\,\mathrm{m/V}$ [12]; elasticity coefficients $c_1 = 10.11 \cdot 10^{10}\,\mathrm{N/m^2}$, $c_2 = 5.61 \cdot 10^{10}\,\mathrm{N/m^2}$, $c_3 = 4.56 \cdot 10^{10}\,\mathrm{N/m^2}$ [12]; photoelasticity coefficients $p_1 = -0.150$, $p_2 = p_3 = -0.115$, $p_4 = -0.056$ [13]; piezoelectric coefficient $e_{14} = 0.13\,\mathrm{C/m^2}$ [14]. As per [15], the following designations has been adopted for the non-zero tensor components of linear electro-optical $(\hat{r})$, photoelastic $(\hat{p})$ and inverse piezoelectric $(\hat{e})$ effects, as well as elasticity tensor components $(\hat{c}^E)$:

$$r^S_{123} = r^S_{132} = r^S_{213} = r^S_{231} = r^S_{312} = r^S_{321} \equiv r_{41},$$

$$e_{123} = e_{132} = e_{213} = e_{231} = e_{312} = e_{321} \equiv e_{14},$$

$$c^E_{11} = c^E_{22} = c^E_{33} \equiv c_1,$$

$$c^E_{12} = c^E_{13} = c^E_{23} = c^E_{21} = c^E_{31} = c^E_{32} \equiv c_2,$$

$$c^E_{44} = c^E_{55} = c^E_{66} \equiv c_3,$$

$$p^E_{11} = p^E_{22} = p^E_{33} \equiv p_1,$$

$$p^E_{12} = p^E_{23} = p^E_{31} \equiv p_2,$$

$$p^E_{13} = p^E_{21} = p^E_{32} \equiv p_3,$$

$$p^E_{44} = p^E_{55} = p^E_{66} \equiv p_4.$$

The order of calculating the dependence of optimized reflection coefficient R^{opt} on the crystal cut orientation is as follows. First set the spatial orientation of the crystal cut plane by using unit vector $\mathbf{e}_3$ normal to it (see [11]) whose direction in crystallographic axes $x_1 \parallel [100]$, $x_2 \parallel [010]$, $x_3 \parallel [001]$ (Fig. 1, *a*) is defined by angles α and β. Use orientation angle θ to specify the angle of turning the crystal cut plane relative to vector $\mathbf{e}_3$. Fix the vector $\mathbf{e}_3$ direction and find reflection coefficients R for various combinations of orientation angles and light wave polarization azimuths. Select from the found series R the highest value corresponding to optimized reflection coefficient R^{opt} for the given crystal cut. By searching through angles α and β, find R^{opt} for all the vector $\mathbf{e}_3$ directions. To visualize the obtained results in crystallographic axes (x_1, x_2, x_3), construct surface $R^{opt}(\mathbf{e}_3)$ whose radius vector is proportional to the R^{opt} values in the given vector $\mathbf{e}_3$ direction. Since the reflection coefficient may take only positive values, the surface is painted in one color. Surface $R^{opt}(\mathbf{e}_3)$ extreme directions along which the optimized reflection coefficient reaches its absolute maximum $R^{\max} = R^{opt}$ are perpendicular to the plane of the desired optimal cut of the InP crystal which, being selected, ensures the maximum wavefront conjugation efficiency.

Consider dependence $R^{opt}(\mathbf{e}_3)$ calculated for the InP crystal 4 mm thick, which is presented in Fig. 1, *a*. In the constructed plot, the cube edges depicted by thin solid lines are used to display the graduated scale for the measured physical quantity. Diagonal lines in the cube coincide with third-order rotation axes and are drawn for visual convenience. Optimized reflection coefficient R^{opt}

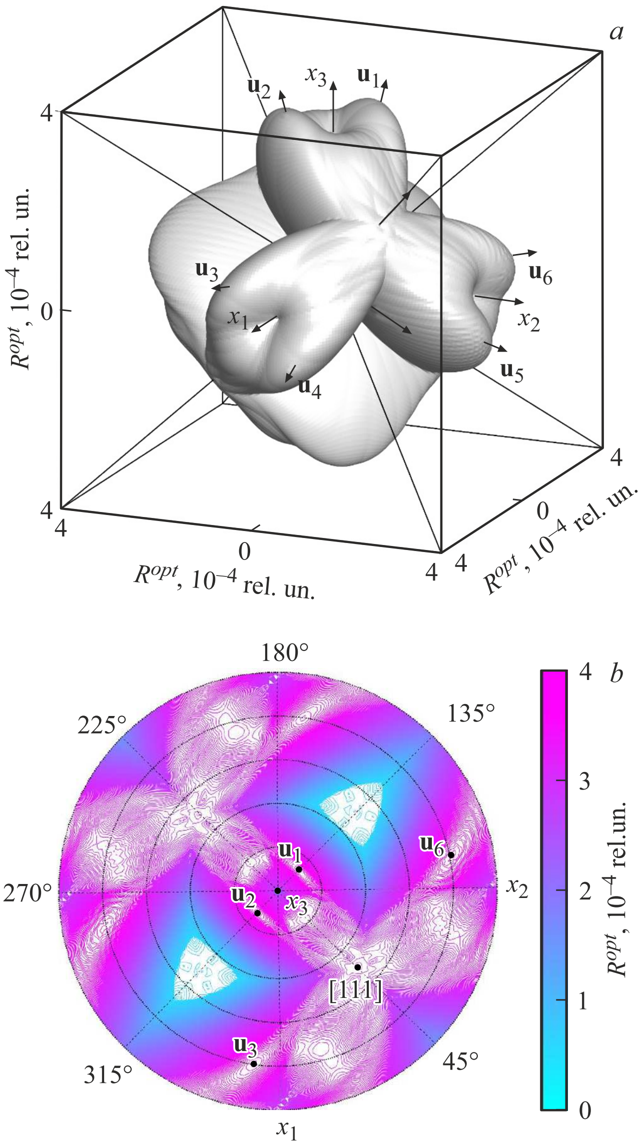

Figure 1. Surface plot of the dependence of optimized reflection coefficient R^{opt} on vector $\mathbf{e}_3$ orientation in the crystallographic coordinate system (a) and surface $R^{opt}(\mathbf{e}_3)$ in the stereographic projection onto plane x_1x_2 (b).

reaches its absolute maximum $R^{\max}$ along symmetrically equivalent directions $\langle 115 \rangle$. Vectors $\mathbf{u}_1 - \mathbf{u}_6$ denote the extreme directions of surface $R^{opt}(\mathbf{e}_3)$ shown in Fig. 1, *a*, which correspond to the crystal lattice edges whose indices are $\mathbf{u}_1 \parallel [\bar{1}15]$, $\mathbf{u}_2 \parallel [1\bar{1}5]$, $\mathbf{u}_3 \parallel [5\bar{1}1]$, $\mathbf{u}_4 \parallel [51\bar{1}]$, $\mathbf{u}_5 \parallel [15\bar{1}]$, $\mathbf{u}_6 \parallel [\bar{1}51]$. Since radius vectors to the surface $R^{opt}(\mathbf{e}_3)$ points laid out in opposite directions along lines $\langle 115 \rangle$ have modules different in values (e.g. $[1\bar{1}5]$ — $R^{\max} = 3.6 \cdot 10^{-4}$, $[\bar{1}1\bar{5}]$ — $R^{opt} = 3.3 \cdot 10^{-4}$) and cannot be brought in coincidence by any symmetry transformations, it is possible to speak of the extreme directions' polarity, which is because the InP crystal have no symmetry center. As is known [16], direction $[hrl]$ in cubic-syngony crystals is always normal to plane (hrl); therefore, a family of

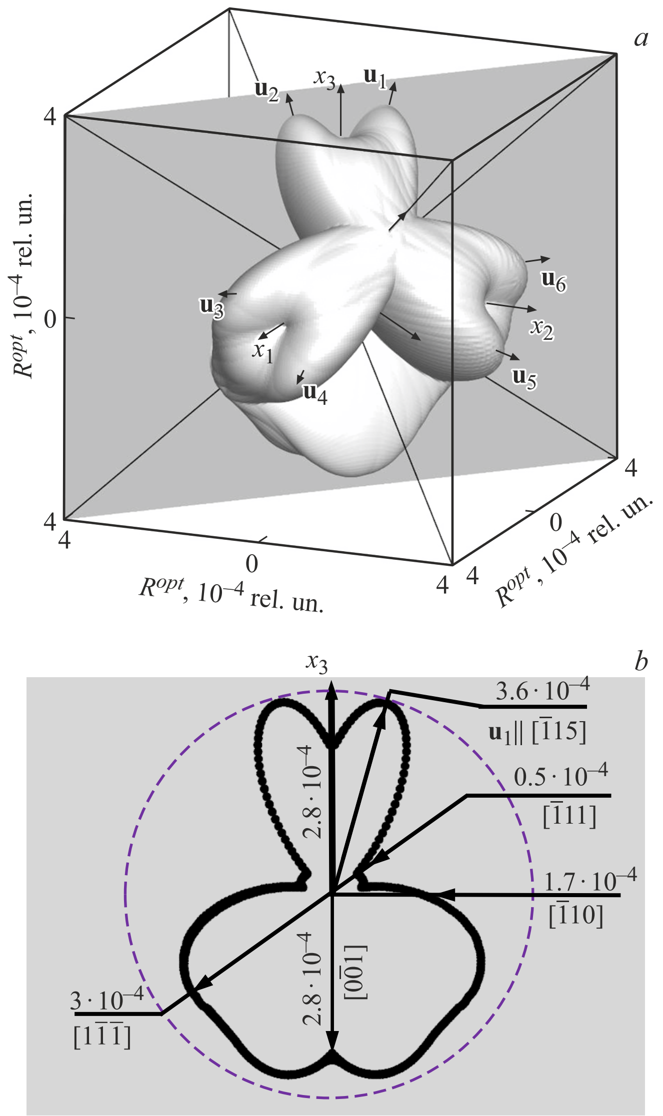

Figure 2. *a* — mutual arrangement of surface $R^{opt}(\mathbf{e}_3)$ and cut plane parallel to (110) ; *b* — surface $R^{opt}(\mathbf{e}_3)$ cross-section.

planes $\{115\}$ corresponds to directions $\langle 115\rangle$. Thus, under the optimal conditions of contra-directional FWM in the InP crystal, the maximum conjugated wave intensity gets achieved when the crystal cut is parallel to the $\{115\}$ plane.

According to the Neumann's principle [16], external symmetry of surface $R^{opt}(\mathbf{e}_3)$ includes a complete combination of symmetry elements of the cubic-syngony planar-class crystalline polyhedron. To substantiate this statement, consider the $R^{opt}(\mathbf{e}_3)$ surface stereographic projection onto the plane x_1x_2 parallel to (001) (see Fig. 1, *b*). On the plot, projections of vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$, $\mathbf{u}_6$, as well as axes x_3 and directions [111], are marked with black dots. As shown in the plot, surface $R^{opt}(\mathbf{e}_3)$ is associated with fourth-order symmetrical inversion axis oriented parallel to x_3; this means that the figure coincides with itself under a joint symmetric transformation combining rotation by 90° and reflection at the reference point. In addition, the figure coincides with itself being rotated by 120° relative to the lines passing along $\langle 111\rangle$, which corresponds to the third-order symmetry of rotation axis. With the surface, six symmetry planes parallel to $\{110\}$ and passing along the cube diagonals (Fig. 1, *a*) may be associated. The described symmetry elements fully correspond to the point symmetry group of the InP crystalline polyhedron.

Let us compare the R^{opt} values that may be achieved for the InP crystal with cut planes $\{001\}$, $\{110\}$, $\{111\}$ or $\{112\}$ typical for the holographic experiment. Figs. 2 and 3 demonstrate, in the form convenient for visual examination, the surface $R^{opt}(\mathbf{e}_3)$ cross-sections parallel to the (110) (Fig. 2) and (111) (Fig. 3) planes. Mutual arrangement of the figure and cut planes is illustrated in Figs. 2, *a* and 3, *a*. Traces of contiguity between the cut planes and figure are shown in Figs. 2, *b* and 3, *b*. Dashed circles are additionally constructed in order to indicate the surface $R^{opt}(\mathbf{e}_3)$ cross-section points having equal R^{opt} values.

In the case of FWM in the InP crystals with cut planes parallel to $\{100\}$ and $\{111\}$, optimized reflection coefficients are almost fully identical since the R^{opt} values corresponding to directions [001] and $[1\,\bar{1}\,\bar{1}]$ are $2.8 \cdot 10^{-4}$ and $3 \cdot 10^{-4}$, respectively; these values are about 20 % lower than $R^{\max}$ (Fig. 2, *b*). Note that along $[\bar{1}11]$ there is obtained value of $R^{opt} = 0.5 \cdot 10^{-4}$ which is 6 times lower than that along $[1\,\bar{1}\,\bar{1}]$, which indicates polarity of the $\langle 111\rangle$ directions. Directions $\langle 100\rangle$ are centrally symmetrical, which follows from equality of the optimized reflection coefficients at the points of surface $R^{opt}(\mathbf{e}_3)$ intersection with the straight line parallel to axis x_3. Directions $\langle 110\rangle$ corresponding to the $\{110\}$-cut crystal are also centrally symmetric; however, in this case the optimized reflection coefficients being achieved along them ($R^{opt} = 1.7 \cdot 10^{-4}$) are 40 % lower than for $\langle 100\rangle$ and $\langle 111\rangle$, and almost twice lower than $R^{\max}$.

To find R^{opt} achievable under FWM in the $\{112\}$-cut InP crystal, consider the surface $R^{opt}(\mathbf{e}_3)$ cross-section shown in Fig. 3, *b* whose plane contains directions $[\bar{1}\,\bar{1}\,2]$, $[2\,\bar{1}\,\bar{1}]$ and $[\bar{1}\,2\,\bar{1}]$. The cross-section is a symmetrical figure able to coincide with itself being rotated by 120° because of perpendicularity to the third-order rotation axis [111]. Evidently, directions $\langle 112\rangle$ are not centrally symmetrical; along them, an optimized reflection coefficient comparable to $\langle 100\rangle$ and $\langle 111\rangle$ gets achieved, that is, $R^{opt} = 3.1 \cdot 10^{-4}$. The cross-section's characteristic feature is the presence of symmetrically arranged arc-shaped areas coinciding with the dashed circle $R^{opt} = 1.3 \cdot 10^{-4}$ in radius shown in Fig. 3, *b*. This means that, when in experiments on wavefront conjugation in the $\{111\}$-cut InP crystal the sample turns relative to vector $\mathbf{e}_3$, the optimized reflection coefficient takes equal values in certain orientation angle ranges.

Thus, absolute maximum of the dependence of the reflection coefficient optimized for the orientation angle and azimuths of the light wave linear polarization under FWM in the InP crystal may be achieved when the crystalline sample is cut parallel to the $\{115\}$ plane. If the InP

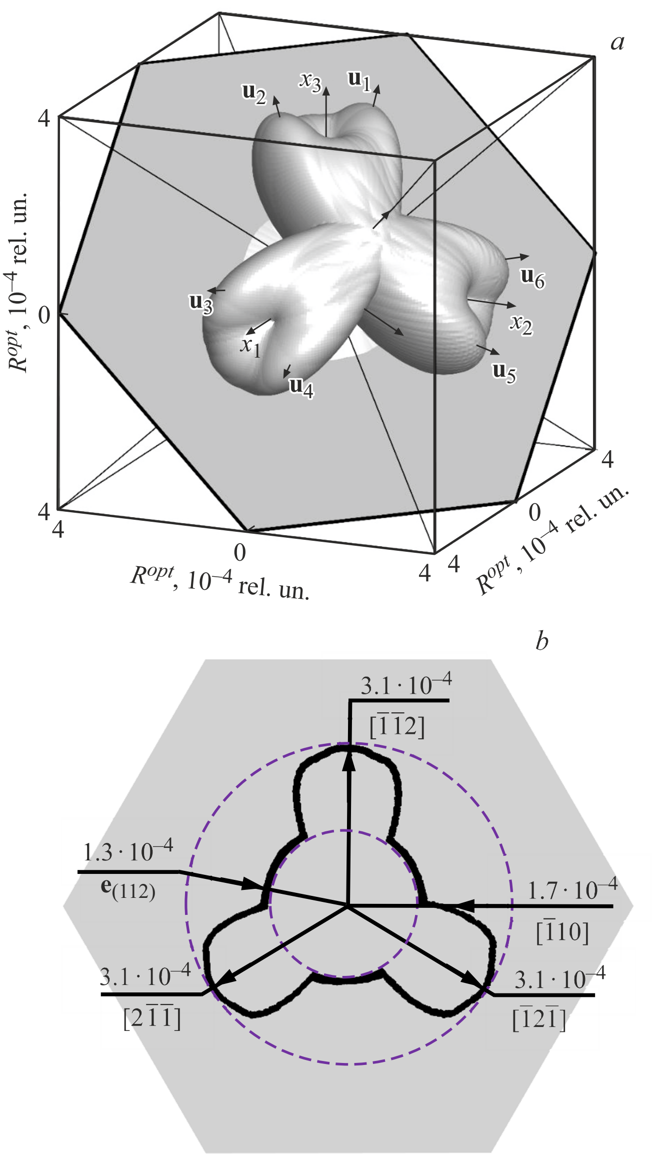

Figure 3. *a* — mutual arrangement of surface $R^{opt}(\mathbf{e}_3)$ and cut plane parallel to (111); *b* — surface $R^{opt}(\mathbf{e}_3)$ cross-section.

crystal working faces are cut parallel to {100}, {111} or {112}, then the optimized reflection coefficient takes approximately equal values, which are 20 % lower than the absolute maximum. The optimized reflection coefficient under FWM in the {110}-cut InP crystal is twice lower than the absolute maximum. The obtained results are valid for the InP crystal material parameters used in this study and should be recalculated if the parameters change.

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Conflict of interests

The author declares that he has no conflict of interests.

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