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Quantum magnetotransport of an electron gas in a triangular quantum well

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Expressions for the Hall coefficients and magnetoresistance of a thin semiconductor layer at the boundary of a heterojunction in a transverse magnetic field are obtained using the quantum Liouville equation. The film temperature is selected so that quantum dimensional effects can be considered along the film thickness and Landau levels in the film plane are not considered, that is, in the film plane, the energy of charge carriers changes continuously. The effect of surface scattering of charge carriers is considered through diffuse-mirror Soffer boundary conditions. The dependence of Hall coefficients and magnetoresistance on magnetic field induction, electrochemical potential and roughness parameter is analyzed.

Keywords: quantum Liouville equation, Soffer model, Hall coefficient, magnetoresistance coefficient, quantum triangular well.

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Introduction

When electron motion is limited, their energy is quantized. The electron gas that can freely move in two dimensions, but is limited in the third one, is called two-dimensional (2DEG). The two-dimensional electron gas is applied when developing the optoelectronic [1–3] and ultrahigh-frequency systems [4,5]. In particular, they include high-electron-mobility field-effect transistors (HEMT). In comparison with common field-effect transistors, the HEMTs operate at a higher frequency, with high breakdown field and with less energy consumption in comparison with other heterostructures [6].

If the layer (or the film) includes the two-dimensional electron gas, then the thickness of this film is comparable to the de Broglie wavelength for the average-speed electrons. It this case, layer surface irregularities substantially affect the electrical parameters. With increase of the surface scattering contribution in comparison with charge carrier scattering in a volume, the electrical conductance will decrease, while the Hall coefficient of a thin film will increase. The surface charge carrier scattering is taken into account via the Soffer boundary conditions [7], which are obtained as a result of solution of the quantum-mechanical problem for interaction of the electron wave with the layer surface. Decrease of roughness at the boundary of a heterojunction [8] can increase a HEMT response.

Various approaches are used to take into account quantum size effects with the surface scattering. The work [9] solves the Schrödinger equation for a thin metal film using the Green's functions. The work [10] provides direct calculation of the collision integral in a kinetic equation using autocorrelation functions describing the surface profile. The said work shows an effect of interference between volume and boundary scattering in the film. The interference effects heavily affect temperature (phonon scattering in the volume) or concentration (impurity scattering) dependences of conductance. Another source [11] has developed a conductance model for the metal films by calculating the charge carrier scattering Hamiltonian. The article [12] provides a method of self-consistent solutions of the Kohn-Sham and Poisson system of equations, by which the electron density is determined. The problem solving methods used by the authors lead to cumbersome mathematical calculations. The theoretical studies of transfer phenomenon in the nanofilms are still continuing. The present work determines the Hall coefficient and the magnetoresistance coefficient using the Liouville quantum equation [13], by which elements of the density matrix are found. The work [14] calculates the galvanomagnetic parameters of a semiconductor nanolayer for a rectangular quantum well.

The present work determines the Hall coefficient and the magnetoresistance coefficient in the layer at the boundary of the heterojunction. The potential well of the heterojunction for the electron gas can be approximated by a triangular quantum well [15].

1. Problem formulation

Let us consider the conducting channel (CC) in the GAN buffer layer of HEMT [6], whose gate and drain are energized by the gate voltage V_G and the drain voltage V_{sd}, respectively (Fig. 1). The conducting channel is a semiconductor layer of the thickness a. The drain

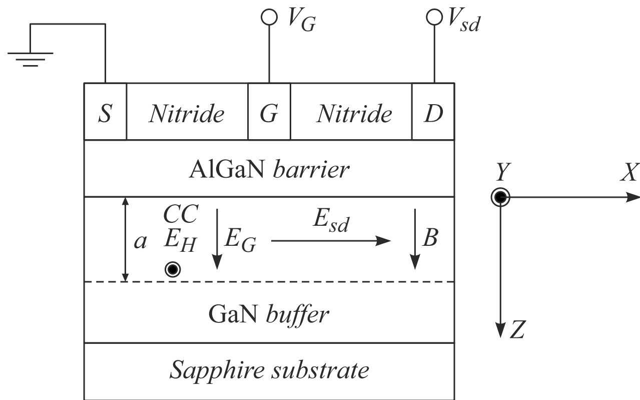

Figure 1. HEMT structure. S — the source, G — the gate, D — the drain, Nitride — silicon nitride (Si_3N_4), CC — conducting channel — the two-dimensional electron gas in the GaN layer at the boundary of the heterostructure.

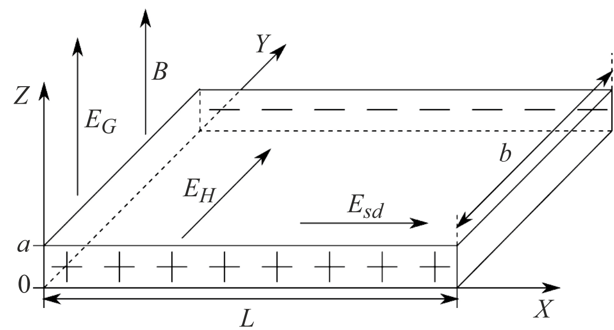

Figure 2. Conducting channel in GaN.

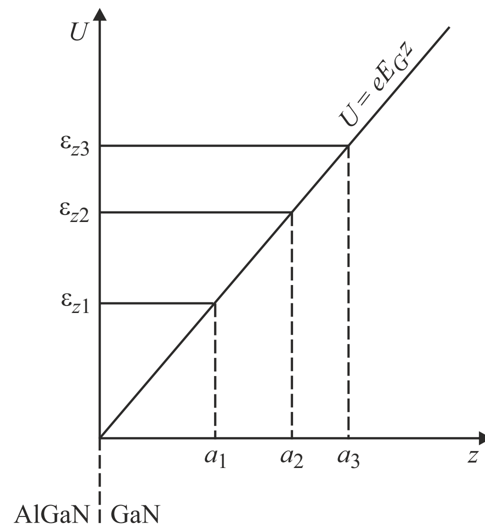

Figure 3. Potential well for the charge carries in the layer.

field with the strength E_{sd} is directed along the layer. The gate field and the heterojunction internal field with the total strength E_G as well as induction of the external magnetic field B are recorded perpendicular to the layer. Under action of the Lorentz force, the side facets of the conducting channel accumulate charges, which form the electric field of the strength E_H.

The process of heterostructure production is quite complicated. It includes lithography and molecular-beam epitaxy, which allow sputtering of the film layer-by-layer, checking the impurity quantity. The form of the triangular quantum well depends of the AlGaAs composition. However, the impurities in the quantum well result in widening of levels of quantization, which can be determined from the Heisenberg uncertainty principle as h/τ, where τ — the typical scattering time. If this value is comparable to the energy difference of the adjacent levels, then the quantization effect does not occur.

Let us determine the Hall coefficient and transverse magnetoresistance of the conducting channel (the semiconductor layer). For this purpose, we introduce the Cartesian system of coordinates, in which the plane XY is directed along the layer plane, the axis X — is directed along the strength of the longitudinal electric field E_{sd}, the axis Y — is directed along the „Hall" strength E_H, the axis Z — is directed along the strength of the transverse electric fieldE_G and along induction of the external magnetic field B (Fig. 2). We assume that in the plane $z = 0$ the GAN buffer layer borders with the AlGAN barrier layer.

It is known from the quantum mechanics that the electron energy spectrum in the quantum well becomes discrete. The electron motion is limited along the film thickness, so the motion is quantized in this direction (Fig. 3). In the first approximation, the conducting layer in the heterojunction for the charge carriers can be considered to be the triangular potential well [15]:

$$U = \begin{cases} \infty, & z < 0, \\ eE_G z, & z > 0, \end{cases}$$

where U — the potential energy, e — the electron charge.

Therefore, the charge carrier energy is as follows:

$$\varepsilon_n = \varepsilon_{\parallel} + \varepsilon_{zn}, \ \varepsilon_{\parallel} = \frac{p_x^2 + p_y^2}{2m}, \ \varepsilon_{zn} = \frac{p_{zn}^2}{2m}, \ n = 1, 2, 3, \ldots .$$

The electron energy along the axis Z [15] has discrete values

$$\varepsilon_{zn} = \frac{\gamma_n}{2m}\left(\frac{heE_G m}{\pi}\right)^{2/3}, \quad n = 1, 2, 3, \ldots ,$$

where h — the Planck constant, m — the effective mass, γ_n — zeros of the Airy function:

$$\gamma_n \approx \left[\frac{3\pi}{2}\left(n - \frac{1}{4}\right)\right]^{2/3}, \quad n > 5.$$

In this case the projection of the carrier pulse to the axis Z is

$$p_{zn} = \sqrt{2m\varepsilon_{zn}} = \sqrt{\gamma_n}\left(\frac{heE_G m}{\pi}\right)^{1/3}, \quad n = 1, 2, 3, \ldots .$$

The thickness $a_n\,(n = 1, 2, 3\ldots)$ is a distance, within which the charge carriers of the level n, can move along the axis Z (Fig. 3). The thickness $a_n\,(n = 1, 2, 3\ldots)$ is determined from the condition $\varepsilon_{zn} = eE_G a_n$ and is

$$a_n = \gamma_n\left(\frac{h^2}{8\pi^2 eE_G m}\right)^{1/3}.$$

For the two-dimensional electron gas along the axis Z there will be only one energy level ε_{z1}. Therefore, the film thickness a with the two-dimensional gas of the charge carriers shall be within $a_1 < a < a_2$.

The layer electrical conductance can be determined using the Liouville quantum equation:

$$\frac{\partial\hat{\rho}}{\partial t} = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}],$$

where $\hat{\rho}$ — the density matrix operator, $\hat{H}$ — the Hamilton operator, $\hbar$ — the Dirac constant, while the brackets mean a commutator. Diagonal components of the density matrix ρ_{nn} correspond to the distribution function f_n, which describes the state of the charge carriers with projection of the pulse to the axis Z equal to p_{zn}.

The present work does not take into account the Landau levels, as small magnetic field or high temperatures are considered, i.e. the following condition is met:

$$k_{\mathrm{B}}T \geq \frac{h\omega_c}{2\pi}, \quad \omega_c = \frac{eB}{m},$$

where k_{B} — the Boltzmann constant, T — the film temperature, ω_c — the cyclotron frequency, B — induction of the external magnetic field.

From the Liouville equation, a kinetic equation was obtained in the works [14]:

$$\frac{\partial f_n}{\partial t} + v\frac{\partial f_n}{\partial r} + F\frac{\partial f_n}{\partial p} = -\frac{2\pi}{\hbar}N_{imp} \times \sum_{n'}|V_{0n'}|^2(f_n - f_{n-n'})\delta(\varepsilon_n - \varepsilon_{n-n'}), \quad (1)$$

where $F = e(E_{sd} + E_H) + e[v \times B]$ — the force acting on the charge carriers, v — the velocity of electrons (holes), N_{imp} — the concentration of impurities, $V_{0n'}$ — the matrix element of scattering from the level n to the level n'.

The wave electron function at the quantization level is a standing wave that is obtained by imposition of two oncoming running waves which move at the group velocities v and $-v$.

In order to take into account charge carrier scattering at the boundary of the triangular well of the semiconductor layer we will use the Soffer model [7]:

$$\begin{cases} f_n^+(z = 0, +v_{zn}) = q_1 f_n^-(z = 0, -v_z n), \\ f_n^-(z = a, -v_z n) = q_2 f_n^+(z = a, +v_z n), \end{cases} \quad (2)$$

$$q_{1,2}(g_{1,2}, \Theta) = \exp(-(4\pi g_{1,2}\cos\Theta)^2), \quad g_{1,2} = g_{s1,2}/\lambda_{BV},$$

where f_n^+ and f_n^- — nonequilibrium functions of distribution of electrons (holes) respectively with positive and negative projections of the pulse to the axis Z; $g_{s1,2}$, $g_{1,2}$ and $q_{1,2}$ — respectively, the root-mean-square height of the surface relief, the roughness parameter and the reflection coefficient for the lower (index 1) and the upper (index 2) surface of the semiconductor layer; λ_{BV} — the de Broglie wavelength of the charge carrier moving at the average velocity; Θ — the angle of incidence of the charge carrier to the internal surface of the layer.

The Soffer model takes into account the dependence of the reflection coefficient on the roughness parameter of the incident angle of the charge carrier. This model will allow to compare the theoretical calculations with the experimental data, as the roughness parameters is associated with the root-mean-square height of the surface relief.

The current density formed by the charge carriers at the level n is

$$j_n = 2e\int\int v f_n \frac{dp_x dp_y}{h^2}\cdot\frac{p_{zn} - p_{z(n-1)}}{h}.$$

Therefore, the full current from the charge carriers at all the energy levels is

$$I_n = b\int_0^{a_n} j_n dz, \quad I = 2\sum_{n=1}^{\infty} I_n,$$

where b — the film width (Fig. 2). The film width b significantly exceeds the de Broglie electron wavelength, which makes it possible to neglect quantization in the rectangular potential well of the width b. The number 2 in the second expression takes into account the charge carriers both with the positive and the negative projection.

The connection between the current and the strength in the transverse magnetic field is as follows:

$$\begin{cases} I_x = G_{xx}LE_{sd} + G_{xy}LE_H, \\ I_y = G_{yx}LE_{sd} + G_{yy}LE_H, \end{cases}$$

where I_x and I_y — the currents along the axes X and Y, L — the film length (Fig. 2). Along the axis Y the Hall current is compensated by the drift current under effect of occurred Hall voltage ($I_y = 0$). Therefore,the full current is equal to the current along the axis X ($I_x = I$).

The following film parameters are determined from these equations:

$$G = \frac{I}{LE_{sd}} = G_x x - \frac{G_{yx}G_{xy}}{G_{yy}},$$

$$R_H = \frac{bE_H}{IB} = -\frac{bG_{yx}}{(G_{xx}G_{yy} - G_{yx}G_{xy})BL},$$

$$\varkappa_{\perp} = \frac{G_0 - G}{G_0 B^2}, \quad (3)$$

where G — the integral conductance of the film, $G_0 = G(B = 0)$, R_H — the Hall coefficient, $\varkappa$ — the coefficient of transverse magnetoresistance.

2. Transformation of kinetic equation

A solution of the kinetic equation (1) may be written as follows

$$f_n = f_n^{(0)} + f_n^{(1)}, \quad f_n^{(0)} = \frac{1}{1 + \exp\left(\frac{\varepsilon_n - \mu}{k_{\mathrm{B}} T}\right)}, \quad (4)$$

$$f_n^{(1)} = -\frac{\partial f_n^{(0)}}{\partial \varepsilon_n} (c_{xn} p_x + c_{yn} p_y) \exp(i\omega t), \quad (5)$$

where $f_n^{(0)}$ — the equilibrium function of the Fermi-Dirac distribution, μ — the electrochemical potential, k_{B} — the Boltzmann constant, T — the temperature, $f_n^{(1)}$ — the non-equilibrium addition to the distribution function f_n, c_{xn} and c_{yn} — the coefficients depending on the coordinate z.

Taking into account (4), the kinetic equation (1) in relaxation time approximation is transformed into the expression

$$\frac{\partial f_n}{\partial t} + v_{zn} \frac{\partial f_n}{\partial z} + F \frac{\partial f_n}{\partial p} = -\frac{f_n - f_n^{(0)}}{\tau}, \quad (6)$$

where τ — the relaxation time that is

$$\tau = \frac{\tau_t}{\tau_\beta}, \quad \tau_\beta = \frac{\beta}{\sin(\beta)}, \quad \beta = \frac{\tau_V e B}{m},$$

where τ_β — the coefficient taking into account curvature of the path of the charge carriers in the external magnetic field ($\tau_\beta = 1$, if there is no magnetic field ($B = 0$)), τ_t and τ_V — the time of electron relaxation of the electron in the triangular well and in a macroscopic sample without the magnetic field, respectively, β — dimensionless induction of the externa magnetic field. The mean free path λ of the charge carriers does not take into account the mechanism of surface scattering, i.e. this length does not depend on the film thickness and is determined via a structure of the nanolayer crystal. Therefore, the relaxation times τ_t and τ_V are related to each other via the mean free path λ:

$$\lambda = v_{0V} \tau_V = v_{0t} \tau_t,$$

where v_{0V} and v_{0t} — characteristic velocities of the charge carriers in the macroscopic sample and in the triangular well, respectively. We assume that τ_V does not depend on the energy (scattering on the neutral impurity atoms in the volume).

Taking into account (5), the kinetic equation (6) in linear approximation is transformed into the system of equations

$$\begin{cases} \frac{\tau_\beta}{\tau_t} c_{xn} + v_{zn} \frac{\partial c_{xn}}{\partial z} = \frac{e E_{0sd}}{m} + \frac{eB}{m} c_{yn}, \\ \frac{\tau_\beta}{\tau_t} c_{yn} + v_{zn} \frac{\partial c_{yn}}{\partial z} = \frac{e E_{0H}}{m} - \frac{eB}{m} c_{xn}. \end{cases}$$

Taking into account (4), (5), the boundary conditions (2) are written as follows:

$$\begin{cases} c_{in}^{+}(z = 0, +v_{zn}) = q_1 c_{in}^{-}(z = 0, -v_{zn}), \\ c_{in}^{-}(z = a, -v_{zn}) = q_2 c_{in}^{+}(z = a, +v_{zn}), \end{cases}$$

where $i = x, y$.

3. Concentrations and characteristic velocities

The concentration n_{cV} and the characteristic velocity v_{0V} in the bulk sample are determined as follows:

$$n_{cV} = 2 \frac{m^3}{h^3} \int f_0 d^3 v, \quad n_{cV} v_{0V}^2 = 2 \frac{m^3}{h^3} \int f_0 v^2 d^3 v,$$

$$f_0 = \frac{1}{1 + \exp \frac{(\varepsilon - \mu)}{k_{\mathrm{B}} T}}, \quad \varepsilon = \frac{m v^2}{2},$$

and are equal to

$$n_{cV} = 2\sqrt{m_1 m_2 m_3} \left(\frac{2\pi k_{\mathrm{B}} T}{h^2} \right)^{3/2} F_{1/2}\left(\frac{\mu}{k_{\mathrm{B}} T} \right),$$

$$v_{0V} = \sqrt{\frac{5 k_{\mathrm{B}} T}{m} \cdot \frac{F_{3/2}\left(\frac{\mu}{k_{\mathrm{B}} T}\right)}{F_{1/2}\left(\frac{\mu}{k_{\mathrm{B}} T}\right)}},$$

$$F_j(x) = \frac{1}{\Gamma(j+1)} \int_0^\infty \frac{t^j dt}{1 + e^{t - x}}, \quad \Gamma(j+1) = \int_0^\infty t^j e^{-t} dt,$$

where $F_j(x)$ — the Fermi-Dirac integral with the index j, $\Gamma(j+1)$ — the gamma function.

The velocity projections may be written as the system of parametric equations:

$$\begin{cases} v_x = \sqrt{2\varepsilon_{\parallel}/m} \cos\varphi, \\ v_y = \sqrt{2\varepsilon_{\parallel}/m} \sin\varphi, \end{cases}$$

where $0 \le \varphi \le 2$ — the angle read from the axis X.

For subsequent calculations, it is more convenient to proceed from the Cartesian coordinates in the pulse space (v_x, v_y) to the new coordinates $(\varepsilon_{\parallel}, \varphi)$. In this case, the transition Jacobian is equal to m.

The concentration n_{cV} and the characteristic velocity v_{0t} in the conducting channel are determined by the formulae

$$n_{ct} = 2 \frac{m^2}{h^2} \sum \int f_n^{(0)} m d\varepsilon_{\parallel} d\varphi \cdot \frac{p_{zn} - p_{z(n-1)}}{h},$$

$$n_{ct} v_0 t^2 = \frac{5}{3} \cdot 2 \frac{m^2}{h^2} \sum \int (v_x^2 + v_y^2 + v_{zn}^2) f_n^{(0)} m d\varepsilon_{\parallel} d\varphi$$

$$\times \frac{p_{zn} - p_{z(n-1)}}{h}$$

and are equal to

$$n_{ct} = \frac{8 k_{\mathrm{B}} T \pi^{2/3} P_2 (E_G h m)^{1/3} m}{h^3},$$

$$v_{0t} = \sqrt{\frac{10k_BT(P_{3t}+P_{4t})}{3mP_{1t}}},$$

$$P_{2t} = \sum_{n=1}^{\infty}(\sqrt{\gamma_n}-\sqrt{\gamma_{n-1}})\cdot\ln\left(1+\exp\left(\frac{\mu-\varepsilon_{zn}}{k_BT}\right)\right),$$

$$P_{3t} = \sum_{n=1}^{\infty}(\sqrt{\gamma_n}-\sqrt{\gamma_{n-1}})\cdot\frac{\varepsilon_{zn}}{k_BT}\cdot\ln\left(1+\exp\left(\frac{\mu-\varepsilon_{zn}}{k_BT}\right)\right),$$

$$P_{4t} = \sum_{n=1}^{\infty}(\sqrt{\gamma_n}-\sqrt{\gamma_{n-1}})\cdot F_1\left(\frac{\mu-\varepsilon_{zn}}{k_BT}\right)$$

4. Calculation of the Hall coefficient and the transverse magnetoresistance coefficient

Let's introduce the dimensionless parameters:

$$x_\lambda = \frac{\lambda}{\lambda_{BV}},\quad u_\parallel = \frac{\varepsilon_\parallel}{(k_BT)},\quad u_\mu = \frac{\mu}{(k_BT)},$$

$$\delta\gamma_n = \sqrt{\gamma_n}-\sqrt{\gamma_{n-1}},$$

$$u_{0V}^2 = \frac{mv_{0V}^2}{(2k_BT)},\quad u_{0t}^2 = \frac{mv_{0t}^2}{(2k_BT)},\quad k_{0t} = \frac{u_{0t}}{u_{0V}},$$

$$E_{G0} = \left(\frac{heE_G}{\pi m^2}\right)^{1/3}/\sqrt{\frac{2k_BT}{m}},$$

where $\lambda_{BV} = h/p_{0V}$ and $p_{0V} = mv_{0V}$ — the de Broglie wavelength and the electron pulse in the bulk sample, E_{G0} — the dimensionless strength of the transverse electric field, $\lambda = v_{0t}\tau = v_{0V}\tau_V$ — the mean free path, τ_V and τ — the times of relaxation in the bulk sample and the conducting channel.

By omitting the intermediate calculations, we obtain the expressions of the Hall coefficient R_H and the transverse magnetoresistance coefficient $\varkappa_\perp$ (3):

$$G = G_V\Sigma_{qt}(x_\lambda, u_\mu, E_{G0}, \beta, g_2),\ G_0 = \frac{\sigma_V\lambda b}{L},\ \sigma_V = \frac{ne^2\tau_V}{m},$$

$$R_H = R_{HV}A_{Hqt}(x_\lambda, u_\mu, E_{G0}, \beta, g_2),\quad R_{HV} = \frac{1}{ne},$$

$$\varkappa_\perp = \varkappa_{\perp V}D_{qt}(x_\lambda, u_\mu, E_{G0}, \beta, g_2),\quad \varkappa_{\perp V} = \frac{(\sigma_V R_{HV})^2}{6},$$

$$\Sigma_{qt} = \frac{k_{0t}^3}{F_{1/2}(u_\mu)\pi^{3/2}x_\lambda u_{0V}}$$
$$\times\frac{(a_{qt}K_{H1}-K_{H2}\frac{\tau_\beta}{k_{0t}})^2+(2b_{qt}K_{H1}-K_{H2}\beta)^2}{K_{H2}\tau_\beta(\tau_\beta^2+k_{0t}^2\beta^2)-a_{qt}K_{H1}k_{0t}(\tau_\beta^2-k^2\beta^2)-4K_{H1}k_{0t}^2\tau_\beta\beta b_{qt}},$$

$$A_{Hqt} = \frac{F_{1/2}(u_\mu)\pi^{3/2}x_\lambda u_{0V}}{k_{0t}^2\beta}$$
$$\times\frac{K_{H2}\beta(\tau_\beta^2+k_{0t}^2\beta^2)+2b_{qt}K_{H1}(\tau_\beta^2-k_{0t}^2\beta^2)-2K_{H1}k_{0t}\tau_\beta\beta a_{q}t}{(a_{qt}K_{H1}-K_{H2}\frac{\tau_\beta}{k_{0t}})^2+(2b_{qt}K_{H1}-K_{H2}\beta)^2},$$

$$D_{qt} = \frac{6(\Sigma_{0qt}-\Sigma_{qt})}{\Sigma_{0qt}\beta^2},\quad \Sigma_{0qt} = \Sigma_{qt}(\beta=0),$$

$$K_{H1} = \pi E_{g0}^2x_\lambda,\quad K_{H2} = P_{1t}u_{0V}^2,$$

$$a_{qt} = \sum_{n=1}^{\infty}\delta\gamma_n\sqrt{\gamma_n}\int_0^{\infty}\left(\frac{-\partial f_n^{(0)}}{\partial u_\parallel}\right)u_\parallel A_1(\Omega_{1n},\Omega_{2n})du_\parallel$$

$$b_{qt} = \sum_{n=1}^{\infty}\delta\gamma_n\sqrt{\gamma_n}\int_0^{\infty}\left(\frac{-\partial f_n^{(0)}}{\partial u_\parallel}\right)u_\parallel A_2(\Omega_{1n},\Omega_{2n})du_\parallel$$

$$P_{1t} = \sum_{n=1}^{\infty}\delta\gamma_n\cdot\gamma_n\cdot\ln(1+\exp(u_\mu-\Delta_{tn})),$$

$$P_{2t} = \sum_{n=1}^{\infty}(\sqrt{\gamma_n}-\sqrt{\gamma_{n-1}})\cdot\ln(1+\exp(u_\mu-\Delta_{tn})),$$

$$P_{3t} = \sum_{n=1}^{\infty}(\sqrt{\gamma_n}-\sqrt{\gamma_{n-1}})\cdot\Delta_{tn}\cdot\ln(1+\exp(u_\mu-\Delta_{tn})),$$

$$P_{4t} = \sum_{n=1}^{\infty}(\sqrt{\gamma_n}-\sqrt{\gamma_{n-1}})\cdot F_1(u_\mu-\Delta_{tn}).$$

$$\Delta_{tn} = \frac{\varepsilon_{zn}}{k_BT} = \gamma_nE_{G0}^2,$$

$$u_{0V} = \sqrt{\frac{5}{2}\frac{F_{3/2}(u_\mu)}{F_{1/2}(u_\mu)}},\quad u_{0t} = \sqrt{\frac{5(P_{3t}+P_{4t})}{3P_{2t}}}$$

$$\begin{aligned}A_1(\Omega_{1n},\Omega_{2n}) =& \{(2-q_1-q_2)+q_1q_2(q_1+q_2-2q_1q_2)\\ &\times\exp(-4\Omega_{1n})-2(1-q_1)(1-q_2)\\ &\times\exp(-\Omega_{1n})(1-q_1q_2\exp(-2\Omega_{1n}))\cos\Omega_{2n}\\ &-[q_1+q_2-q_1q_2(q_1+q_2)]\\ &\times\exp(-2\Omega_{1n})\cos(2\Omega_{2n})\}/(1+q_1^2q_2^2\\ &\times\exp(-4\Omega_{1n})-2q_1q_2\exp(-2\Omega_{1n})\\ &\times\cos(2\Omega_{2n}),\end{aligned}$$

$$\begin{aligned}A_2(\Omega_{1n},\Omega_{2n}) =& \{(1-q_1)(1-q_2)\exp(-\Omega_{1n})\\ &\times(1+q_1q_2\exp(-2\Omega_{1n}))\\ &+[q_1+q_2+q_1q_2(q_1+q_2-4)]\\ &\times\exp(-2\Omega_{1n})\cos(\Omega_{2n})\}\sin(\Omega_{2n})/(1+q_1^2q_2^2\\ &\times\exp(-4\Omega_{1n})-2q_1q_2\exp(-2\Omega_{1n})\\ &\times\cos(2\Omega_{2n}),\end{aligned}$$

$$\Omega_{1n} = \frac{\tau_\beta u_{0V}^2\sqrt{\gamma_n}}{2k_{0t}\pi E_{G0}^2x_\lambda},\quad \Omega_{2n} = \frac{\beta u_{0V}^2\sqrt{\gamma_n}}{2\pi E_{G0}^2x_\lambda},$$

$$q_2 = \exp\left(-\left[4\pi g_2\frac{\sqrt{\gamma_n}E_{G0}}{\sqrt{u_\parallel+\gamma_nE_{G0}^2}}\right]^2\right),\quad q_1 = 1,$$

where σ_V — the static electrical conductance in the bulk sample, Σ_{qt} — dimensionless conductivity, R_{HV} and $\varkappa_{\perp V}$ — the Hall coefficient and the magnetoresistance coefficient in the bulk sample, A_{Hqt} and D_{qt} — the dimensionless Hall coefficient and the dimensionless magnetoresistance coefficient. The reflection coefficient of the lower surface is equal to unity ($q_1 = 1$), since the conducting layer does not border with another medium below, i.e. the case of mirror reflection from the film boundary is similar to the case of absence of this boundary.

in the GaN semiconductor layer, the electron gas may be assumed to be non-degenerate, i.e. the inequality $e^{u_\mu} = 10^{-30} \ll 1$ is met, and the Fermi-Dirac distribution function is as follows

$$f_n^{(0)} = e^{u_\mu - u_\| - \Delta_{tn}}.$$

In this case, the expression $\exp(u_\mu - \Delta_{tn})$ in the logarithm is a small number. Expanding the logarithm into a Taylor series, we obtain

$$\ln(1 + \exp(u_\mu - \Delta_{tn})) \approx \exp(u_\mu - \Delta_{tn}).$$

Therefore, the expression for conductance is as follows

$$\Sigma_{qt} = \frac{k_{0t}^3}{\pi^{3/2} x_\lambda \sqrt{5/2}}$$

$$\times \frac{(a_{qt} K_{H1} - K_{H2} \frac{\tau_\beta}{k_{0t}})^2 + (2b_{qt} K_{H1} - K_{H2}\beta)^2}{(K_{H2}\tau_\beta(\tau_\beta^2 + k_{0t}^2\beta^2) - a_{qt} K_{H1} k_{0t}(\tau_\beta^2 - k^2\beta^2) - 4K_{H1} k_{0t}^2 \tau_\beta \beta b_{qt}},$$

$$A_{Hqt} = \frac{\pi^{3/2} x_\lambda \sqrt{5/2}}{k_{0t}^2 \beta}$$

$$\times \frac{K_{H2}\beta(\tau_\beta^2 + k_{0t}^2\beta^2) + 2b_{qt} K_{H1}(\tau_\beta^2 - k_{0t}^2\beta^2) - 2K_{H1} k_{0t} \tau_\beta \beta a_{qt}}{\left(a_{qt} K_{H1} - K_{H2} \frac{\tau_\beta}{k_{0t}}\right)^2 + (2b_{qt} K_{H1} - K_{H2}\beta)^2},$$

$$K_{H1} = \pi E_{g0}^2 x_\lambda, \quad K_{H2} = \frac{5}{2} P_{1t},$$

$$a_{qt} = \sum_{n=1}^{\infty} \delta\gamma_n \sqrt{\gamma_n} e^{-\Delta_{tn}} \int_0^\infty e^{-u_\|} u_\| A_1(\Omega_{1n}, \Omega_{2n}) du_\|$$

$$b_{qt} = \sum_{n=1}^{\infty} \delta\gamma_n \sqrt{\gamma_n} e^{-\Delta_{tn}} \int_0^\infty e^{-u_\|} u_\| A_2(\Omega_{1n}, \Omega_{2n}) du_\|$$

$$P_{1t} = \sum_{n=1}^{\infty} \delta\gamma_n \cdot \gamma_n \cdot e^{-\Delta_{tn}},$$

$$P_{2t} = P_{4t} = \sum_{n=1}^{\infty} \delta\gamma_n \cdot e^{-\Delta_{tn}},$$

$$P_{3t} = \sum_{n=1}^{\infty} \delta\gamma_n \cdot \Delta_n \cdot e^{-\Delta_{tn}}.$$

In case of the two-dimensional electron gas, the n level will be equal to unity only.

5. Analysis of results

We assume that the mean free path of electrons exceeds the de Broglie wavelength in 10 times: $x_\lambda = 10$. The dimensionless strength of the transverse electric field E_{G0} is related to the dimensional strength by the relationship

$$E_G = 2.1 \cdot 10^7 E_{G0}^3 \, [\text{V/m}].$$

The strength of the internal electric field at the boundary of the heterojunction is about 10^4 V/m, which corresponds to the dimensionless value $E_{G0} = 0.1$. In laboratory plants, value of the magnetic field is $B = 100$ mT, which corresponds to the dimensionless value $\beta = 1.7 \cdot 10^{-3}$. Relation between the dimensional and the dimensionless value of the magnetic field is as follows

$$B = 57\beta \, [\text{T}].$$

Figures 4 and 5 show the dependences of the dimensionless Hall coefficient A_{Hqt} and the dimensionless magnetoresistance coefficient D_{qt} on dimensionless induction β. In case of the mirror boundaries (the curves 1) the Hall coefficient ceases to depend on the external magnetic field. With increase of dimensionless induction of the magnetic field, the curvature radius of the charge carrier path decreases. It results in increase of a relative number of the charge carriers not scattering on the film surfaces and

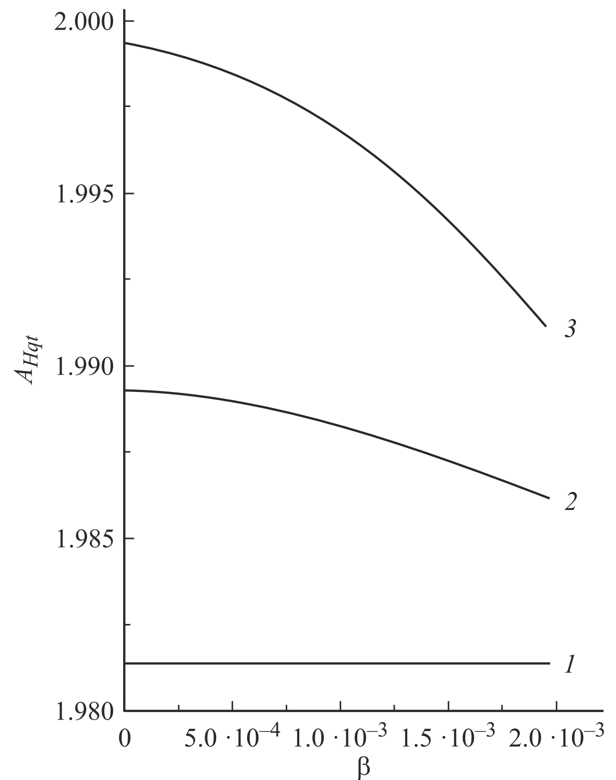

Figure 4. Dependence of A_{Hqt} on dimensionless induction β when $x_\lambda = 10$, $E_{G0} = 0.1$. The curves *1*, *2*, *3* are obtained when $g_2 = 0$, 0.3, 1, respectively.

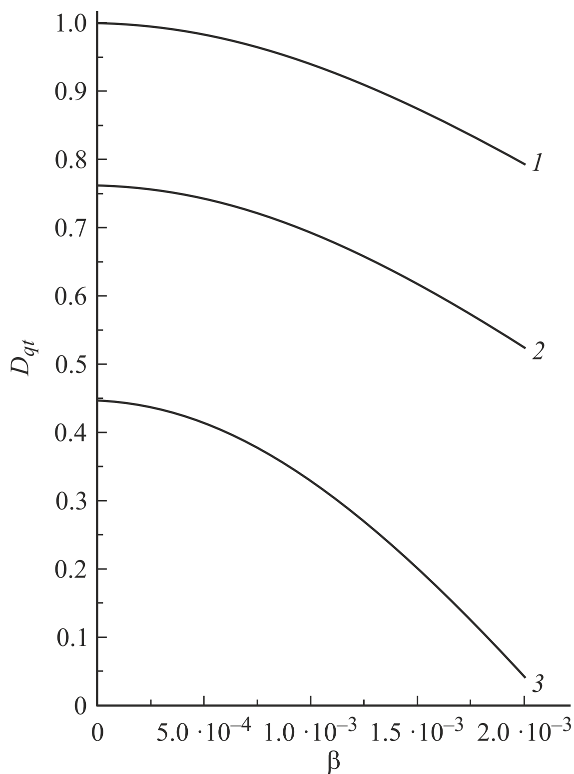

Figure 5. Dependence of D_{qt} on dimensionless induction β when $x_\lambda = 10$, $E_{G0} = 0.1$. The curves *1*, *2*, *3* are obtained when $g_2 = 0$, 0.3, 1, respectively.

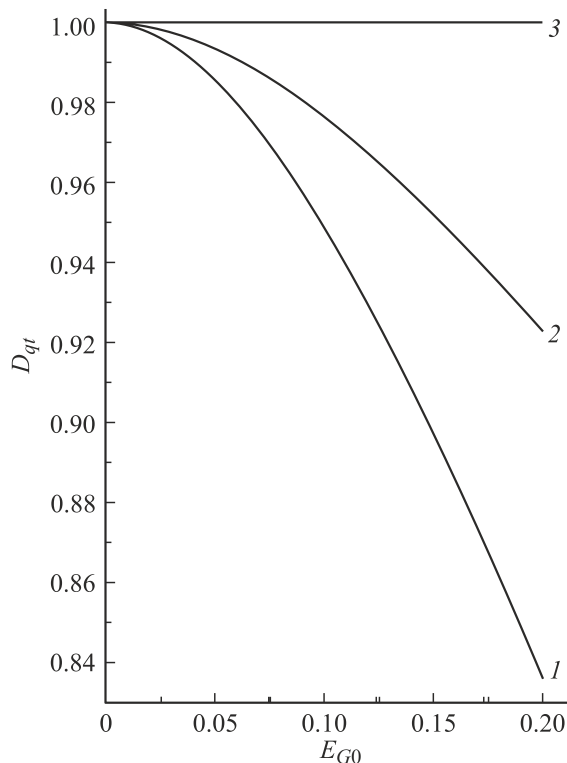

Figure 7. Dependence of D_{qt} on dimensionless strength E_{G0} when $x_\lambda = 10$, $\beta = 0.1$. The curves *1*, *2*, *3* are obtained when $g_2 = 0$, 0.3, 1, respectively.

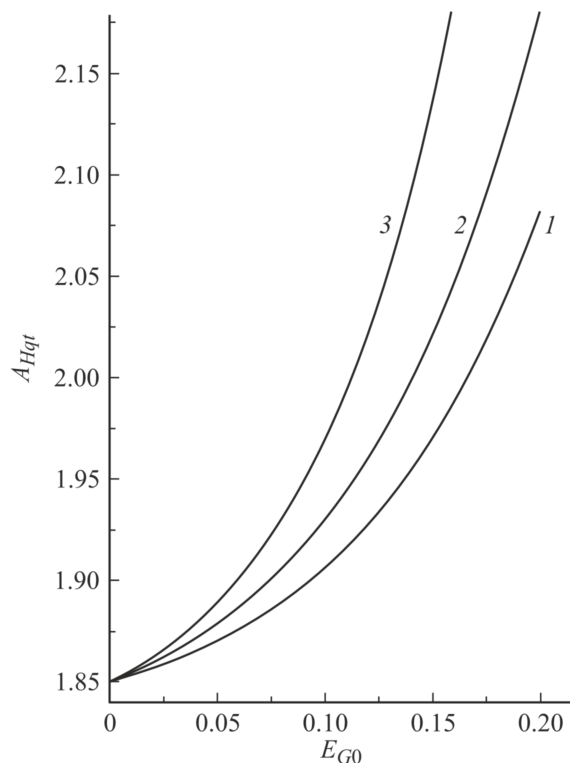

Figure 6. Dependence of A_{Hqt} on dimensionless strength E_{G0} when $x_\lambda = 10$, $\beta = 0.1$. The curves *1*, *2*, *3* are obtained when $g_2 = 0$, 0.3, 1, respectively.

decrease of contribution of surface scattering to the Hall coefficient and magnetoresistance.

Figures 6 and 7 show the dependences of the dimensionless Hall coefficient A_{Hqt} and the dimensionless magnetoresistance coefficient D_{qt} on dimensionless strength E_{G0}. In case of the mirror boundaries (the curves 1) the magnetoresistance coefficient does not depend on the form of the quantum well, i.e. on E_{G0}. With increase of dimensionless strength E_{G0} the distance between the levels increases and the first level energy increases ε_{z1} as well. It is more difficult for electrons to go to the above levels. It results in reduction of the average energy of the charge carriers which causes increase of the Hall coefficient and decrease of magnetoresistance.

Conclusion

The present work has calculate the Hall coefficient and the magnetoresistance coefficient of the nanolayer in the transverse magnetic field in approximation of the triangular potential well. With increase of induction of the external magnetic field, the Hall coefficient and magnetoresistance decrease due to reduction of effect of surface charge carrier scattering. In case of the mirror boundaries the Hall coefficient depends on the transverse strength of the electric field, while magnetoresistance becomes a constant value.

These results can be used when designing the nanoscale Hall sensors and magnetometer for measurements of the magnetic fields in the nanosystems.

Conflict of interest

The authors declare that they have no conflict of interest.

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