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Impact of heat and mass exchange, thermal diffusion and evaporation coefficient on photophoresis of large high-viscose drop

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Photophoretic motion of the large high-viscose spherical drop (no matter circulation inside the particle and no forces of interphase surface tension) inside a viscose binary gas mixture at small relative temperature gradients therearound is theoretically described. In a quasistationary approximation, a system of hydrodynamic equations (the Navier-Stokes equations system) and convective equations of heat and mass transfer at small Reynolds and Peclet numbers have been solved. The obtained formulae allow evaluate the contribution of heat and mass transfer, thermal diffusion and direct impact of the evaporation rate on the photophoresis rate, distribution of velocities, temperatures and concentration of a volatile component. It is shown that for highly thermal conductive particles there is photophoresis due to convective heat and mass transfer.

Keywords: photophoresis of drops, motion of highly-viscose drops in gas, motion of drops in the electromagnetic radiation field, heat and mass exchange.

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Introduction

Photophoresis in a gas is understood as motion of the particles in a uniform electromagnetic field. Two types of photophoresis are distinguished the literature, and they differ by a physical nature of this phenomenon. In the first case, photophoresis occurs due to transfer of a photon pulse to the particle by refraction and reflection, if the particle is transparent and has a higher refractive index than a refractive index of the environment. Here, the photophoretic force depends on the intensity of light and the particle size. In the second case, photophoresis occurs due to absorption of the electromagnetic radiation by the particle surface, thereby resulting in non-uniform heating of one of the sides of the particle surface. An uncompensated pulse occurs, i.e. the gas molecules bounce off the higher-heated particle surface with a higher velocity (pulse) than from the lower-heated one. Here, the photophoretic force depends on many factors, in particular, on intensity and wavelength of incident radiation, thermal & physical properties of the gas and the particle, etc.

The present work focuses on the second case of photophoresis. Here, it is difficult to mathematically describe the photophoresis phenomenon due to the following factors. First of all, it is necessary to take into account an electrodynamic problem of calculating the absorbed electromagnetic radiation within the particle volume; secondly, to take into account the heat problem of calculating temperature fields in the volume and on the particle surface and, thirdly, to take into account the gas kinetics problem of calculating the fields of velocities, pressures, etc.

The study of the photophoresis phenomenon is important in terms of its application and fundamentality despite the fact that it was discovered at the beginning of the twentieth century [1]. New fields of photophoresis application are being discovered [2–8]: a global problem of struggle against contamination of the air environment with natural and man-made aerosols; numerous process applications (processes of particle sedimentation in channels; fine purification of small volumes of a gas); chemical industry; application of special coatings of a given thickness; sampling of aerosols, etc.

The present work is the first one to obtain expressions which can evaluate impact of heat and mass exchange (convective terms in the thermal conductivity and diffusion equations), thermal diffusion and the evaporation coefficient on the force and the velocity of photophoresis of the highly-viscose drop in the binary gas medium. It should be noted that thermal diffusion refers to the so-called „weak“ effects or the effects of the second order of smallness, but it is quite sensitive to parameters of potentials of intermolecular interaction. The thermal diffusion phenomenon is crucial in many processes and has an independent nature, for example, when purifying and separating mixtures of gases and isotopes, producing especially pure substances, etc.

1. Main equations and boundary conditions

The attention is focused on the large [9] highly-viscose evaporating spherical drop, which is suspended in the gas mixture and has a radius R, with a density ρ_i, thermal conductivity λ_i and viscosity μ_i, and which stays in the field of flat wave of monochromatic radiation of the intensity I_0 (Fig. 1). The binary gas mixture is described by two components C_1 and C_2, the density ρ_e, the thermal conductivity λ_e, the mutual diffusion coefficient D_{12} and the viscosity μ_e. $C_1 = n_1/n_e$, $C_2 = n_2/n_e$, $n_e = n_1 + n_2$, $\rho_e = \rho_1 + \rho_2$, $\rho_1 = n_1 m_1$, $\rho_2 = n_2 m_2$; m_1, n_1 and m_2, n_2 — the weight and the numerical concentration of molecules of the first and second components of the mixture. Hereinafter, the indices „e“ and „i“ refer to the mixture and the particle, respectively; the index „S“ designates values of the physical quantities taken at the average temperature of the drop surface and the index „∞“ designates the physical quantities which characterize the gas medium far away from the drop.

When describing the photophoresis, evaporation is assumed to be slow [10], and the first component C_1 in terms of its physical & chemical composition coincides with the liquid drop substance, whereas the second component C_2 is believed to be a basic (carrier) one and a diffusion evaporation mode ($C_1 \ll C_2$) is considered [11].

The molecules of the condensed phase evaporate or condense an Mach number much smaller than unity. The drop radius is considered to be invariable (the time of noticeable change of the drop radius is significantly higher than the time of relaxation of diffusion and heat nonuniformities nearby). When moving, the drop keeps its spherical form, i.e. the forces of surface tension significantly exceed the force of viscose resistance. An evaporation-induced reactive effect is taken into account. Due to smallness of time of thermal and diffusional relaxation, the process of heat and mass transfer in the particle-gas system undergoes in a quasi-steady-state and free convection is neglected (the Grashof number is low). The problem is solved by a hydrodynamic method, i.e. the equations of hydrodynamics and heat and mass transfer with respective boundary conditions are solved.

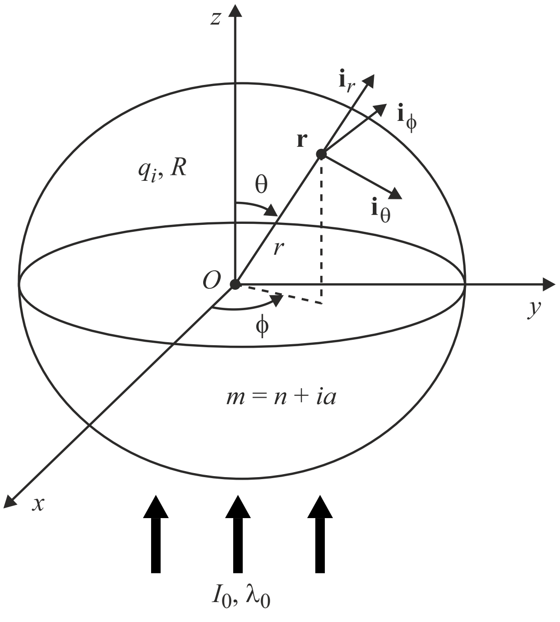

Figure 1. Problem geometry.

Let us direct the axis O_z in the direction of an intensity vector of the electromagnetic field. The problem is solved in the spherical system of coordinates r, θ, φ, whose origin coincides with a mass center of the evaporating drop. The distributions of the velocities, the pressures, the relative concentrations and the temperatures have an axial symmetry in relation to the axis O_z. With the said selection of the origin of the system of coordinates, the drop can be considered to be stationary, and the medium (gas) can be considered to be moving at the velocity U_∞ in a direction opposite to actual ordered motion of the drop U_p ($U_p = -U_\infty$, $U_\infty \parallel O_z$).

The distributions of the fields of the mass velocity U_e, the pressure P_e, the concentration C_1 and the temperatures T_e, T_i are described by the system of equations (1), (2) [11]:

$$\mu_e \Delta U_e = \nabla P_e, \quad \operatorname{div} U_e = 0, \tag{1}$$

$$\rho_e c_p (U_e \nabla) T_e = \lambda_e \Delta T_e, \ \ (U_e \nabla) C_1 = D_{12} \Delta C_1, \ \ \Delta T_i = -\frac{q_i}{\lambda_i}, \tag{2}$$

which was solved with the following boundary conditions (3)–(7):

$$y \to \infty, \ U_e = U_\infty n_z, \ T_e = T_\infty, \ C_1 = C_0, \ P_e = P_\infty,$$

$$U_\infty = |U_\infty|, \tag{3}$$

$$y \to 0, \quad T_i \neq \infty, \tag{4}$$

$$y = 1, \quad T_e = T_i,$$

$$n_1 U_r^{(e)} - D_{12} \frac{n_e^2 m_2}{R\rho_e} \left(\frac{\partial C_1}{\partial y} + \frac{K_T}{T_e} \frac{\partial T_e}{\partial y} \right) = \alpha_0 \nu n_e$$

$$\times \left[C_{1S}^{(H)} + C_{1S}^* \delta T_i - C_1 \right], \tag{5}$$

$$n_2 U_r^{(e)} + D_{12} \frac{n_e^2 m_1}{R\rho_e} \left(\frac{\partial C_1}{\partial y} + \frac{k_T}{T_e} \frac{\partial T_e}{\partial y} \right) = 0,$$

$$U_\theta^{(e)} = K_{TS} \frac{\nu_e}{R T_e} \frac{\partial T_e}{\partial \theta} + K_{DS} \frac{D_{12}}{R} \frac{\partial C_1}{\partial \theta}, \tag{6}$$

$$-\lambda_e \frac{\partial T_e}{\partial y} + \lambda_i \frac{\partial T_i}{\partial y} = -L m_1 R \alpha_0 \nu n_e \left[C_{1S}^{(H)} + C_{1S}^* \delta T_i - C_1 \right]$$

$$-\sigma_0 \sigma_1 R (T_i^4 - T_\infty^4) + L D_{12} \frac{n_e^2 m_1 m_2}{\rho_e} \frac{k_T}{T_e} \frac{\partial T_e}{\partial y}, \tag{7}$$

$$C_{1S}^{(H)} = \left. \frac{n_{1S}^{(H)}}{n_e} \right|_{T_i = T_{iS}}, \quad C_{1S}^* = \left. \frac{1}{n_e} \frac{\partial n_{1S}^{(H)}}{\partial T_i} \right|_{T_i = T_{iS}},$$

$$\nu = \sqrt{k_B T_e / (2\pi m_1)}, \quad y = r/R.$$

Here k_B — the Boltzmann constant; $U_r^{(e)}$, $U_\theta^{(e)}$ — the components of the mass velocity U_e in the spherical system of coordinates; σ_0, σ_1, n_z — the Stefan-Boltzmann constant, integral emissivity factor of the drop substance and a unit vector along the axis O_z; c_p, L, ν_e — specific heat capacity with the constant pressure, specific evaporation heat of liquid and the kinematic viscosity coefficient; ν — one quarter of the average arithmetic velocity of thermal motion of gas molecules of the first class [12]; α_0 — the evaporation coefficient of the liquid drop [10,12]. The experimental data of the literature shown that the evaporation coefficient $\alpha_0 \le 1$; $n_{1s}^{(H)}(T_{is})$ — the saturated concentration of the molecules of the first component of the binary gas mixture, which depends on the average temperature of the drop surface T_{iS}; K_{TS}, K_{DS} — the coefficients of thermal and diffusional gliding [13,14]. With coefficients of accommodation for energy and tangential pulse equal to unity, the numerical values of these coefficients have the following values: $K_{TS} = 1.161$, $K_{DS} = 0.3$ [13,14]; $n_1 U_r^{(e)}$, $n_2 U_r^{(e)}$, $D_{12}\frac{n_e^2 m_2}{R\rho_e}\left(\frac{\partial C_1}{\partial y} + \frac{k_T}{T_e}\frac{\partial T_e}{\partial y}\right)$, $D_{12}\frac{n_e^2 m_1}{R\rho_e}\left(\frac{\partial C_1}{\partial y} + \frac{k_T}{T_e}\frac{\partial T_e}{\partial y}\right)$ — radial convective, radial diffusional and thermo-diffusional flows of the respective components, k_T — the thermal diffusion ratio [11]. The value of C_0 in the edge condition (3) is determined via numerical concentrations n_1 and n_2, while the undisturbed parameters $(T_\infty, P_\infty, C_0)$ are determined in a location of the geometrical center of the drop without it.

The volume density of the internal heat sources q_i, which cause non-uniform heating of the drop surface, is determined from the solution of the electrodynamic problem, for example, in [15,16]. Generally, the volume density can be represented as $q_i = \frac{4\pi n a}{n_s \lambda_0} I_0 B_k$. Here $m = n + ia$ — the complex refractive index of the drop, n_s — the refractive index of the medium, λ_0, I_0 — the wavelength and intensity of incident radiation, B_k — the coordinate function calculated by the Mi theory [15,16].

The function q_i has the simplest form when the drop absorbs electromagnetic radiation as a black body (radiation is absorbed in a thin layer with the thickness $\delta \ll R$, which is adjacent to the heated part of the drop surface) [15,16]:

$$q_i = \begin{cases} -\frac{I_0}{\delta}\cos\theta, & \frac{\pi}{2} \le \theta \le \pi,\ R - \delta \le r \le R, \\ 0, & 0 \le \theta \le \frac{\pi}{2}. \end{cases}$$

When describing the photophoresis phenomenon, we use the disturbance theory [17]. The crucial parameters of the problem can be taken to compose three dimensionless combinations: the Reynolds number, the thermal and diffusion Peclet numbers [11]. The last two, in turn, are expressed via the Reynolds number [11], so the Reynolds number is used as a small problem parameter $\varepsilon = \mathrm{Re} = (\rho_e R U_\infty)/\mu_e \ll 1$. When finding the force and the velocity of photophoresis, we will be limited by the first small correction by ε.

Let us explain the physical meaning of the edge conditions on the drop surface ($y = 1$). The temperature equalities and the continuity of the radial flow of the first component through the drop surface are taken into account in the edge condition (5); the edge condition (6) correspondingly takes into account non-permeability of the drop surface for the second component of the gas mixture and known phenomena of thermal and diffusional glidings proportional to the coefficients K_{TS}, K_{DS} [9,13,14]; and the condition (7) takes into account continuity of radial flow of heat with taking into account heat for the phase transition and radiation.

2. Fields of the velocity, the pressure, the temperatures and the concentration of the first component of the binary gas mixture

The general solutions of the hydrodynamics and heat & mass transfer when $\varepsilon \ll 1$, which satisfy the edge conditions (3)–(7), are as follows

$$t_{e0}(y) = 1 + \frac{\Gamma_0}{y},\quad U_r^{(e)}(y,\theta) = U_\infty\cos\theta\left(1 + \frac{A_1}{y^3} + \frac{A_2}{y}\right),$$

$$U_\theta^{(e)}(y,\theta) = -U_\infty\sin\theta\left(1 - \frac{A_1}{2y^3} + \frac{A_2}{2y}\right),$$

$$P_e(y,\theta) = P_\infty + \mu_e\frac{U_\infty}{R}\cos\theta\frac{A_2}{y^2},$$

$$C_{10}(y) = C_0 + \frac{M_0}{y},\quad t_e = T_e/T_\infty,$$

$$t_i = T_i/T_\infty,\quad H_0 = \frac{R^2}{3\lambda_i T_\infty}J_0,$$

$$t_e(y,\theta) = t_{e0}(y) + \varepsilon t_{e1}(y,\theta),$$

$$t_e^*(\xi,\theta) = t_{e0}^*(\xi) + \varepsilon t_{e1}^*(\xi,\theta),$$

$$t_i(y,\theta) = t_{i0}(y) + \varepsilon t_{i1}(y,\theta),$$

$$H_1 = \frac{R}{3\lambda_i T_\infty}J_1,$$

$$t_{e1}(y,\theta) = \frac{\omega_0^{(T)}}{2y}(N_2 - y) + \cos\theta\left[\frac{\Gamma_1}{y^2}\frac{\omega_0^{(T)}}{2}\left(A_3 + \frac{A_2}{y} - \frac{A_1}{y^3}\right)\right],$$

$$t_{e0}^*(\xi) = 1, \tag{8}$$

$$t_{e1}^*(\xi,\theta) = \frac{\Gamma_0}{\xi}\exp\left\{\frac{Pr_T}{2}\xi(x-1)\right\},$$

$$C_1(y,\theta) = C_{10}(y) + \varepsilon C_{11}(y,\theta),$$

$$C_1^*(\xi,\theta) = C_{10}^*(\xi) + \varepsilon C_{11}^*(\xi,\theta),$$

$$C_{10}^*(\xi) = C_0,\quad J_0 = \frac{1}{V}\int_V q_i\,dV,$$

$$C_{11}(y,\theta) = \frac{\omega_0^{(D)}}{2y}(N_3 - y)$$

$$+ \cos\theta\left[\frac{M_1}{y^2} + \frac{\omega_0^{(D)}}{2}\left(A_3 + \frac{A_2}{y} - \frac{A_1}{y^3}\right)\right],$$

$$C_{11}^*(\xi,\theta) = \frac{M_0}{\xi}\left\{\frac{Pr_D}{2}\xi(x-1)\right\},\quad x = \cos\theta,$$

$$\psi_0(y) = -\frac{R^2}{2\lambda_i T_\infty} y^2 \int\limits_{-1}^{-1} q_i dx,$$

$$J_1 = \frac{1}{V} \int\limits_V q_i z dV,$$

$$t_{i0}(y) = B_0 + \frac{H_0}{y} - \frac{1}{y} \int\limits_1^y \psi_0(y) dy + \int\limits_1^y \frac{\psi_0(y)}{y} dy,$$

$$\psi_1(y) = -\frac{3R^2}{2\lambda_i T_\infty} y^2 \int\limits_{-1}^{+1} q_i x dx,$$

$$V = \frac{4}{3}\pi R^3,$$

$$t_{i1}(y, \theta) = N_4 + \cos\theta \left\{ B_1 y + \frac{H_1}{y^2} + \frac{1}{3} \right.$$
$$\left. \times \left[y \int\limits_1^y \frac{\psi_1(y)}{y^2} dy - \frac{1}{y^2} \int\limits_1^y y \psi_1(y) dy \right] \right\},$$

$$P_{r_T} = \frac{\mu_e c_p}{\lambda_e}, \quad P_{r_D} = \frac{\mu_e}{\rho_e D_{12}}$$

— the thermal and the diffusional Prandtl number,

$$\omega_0^{(T)} = \Gamma_0 P_{r_T}, \ \omega_0^{(D)} = \Gamma_0 P_{r_D}, \ z = r\cos\theta,$$

$$\delta T_i(y, \theta)\Big|_{y=1} = T_\infty t_{i1}(y, \theta)\Big|_{y=1},$$

$\int\limits_V q_i z dV$ — the dipole moment of the density of the thermal sources inside the evaporating drop [1–3,16]. The integration is across the total volume of the particle and $dV = r^2 \sin\theta dr d\theta d\varphi$. The constants A_1, A_2, Γ_0, Γ_1, etc., which are included in (8), are unambiguously determined from the boundary conditions (3)–(7).

It should be noted that the convective equations of heat and mass transfer have been solved by a method of splicing of asymptotic decompositions [17,18]. It is related to convective terms in the heat and mass transfer equations. It is known that [17,18] the usual method of expansion in the small parameter can not strictly satisfy the edge conditions at infinity and obtain an exact unified solution that is uniformly valid for the entire region of the problem.

The splicing method consists of three procedures: construction of external decomposition, construction of internal decompositions and splicing the external decomposition with the internal one [17–19]. This procedure made it possible to obtain asymptotic solutions of the convective equations of heat and mass transfer (2), which are presented in (8).

The average value of the temperature on the drop surface $T_{iS} = T_\infty t_{iS}$ is determined from solving the following system of equations:

$$\begin{cases} t_{eS} = t_{is}, \Gamma_0 = t_{eS} - 1, M_0 = \dfrac{C_{1S}^{(H)} - C_0 - D_{12} \frac{n_e}{R\alpha_0 \nu n_2} \frac{k_T}{t_{eS}} (t_{eS} - 1)}{1 + \frac{n_e}{R\alpha_0 \nu n_2} D_{12}}, \\ (t_{eS} - 1)\left[1 + L \frac{n_e^2 m_1 m_2}{\rho_e \lambda_e T_\infty} D_{12} \frac{k_T}{t_{eS}}\right] = \frac{R^2 J_0}{3\lambda_e T_\infty} + L \frac{n_e m_1 R \alpha_0 \nu}{T_\infty \lambda_e} \\ \times \left[C_0 + M_0 - C_{1S}^{(H)}\right] - \sigma_0 \sigma_1 \frac{R T_\infty^3}{\lambda_e} \left(t_{eS}^4 - 1\right). \end{cases}$$

Here $t_{eS} = t_{e0}(y = 1)$, $t_{iS} = t_{i0}(y = 1)$.

3. Photophoretic force and velocity. Analysis of the obtained results

We integrate the stress tensor [11] across the surface, which makes it possible to find the resultant force acting thereon. This force F is summed by four forces:

$$F = F_\mu + F_{ph} + F_{cht} + F_{cmt}. \tag{9}$$

Here F_μ — the force of viscose resistance of the medium; F_{ph} — the photophoretic force, which is proportional to the dipole moment of the density of the thermal sources J_1; the force F_{cht}, which is proportional to the coefficient (due to impact of convective thermal exchange on the photophoresis) and the force F_{cmt}, which is proportional to the coefficient $\omega_0^{(D)}$ (due to impact of convective mass exchange on the photophoresis).

At the same time

$$F_\mu = 6\pi R \mu_e U_\infty n_z, \quad F_{ph} = -6\pi R \mu_e f_{ph} J_1 n_z,$$

$$F_{cht} = -6\pi R \mu_e f_{cht} \omega_0^{(T)} n_z, \quad F_{cmt} = -6\pi R \mu_e f_{cmt} \omega_0^{(D)} n_z,$$

$$f_{ph} = \frac{2}{3\delta\lambda_i T_\infty} \left\{ \frac{V_e}{t_{eS}} K_{TS} + K_{DS} \frac{D_{12}}{a_1} \left[C_{1S}^* T_\infty \right. \right.$$
$$\left. - 2 \frac{k_T}{t_{eS}} D_{12} \frac{n_e}{R\alpha_0 \nu n_2} \frac{t_{eS} + 1}{2t_{eS}} \right] + 2D_{12} \frac{m_1 n_e^2}{\rho_e n_2 a_1}$$
$$\left. \times \left[C_{1S}^* T_\infty + \frac{k_T}{t_{eS}} \frac{t_{eS} + 1}{2t_{es}} \right] \right\},$$

$$f_{cht} = \frac{1}{4R\delta} \left\{ \frac{V_e}{t_{eS}} K_{TS} \left[\frac{\lambda_e}{\lambda_i} + \frac{k_T}{t_{eS}} L D_{12} \frac{m_1 n_e}{\lambda_i T_\infty} \left(\frac{m_2}{\rho_e} + \frac{1}{n_2 \beta_1} \right) \right] \right.$$
$$+ K_{DS} \frac{D_{12}}{\beta_1} \left[\frac{\lambda_e}{\lambda_i} C_{1S}^* T_\infty + \frac{k_T}{t_{eS}} \left(L \frac{m_1 n_e^2}{\lambda_i} D_{12} C_{1S}^* \left(\frac{m_2}{\rho_e} + \frac{1}{n_2 \beta_1} \right) \right. \right.$$
$$\left. \left. + D_{12} \frac{n_e}{R\alpha_0 \nu n_2} \left(\frac{\lambda_e}{\lambda_i} \Gamma_0 + \beta_0 + 2L \frac{m_1 n_e^2}{\lambda_i} D_{12} C_{1S}^* \right) \right) \right]$$
$$+ D_{12} \frac{m_1 n_e^2}{\beta_1 \rho_e n_2} \left[2 \frac{\lambda_e}{\lambda_i} C_{1S}^* T_\infty - \frac{k_T}{t_{eS}} \left(\frac{\lambda_e}{\lambda_i} \frac{\Gamma_0}{t_{eS}} + \beta_0 \right. \right.$$
$$\left. \left. \left. - 2L \frac{m_1 m_2 n_e^2}{\rho_e \lambda_i} D_{12} C_{1S}^* \right) \right] \right\},$$

$$f_{cmt} = \frac{D_{12}}{4R\delta\beta_1}\left\{\frac{\nu_e}{t_{eS}} K_{TS} L\,\frac{m_1 n_e^2}{\lambda_i T_\infty n_2} + K_{DS}\left[D_{12}\frac{n_e}{R\alpha_0 \nu n_2}\right.\right.$$

$$\times\left(\beta_0 + 2\,\frac{\lambda_e}{\lambda_i} + 2L\,\frac{m_1 n_e^2}{\lambda_i n_2 \beta_1}\,D_{12}C_{1S}^* + 2\,\frac{k_T}{t_{eS}}\,L\,\frac{m_1 m_2 n_e^2}{\lambda_i \rho_e T_\infty}\right.$$

$$\left.\left.\times D_{12}\left(1 - \frac{t_{eS}-1}{2t_{eS}}\right)\right) + L\,\frac{m_1 n_e^2}{\lambda_i n_2 \beta_1}\,D_{12}C_{1S}^*\right]$$

$$-\frac{m_1 n_e^2}{\rho_e n_2}\left[\beta_0 + 2\,\frac{\lambda_e}{\lambda_i} + 2L\,\frac{m_1 n_e^2}{\lambda_i n_2 \beta_1}\,D_{12}C_{1S}^*\right.$$

$$\left.\left.+ 2\,\frac{k_T}{t_{eS}}\,L\,\frac{m_1 m_2 n_e^2}{\lambda_i \rho_e T_\infty}\,D_{12}\left(1 - \frac{t_{eS}-1}{2t_{eS}}\right)\right]\right\},$$

$$\delta = \beta_0 + 2\,\frac{\lambda_e}{\lambda_i} + 2L\,\frac{m_1 n_e^2}{\lambda_i n_2 \beta_1}\,D_{12}C_{1S}^* + 2\frac{k_T}{t_{eS}}\,L\,\frac{m_1 m_2 n_e^2}{\lambda_i \rho_e T_\infty}$$

$$\times D_{12}\,\frac{t_{eS}+1}{2t_{eS}}\left(\frac{m_2}{\rho_e} + \frac{1}{n_2\beta_1}\right),$$

$$\beta_0 = 1 + 4\sigma_0\sigma_1\,\frac{RT_\infty^3}{\lambda_i}\,t_{eS}^3,$$

$$\beta_1 = 1 + 2D_{12}\,\frac{n_e}{R\alpha_0 \nu n_2}.$$

The general expression for the velocity of ordered motion of the large evaporating drop is so taken to be zero of the full force (the drop moves uniformly):

$$U_p = U_{ph} + U_{cht} + U_{cmt},$$

$$U_p = -(f_{ph}J_1 + f_{cht}\omega_0^{(T)} + f_{cmt}\omega_0^{(D)})n_z. \qquad (10)$$

The expressions (9), (10) make it possible to evaluate impact of heat and mass transfer, thermal diffusion and the evaporation coefficient on the photophoretic force and the velocity of the highly-viscose drop in the binary gas medium and they are the most general.

The coefficients f_{ph}, f_{cht} and f_{cmt} which are included in the force and the velocity of ordered motion of the evaporating drop, consist of the sub of three summands. The first summand that is proportional to the thermal gliding coefficient K_{TS} due to which the evaporating drop tends to move towards temperature decrease in the external medium, i.e.from an area of the higher temperature into the area of the lower temperature; due to the second summand (diffusion gliding, which is proportional to the coefficient K_{DS}), the drop can move both towards increase and decrease of the temperature, depending on the weights of the components of the binary gas mixture (if the weight of the molecule of the component of the external mixture subject to the phase transition on the drop surface, $m_1 < m_2$, then $K_{DS} > 0$, otherwise — $K_{DS} < 0$); the third summand is related to the phase transition and describes impact of a reactive part of the pulse on the drop. The temperature dependence of relative concentration of saturated vapor of the volatile substance of the drop (C_{1S}^*) and the thermodiffusional phenomena within the volume of the gas mixture cause non-uniform evaporation along the boundary of the condensed phase and, as a result, the reactive effect.

For the highly thermal conductive large evaporating drops ($\lambda_i \to \infty$) we see that the „pure“ photophoresis is almost absent, i.e. $f_{ph} \to 0$, which is confirmed in the experiments. As it is seen from the formulae for the coefficients f_{cht} and f_{cmt}, they do not tend to zero when $\lambda_i \to \infty$, i.e. there is photophoresis, but now it is due to convective heat and mass transfer. The photophoresis value related to convective heat transfer depends on the value of the diffusion constant, diffusion gliding, thermal diffusion and the evaporation coefficient, whereas the photophoresis value related to the mass transfer depends on the vale of the diffusion constant, diffusion gliding, and the evaporation coefficient. In this case the coefficients f_{cht} and f_{cmt} can be evaluated by the following formulae: $f_{cht} = \frac{n_e}{4Rn_2\beta_1}\,\frac{k_T}{t_{eS}}\,D_{12}\left(K_{DS}\,\frac{D_{12}}{R\alpha_0\nu} - \frac{m_1 n_e}{\rho_e}\right)$, $f_{cmt} = \frac{n_e}{4Rn_2\beta_1}\,D_{12}\left(K_{DS}\,\frac{D_{12}}{R\alpha_0\nu} - \frac{m_1 n_e}{\rho_e}\right)$. Thus, taking into account the convective terms in the equations of heat and mass transfer for the highly thermal conductive evaporating drops does not result in disappearance of photophoresis as in case of „pure“ photophoresis, but its physical nature is different. But in case of experimental check, it should be borne in mind that thermal diffusion is related to the so-called „weak“ effects or the effects of the second order of smallness (it is quite sensitive to the parameters of potentials of intermolecular interaction, etc.) [20].

The value and the direction of the force and the velocity of „pure“ photophoresis are also determined by the value and the direction of the dipole moment of the density of the thermal sources $\int\limits_V q_i z\,dV n_z$, i.e. it can be positive as well as negative photophoresis. When the dipole moment is negative (when the larger part of thermal energy is released in the particle part facing the radiation flow), the drop can move along the direction of incident radiation, and, otherwise — against the direction of radiation propagation.

In relation to the coefficients $\omega_0^{(T)} = \Gamma_0 Pr_T$ and $\omega_0^{(D)} = M_0 Pr_D$, where $Pr_T = \frac{\mu_e c_p}{\lambda_e}$ — the thermal Prandtl number, $Pr_D = \frac{\mu_e}{\rho_e D_{12}} = \frac{\nu_e}{D_{12}}$ — the diffusional Prandtl number, we note the following. The Prandtl number is about unity ($P_r \cong 1$, for the majority of the gases). It means that $\omega_0^{(T)} \sim \Gamma_0$ and $\omega_0^{(D)} \sim M_0$, which in turn are proportional to relative temperature gradients around the drop. The problem deals with the photophoresis at small relative temperature gradients, i.e. there is the inequality $(T_S - T_\infty)/T_\infty \ll 1$. Here, T_S — the average value of temperature of the surface of the evaporating drop, T_∞ — the value of temperature of the gas medium far away from the drop. Taking into account the above said, we have the following estimate for the coefficients f_{cht} and $f_{cmt} \le f_{ph}$.

Thus, the qualitatively above-considered summands show that the velocity of ordered motion of the large evaporating drop can change not only in magnitude, but in a direction, too, - depending on specific values of the physical quantities included in the expressions (9), (10).

It is interesting to study direct impact of the evaporation coefficient α_0 on the velocity of „pure" photophoresis, for example, as in thermophoresis [21]. In this case, the coefficient f_{ph} can be presented as follows ($k_T = 0$):

$$f^*_{ph} = \frac{2}{3\lambda_i T_\infty \Omega}\left[K_{TS}\,\frac{\nu_e}{t_{eS}}\left(1+\frac{R\alpha_0 \nu n_2}{2D_{12}n_e}\right) + \frac{R\alpha_0 \nu n_2 T_\infty}{2n_e}\left(K_{DS}+2\frac{m_1 n_e^2}{\rho_e n_2}\right)\right], \qquad (11)$$

$$\Omega = \left(\beta_0 + 2\,\frac{\lambda_e}{\lambda_i}\right)\left(1+\frac{R\alpha_0 \nu n_2}{2D_{12}n_e}\right) + L\,\frac{R\alpha_0 \nu m_1 n_e}{\lambda_i}\,C^*_{1S}.$$

The numerical estimates have been performed as per the formula (11) for the water drop in a vapor-air mixture within the temperatures $273\,\mathrm{K} \leq T_\infty \leq 323\,\mathrm{K}$, $P_\infty = 10^5\,\mathrm{Pa}$, $C_0 = 0.01$. Fig. 2 ($R = 50\,\mu$m) and Fig. 3 ($R = 30\,\mu$m) shows plots of the dependences $f^{(1)}_{ph} = f^*_{ph}/f^*_{ph}\Big|_{T_{is}=273}$ ($\alpha_0 = 0.034$ [20]) and $f^{(2)}_{ph} = f^*_{ph}/f^*_{ph}\Big|_{T_{is}=273}$ ($\alpha_0 = 0.07$) on the average temperature of the surface for the various values of the evaporation coefficient and the drop radius. The numerical values of the coefficients are taken from [22–24].

The numerical estimates have shown that with weak evaporation of the drop (the evaporation coefficient $\alpha_0 \leq 0.07$) the dependence of the force and the velocity of the photophoresis on the evaporation coefficient and the drop sizes is very weak. With increase of the average temperature of the drop surface, there is increase of evaporation intensity (as it is clear from the formula (11)) and, respectively, the force and the velocity of the photophoresis will increase, too. In this case, the numerical estimates require, first of all, knowledge of the numerical value of the evaporation coefficient (the literature has contradicting experimental data) and, secondly, now, taking into account the dependences of coefficients of molecular transfer (viscosity, thermal conductivity, diffusion) and the density of the gas medium on the temperature in the equations of hydrodynamics and heat and mass transfer [4,6].

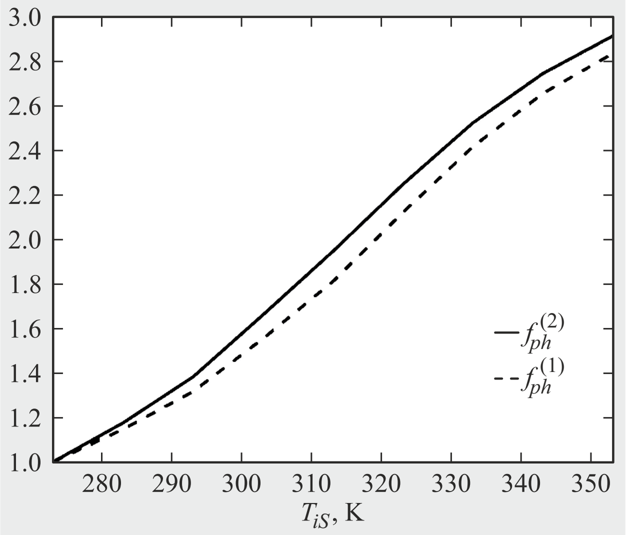

Figure 2. Dependences of the functions $f^{(1)}_{ph}$ and $f^{(2)}_{ph}$ on the average temperature of the surface of the drop ($R = 50\,\mu$m, $\alpha_0 = 0.034$ ($f^{(1)}_{ph}$), $\alpha_0 = 0.07$ ($f^{(2)}_{ph}$)).

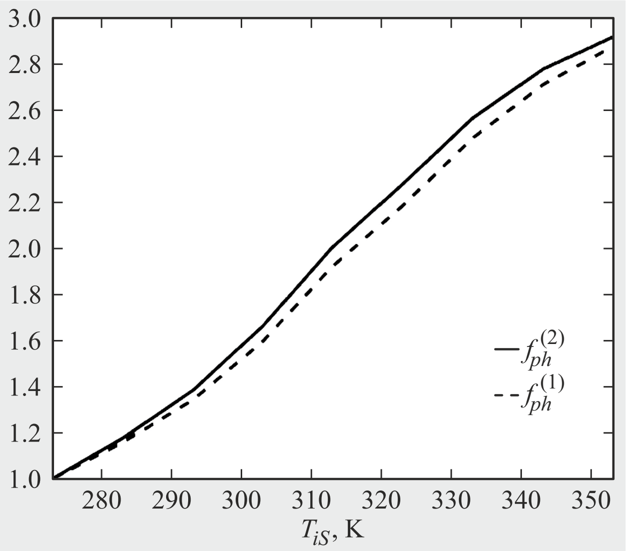

Figure 3. Dependences of the functions $f^{(1)}_{ph}$ and $f^{(2)}_{ph}$ on the average temperature of the surface of the drop ($R = 30\,\mu$m, $\alpha_0 = 0.034$ ($f^{(1)}_{ph}$), $\alpha_0 = 0.07$ ($f^{(2)}_{ph}$)).

Conclusion

The work has theoretically described the photophoretic motion of the large high-viscose spherical drop (no matter circulation inside the particle and no forces of interphase surface tension) in the binary gas mixture. We have obtained the formulae, which can be used for evaluating the contributions to the force and the velocity of photophoresis of the high-viscose drop by the convective terms of heat and mass transfer, thermal diffusion and direct impact of the evaporation rate at small relative gradients of the temperature in its neighborhood The numerical estimates have shown that with weak evaporation of the drop the dependence of the force and the velocity of the photophoresis on the evaporation coefficient and the drop sizes is very weak. For the highly thermal conductive drops there is the photophoresis due to convective heat and mass transfer.

Conflict of interest

The authors declare that they have no conflict of interest.

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