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Method for investigating the spatial correlation properties of a stochastic wave field

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A method for studying the spatial correlation properties of a stochastic wave field with a wide angular spectrum based on a correlation analysis of the spatial distribution of the complex amplitude of this field formed using numerical modelling is proposed and tested. A comparison of the results of determining the transverse correlation properties of a monochromatic field in its various sections based on the proposed method with the results obtained based on analytical formulas showed their very good agreement. The distribution of the complex amplitude of the optical wave field with a wide angular spectrum of spatial harmonics was numerically simulated for various intervals of variation of the random initial phases of the harmonics in the range from 0 to 2π radians. The correlation properties of the fluctuation components of the generated disturbance fields in the lateral plane of the wave field are investigated numerically. It is established that the length of the lateral correlation of field fluctuations does not change with a variation of the interval of the initial phase difference. Studies have shown that the lateral spatial coherence — the shape of the coherence function and the length of the coherence, of a quasi monochromatic wave field is determined by the numerical aperture value of the field and the shape of its angular spectrum.

Keywords: angular spectrum, correlation analysis, spatial distribution of complex amplitude, numerical simulation.

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Introduction

The surfaces of natural, technical, and biological objects are usually not smooth in terms of optics. Surface irregularities are often comparable to the wavelength of optical radiation, and therefore they scatter light. When such surfaces are illuminated by laser (coherent) radiation, speckle structures appear in the reflected field, which are the result of wave interference from individual surface inhomogeneities [1–5]. Spatial variations in the thickness of an object, refractive index, and reflectivity or absorbance of the medium also lead to light scattering. Laser radiation has a high degree of spatial and temporal coherence, therefore, light waves formed when such radiation is scattered by an optically inhomogeneous object are capable of interfering, since they turn out to be mutually coherent. Therefore, during the scattering of laser radiation, the wave fields formed in free space and in optical systems are speckle-modulated [1–5].

In practice, which is always the case, radiators emit non-monochromatic waves of different frequencies, and a dynamic speckle structure appears in the radiation of such a source, which has a stochastic character with a fluctuation time (coherence time) τ_c, inversely proportional to the width $\Delta\omega$ of the frequency spectrum of such radiation, $\tau_c \approx 2\pi/\Delta\omega$ [6–8]. This is the approximate time of quasi-stationarity of instantaneous speckle structures. The radiation of a quasi-monochromatic source at frequency spectrum width $\Delta\lambda = 2 \cdot 10^{-4}\,\mu$m and average wavelength $\lambda_0 = 0.55\,\mu$m has a coherence time $\tau_c = l_c/c = \lambda_0^2/c\Delta\lambda \approx 10^{-11}$ s. In real conditions, instantaneous speckle structures turn out to be experimentally unobservable due to the short coherence time τ_c and the relatively long reaction time of existing photodetectors. During one microsecond (this is approximately the minimum reaction time of existing photodetectors), $N \approx 10^5$ realizations of instantaneous speckle-modulated patterns change, and an image averaged over N realizations of instantaneous speckle-modulated patterns is recorded.

However, instantaneous speckle structures can be numerically simulated and investigated qualitatively and quantitatively using integral diffraction transformations of fields [9,10]. In [11], it was shown using numerical experiment that in a partially coherent optical wave field, instantaneous speckle structures are always formed that change over time, which determine the spatiotemporal fluctuations of the field and, accordingly, its spatial coherent properties.

Mathematical modeling is a necessary tool for studying the processes occurring in various optical systems [12–16]. Modeling the formation of wave fields in free space and in optical systems makes it possible to study the properties of these fields and the optical systems that form them, for example, in optical high-resolution interference microscopy [17–19]. The diffraction processes that take place in optical systems are the basis for modeling complex measured signals in the interference measuring systems and obtaining accurate results for system-level studies [20,21].

Currently, modeling is the basis for the development of fairly new methods of imaging, such as lens-free image acquisition methods, for example, ptychography jcite22, and ghost imaging [23]. At the same time, the frequently used paraxial approximation of diffraction integrals used to describe wave fields no longer satisfies the tasks of describing wave fields with wide angular spectra [22].

Quantification of the decorrelation of optical wave fields with wide frequency and angular spectra in optical systems is critical in terms of application method when such fields pass through the interface of media with different refractive indices [24,25], for the development of correlation interference microscopy [18,19,26,27], determination of geometric and optical parameters of objects[28.29], assessment of optical fields effect on biological objects at the cell level [30], analysis of terahertz wave fields, when the fields with a large numerical aperture are formed [31].

The purpose of this work was to develop and test a method for studying the spatial correlation properties of a quasi-monochromatic wave field with a wide angular spectrum based on numerical modeling and correlation analysis of spatial distributions of the complex amplitude of this field in its cross sections without using paraxial approximation and to develop methods for studying the spatial correlation properties of wave fields based on computer modeling as an alternative or augmentation to the in-situ experiment. The work also aimed at studying the influence of the width of the wave field angular spectrum on the transverse correlation properties of this field using the proposed method of correlation analysis.

1. Method for numerical modelling of optical speckle-modulated wave fields and diffraction structures in the far diffraction region

A spatial partially coherent wave field is created by an extended spatially incoherent or partially coherent polychromatic light source [6,7]. In the diffraction region of such sources, a spatial partially coherent wave field is formed with limited spatial coherence lengths, which are determined by both the width of the angular spectrum and the width of the frequency spectrum of this field [32]. Spatial incoherent self-luminous primary sources of the wave field are thermal light sources, gas-discharge lamps, and LEDs having a wide frequency spectrum of radiation. In accordance with Huygens-Fresnel principle, secondary extended light sources are formed, in practice, for example, a thin diffuser or pupil of an optical system, which can serve as a source of a partially coherent wave field.

With coherent illumination of the diffuser, elementary scattered secondary waves turn out to be mutually coherent and interfere in space. In the scattered wave field there is formed an irregular interference speckle-modulated structure which is distinguished by a pronounced stochastic character due to the random spatial arrangement of scattering centers and their random optical properties [1–5]. If the diffuser is stationary and its scattering elementary centers are unchanged in time, then the speckle-modulated structures formed in the scattered field have a stationary character. Otherwise, a dynamic speckle structure is formed in the scattered field [1,4]. Figure 1 shows an optical scheme for the formation of a speckle-modulated pattern in x, y plane in the far diffraction region during scattering of laser coherent radiation.

The field of complex amplitudes $E(x, y, z, t = 0)$ in the far diffraction region can be defined as a superposition of plane waves for all components of the frequency spectrum of this field with all possible spatial frequencies (k_x, k_y, k_z) within the field angular spectrum with a given numerical aperture NA_i with random initial phases of spatial harmonics $\varphi_0(k_x, k_y)$ from 0 to 2π radians:

$$E(x, y, z, t = 0) \sim \int_{k_1}^{k_2} g(k)\Big[\iint E(k_x, k_y)P(k_x, k_y)$$

$$\times \exp[i(k_x x + k_y y + k_z z)]dk_x dk_y\Big]dk, \quad (1)$$

where $g(k)$ — frequency spectrum, $P(k_x, k_y)$ — aperture function of the wavefield angular spectrum, $E(k_x, k_y) = A(k_x, k_y)\exp(\varphi_0(k_x, k_y))$ — complex amplitude of plane wave, $k = 2\pi/\lambda$ — wavenumber corresponding to the wavelength λ of monochromatic wave field; k_x, k_y and k_z — spatial frequencies — projections of wave vector $\mathbf{k}$ on axis X, Y and Z respectively, that may be expressed as follows:

$$k_x = k\cos\alpha\sin\theta,$$

$$k_y = k\sin\alpha\sin\theta,$$

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2} = k\cos\theta, \quad (2)$$

where α and θ — azimuth and zenith angles which specify the direction of wave vector $\mathbf{k}$ in space (Fig. 2). Azimuth angle α varies from 0 to 2π, zenith angle θ — from 0 to

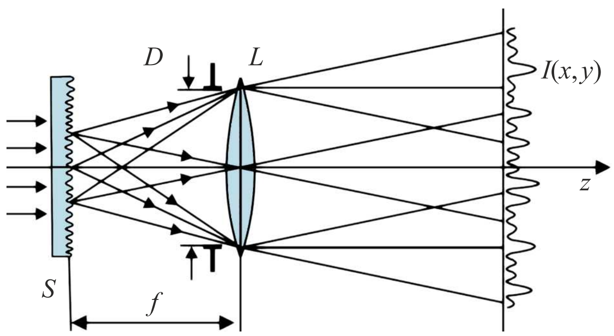

Figure 1. Scheme of speckle structure formation in the far diffraction region.

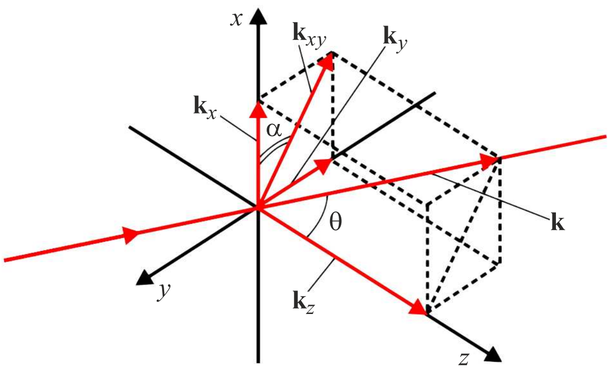

Figure 2. Wave vector of plane wave incident at an angle of θ to the axis z, and its components in 3D space.

some aperture angle θ_A, equal $\theta_{\max}$ — maximal angle of plane waves propagation, the superposition of which makes the wave field (1).

The formula (1) defining the field of complex amplitudes of the wave field $E(x, y, z, t = 0)$ can be written in discrete form

$$E(x,y,z,t=0) \sim \sum_{k=k_1}^{k_2} \sum_{k_y=k_{y\,\min}}^{k_{y\,\max}} \sum_{k_x=k_{x\,\min}}^{k_{x\,\max}} g(k)A(k_x,k_y)P(k_x,k_y) \times \exp(\varphi_0(k_x,k_y)) \exp[i(k_x x + k_y y + k_z z)]. \quad (3)$$

Based on formula (3) numerical modelling of speckle-modulated diffraction structures was performed for the discrete representation of the wave field. The wave field was formed by adding and interfering plane waves of various directions within the numerical aperture of the wave field for all components of the frequency spectrum. To simulate the rectangular shape of the aperture, plane waves with all discrete values k_y and k_x were formed during the formation of the wave field in the intervals $k_{y\,\min}-k_{y\,\max}$ and $k_{x\,\min}-k_{x\,\max}$, respectively, within the boundaries of the rectangular numerical aperture NA_i, which corresponds to the wave vector $\mathbf{k}_{NAi}$. To model the curved shape aperture, e.g. round shape, first a square area with a given numerical aperture NA_i on the square's sides was determined, then all points within the circle inscribed in this area with discrete values k_y and k_x were selected in this area, which meets the condition $k_{xy} \le |\mathbf{k}_{NAi}|$, $k_{xy} = \sqrt{k_x^2 + k_y^2}$ — projection of the wave vector $\mathbf{k}$ onto the plane (x, y) (Fig. 2). The angular apertures of a more complex-shape field can be simulated in a similar way. Figure 3 shows fragments of simulated images of speckle structures in the plane (x, y) for the quasi-monochromatic wave field $\Delta k \approx 0$, which represent the intensity distribution of the speckle modulated field $I(x, y) = |E(x, y)|^2$.

2. Method for studying the wave field spatial coherence based on correlation analysis of the spatial distribution of its complex amplitude

Minimal transverse $\varepsilon_\perp$ and longitudinal $\varepsilon_\parallel$ sizes of instantaneous speckles are defined, accordingly, by the lengths of transverse $\rho_\perp$ and longitudinal L_c spatial coherence of the wavefield [33]. The size of the speckles coincides with the size of the cross-correlation region of the scattered field, since the amplitude and phase of the field remain approximately unchanged within a single speckle. The transverse coherence length $\rho_\perp$ is determined by the average wavelength λ_0 of frequency spectrum and the width of angular spectrum $2\theta_i$ of the wave field [7,9]. The dependence $\rho_\perp$ on the frequency spectrum width is manifested only with a sufficiently large width of frequency spectrum [32,34] and a small numerical aperture NA_i. The magnitude of transverse correlation of the wave field with the square aperture of angular spectrum is determined by the formula [32,34]:

$$\rho_\perp \approx \frac{\lambda_0^2}{n_1(2\lambda_0 + \Delta\lambda)\sin(\theta_i)} \approx \frac{\lambda_0}{2n_1 \sin(\theta_i)} = \frac{\lambda_0}{2NA_i}, \quad (4)$$

where $\Delta\lambda$ — frequency spectrum width in the wavelength scale. For the length of the transverse correlation of a monochromatic wave field with a circular aperture of angular spectrum, we may write the expression [32,34]:

$$\rho_\perp \approx 1.22\frac{\lambda_0}{2n_1 \sin(\theta_i)} = 1.22\frac{\lambda_0}{2NA_i}, \quad (5)$$

Minimal longitudinal size of instantaneous speckles $\varepsilon_\parallel$, and, hence, longitudinal length of coherence L_c are dependent on both, the width of frequency spectrum and width of angular spectrum of field [32,34]:

$$L_c \approx \left[\frac{\Delta\lambda}{\lambda_0^2}n_1 + \frac{2n_1}{\lambda_0}\sin^2\left(\frac{\theta_i}{2}\right)\right]^{-1} \approx \left[\frac{1}{l_c} + \frac{1}{\rho_\parallel}\right]^{-1}, \quad (6)$$

where l_c — length of the wavefield temporal coherence in the medium with refractive index of n_1

$$l_c \approx \upsilon\tau_c \approx \frac{\lambda_0^2}{n_1\Delta\lambda} = \frac{l_{c\upsilon}}{n_1}, \quad (7)$$

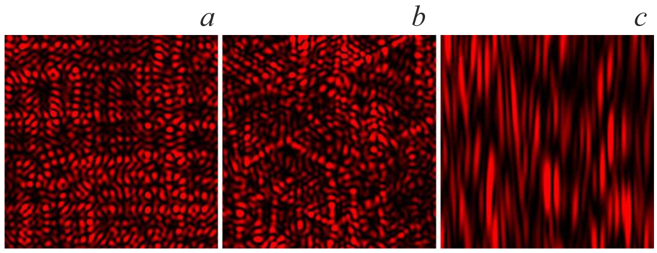

Figure 3. Fragments of speckle-modulated diffraction structures modulated in the numerical experiment $\lambda_0 = 0.63\,\mu$m, $\Delta k \approx 0$, with the aperture of the field angular spectrum: a — circular square, b — circular triangular, c — rectangular.

υ — light speed in the medium, τ_c — time of coherence, $l_{c\upsilon}$ — length of wavefield temporal coherence in vacuum, $\rho_{\parallel}$ — length of the wavefield longitudinal coherence in the medium, depending only on the width of angular spectrum:

$$\rho_{\parallel} \approx \frac{\lambda_0}{2n_1 \sin^2(\theta_i/2)} = \frac{\lambda_0}{n_1 - \sqrt{n_1^2 - NA_i^2}}. \quad (8)$$

The correlation properties of a speckle-modulated field, as a kind of spatial random field, can be established using an autocorrelation function according to the autocorrelation theorem [5,10]. To find the function of autocorrelation $\Gamma(\Delta x, \Delta y)$ of the wave field $E(x, y)$ in x, y plane the following expression may be written

$$\begin{aligned} \mathbf{F}\{\Gamma(\Delta x, \Delta y)\} &= \mathbf{F}\left\{\iint E(x, y)E^*(x - \Delta x, y - \Delta y)dxdy\right\} \\ &= \mathbf{F}\{E(x, y)\} \cdot [\mathbf{F}\{E(x, y)\}]^* = |\mathbf{F}\{E(x, y)\}|^2 \\ &= |E(k_x, k_y)|^2 = I(k_x, k_y), \end{aligned} \quad (9)$$

where $\Gamma(\Delta x, \Delta y) = \iint[E(x, y)E^*(x - \Delta x, y - \Delta y)dxdy$ — an autocorrelation function of complex perturbations $E(x, y)$, $E(k_x, k_y)$ — a complex spatial frequency spectrum of the perturbation field, $I(k_x, k_y)$ — spatial frequency spectrum in terms of intensity (power), $\mathbf{F}$ — forward Fourier transformation, $*$ — complex conjugation sign.

To both sides of the equation (9) apply the inverse Fourier transformation

$$\begin{aligned} \mathbf{F}^{-1}\{\mathbf{F}\{\Gamma(\Delta x, \Delta y)\}\} &= \mathbf{F}^{-1}\left\{\mathbf{F}\{E(x, y)\} \cdot [\mathbf{F}\{E(x, y)\}]^*\right\} \\ &= \mathbf{F}^{-1}\left\{|\mathbf{F}\{E(x, y)\}|^2\right\}, \end{aligned} \quad (10)$$

from where we get the formula for calculating the autocorrelation function from the calculated field of complex amplitudes $E(x, y)$:

$$\begin{aligned} \Gamma(\Delta x, \Delta y) &= \mathbf{F}^{-1}\left\{\mathbf{F}\{E(x, y)\} \cdot [\mathbf{F}\{E(x, y)\}]^*\right\} \\ &= \mathbf{F}^{-1}\left\{|\mathbf{F}\{E(x, y)\}|^2\right\} = \mathbf{F}^{-1}\left\{|E(k_x, k_y)|^2\right\} \\ &= \mathbf{F}^{-1}\{I(k_x, k_y)\}, \end{aligned} \quad (11)$$

where

$$\mathbf{F}^{-1}\{I(k_x, k_y)\} = \iint I(k_x, k_y) \exp[-i(k_x\Delta x + k_y\Delta y)]dk_xdk_x.$$

The expression (11), written in shortened form, is the Wiener-Khinchin theorem [6]:

$$\Gamma(\Delta x, \Delta y) = \iint I(k_x, k_y) \exp[-i(k_x\Delta x + k_y\Delta y)]dk_xdk_x. \quad (12)$$

The function (11) of the wave field autocorrelation in XY plane passing through the pointz $= 0$, at a moment of time $t = 0$, may be written in discrete form

$$\begin{aligned} &\Gamma(\Delta x, \Delta y, z = 0, t = 0) \\ &= \sum_{x=x_1}^{x_2}\sum_{y=y_1}^{y_2} E(x, y, z = 0, t = 0) \\ &\times E^*(\Delta x - x, \Delta y - y, z = 0, t = 0) \\ &= \mathbf{F}^{-1}\Big\{\mathbf{F}(E(x, y, z = 0, t = 0)) \\ &\times [\mathbf{F}(E(x, y, z = 0, t = 0))]^*\Big\}. \end{aligned} \quad (13)$$

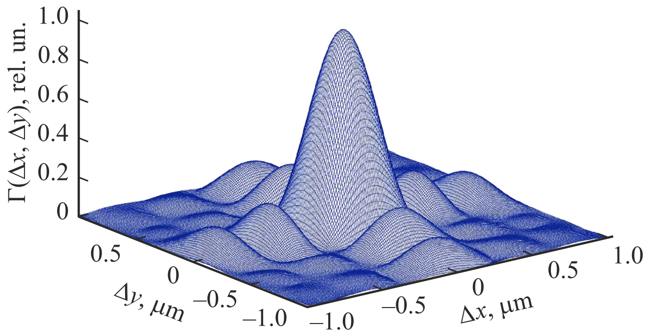

Figure 4. Autocorrelation function of the complex amplitude of wave field in cross-section; $\lambda_0 = 0.55\,\mu$m, $\Delta k \approx 0$, square aperture of the field angular spectrum, numerical aperture $NA_i = 0.5$.

Formula (13) was used in numerical calculations of the autocorrelation function of a speckle-modulated wave field. The normalized three-dimensional autocorrelation function of the speckle field, the aperture function of the angular spectrum of which has the shape of a square ring found from the formula (13), is shown in Fig. 4.

In full-scale and numerical experiments, the transverse sizes of speckles and, consequently, the lengths of transverse $\rho_{\perp}$ and longitudinal $L_c$spatial coherence of the wave field can be determined from the width of the speckle field autocorrelation function, as half the width of the function central maximum. The use of fast Fourier transform algorithm in numerical experiment to calculate the autocorrelation function significantly reduces the calculation time of the wave field correlation parameters.

3. Study of cross-correlation of a quasi-monochromatic wave field with a wide angular spectrum

Section 3 outlines the findings of numerical modeling of correlation properties of a wave field in its cross-section. The cross-sectional field of complex amplitudes $E(x, y)$ simulated in a numerical experiment, according to formula (3), was formed as a superposition of plane waves with various spatial frequencies k_x, k_y within the spatial spectrum of a field with a given angular numerical aperture NA and random initial phases $\varphi_0(k_x, k_y)$ in the range

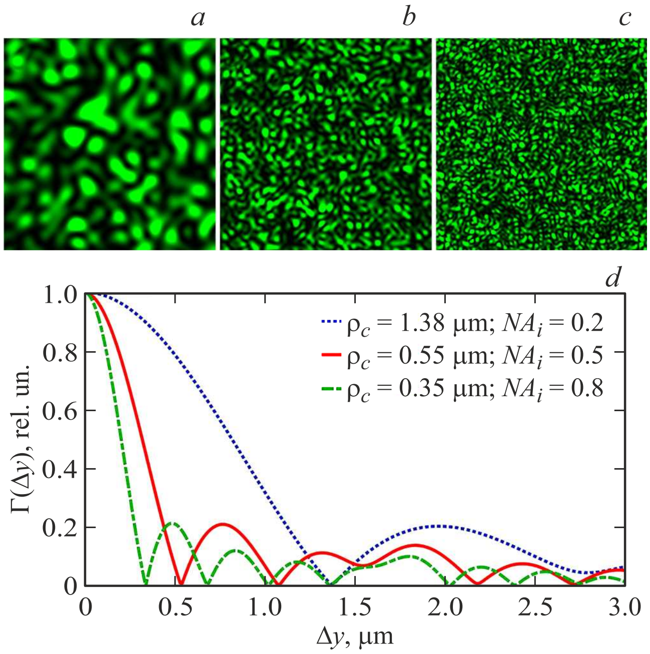

Figure 5. Modelled speckle patterns of wavefield in cross-section for the square aperture of angular spectrum with numerical apertureNA_i, equal: a — 0.2, b — 0.5, c — 0.8; size of fragments of speckle patterns $20 \times 20\,\mu$m. Functions of field transverse correlation (d): ρ_c — length of field transverse correlation (coordinate of the first minimum $\Gamma(\Delta y)$), $\lambda_0 = 0.55\,\mu$m, $\Delta k \approx 0$.

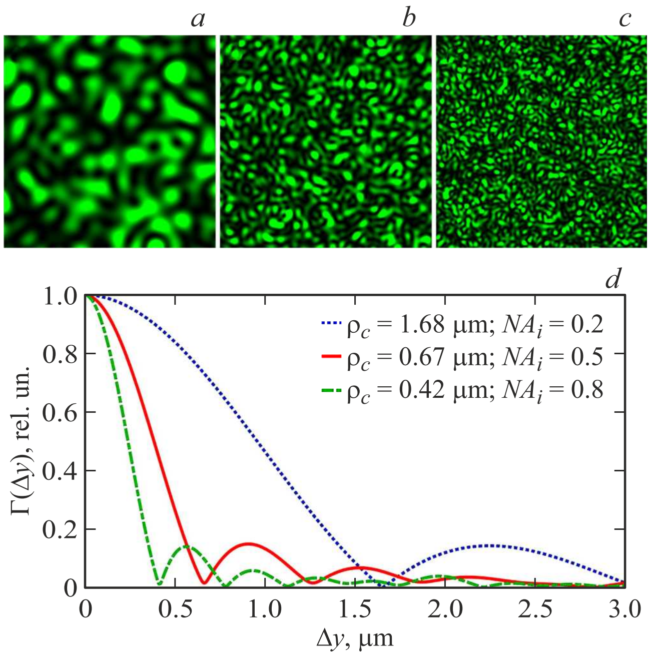

Figure 6. Modelled speckle patterns of wavefield in cross-section for round aperture of angular spectrum with numerical aperture NA_i, equal: a — 0.2, b — 0.5, c — 0.8; size of fragments of speckle patterns $20 \times 20\,\mu$m. Functions of field transverse correlation (d): ρ_c — length of field transverse correlation (coordinate of the first minimum $\Gamma(\Delta y)$), $\lambda_0 = 0.55\,\mu$m, $\Delta k \approx 0$.

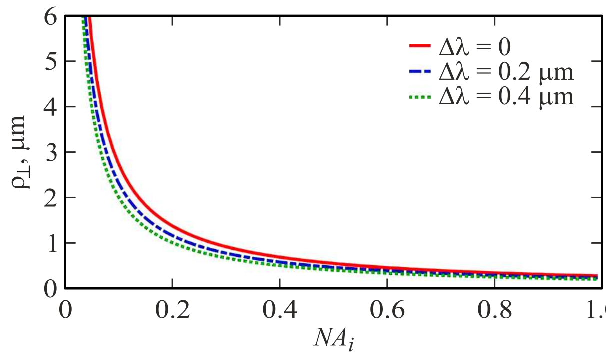

Figure 7. Coherence transverse length versus various frequency spectrum widths.

from 0 to 2π radians. The fragments of modelled speckle-modulated patterns are given in Fig. 5, $a-c$ and 6, $a-c$.

The wave field cross-correlation function was calculated using the formula (11). In the numerical experiment, the effect of numerical aperture on the magnitude of field correlation was examined. Figures 5, d and 6, d show the normalized cross-correlation functions of the simulated wave field with a numerical aperture having a different magnitude and shape.

Figure 7 shows curves of the transverse coherence length versus numerical aperture $\rho_\perp(\sin\theta) = \rho_\perp(NA_i)$, plotted using formula (4), for different widths of the frequency spectrum in the wavelength scale $\Delta\lambda$.

The coherence lengths ρ_c, determined from the curves of coherence function $\Gamma(\Delta y)$ (Fig. 5, d and 6, d), correspond to the theoretical values with high accuracy in the numerical experiment, defined by the formula (4). The accuracy of determining ρ_c depends on the ratio between the size B_y of the analyzed field and the size of spatial fluctuations — field speckles, $\varepsilon_y \approx \rho_c$. The spread $\Delta\rho_c$ of values ρ_c obtained for different field realizations decreases with increasing ratio B_y/ε_y, since in this case the efficiency of the averaging operation used to calculate the correlation function of $\Gamma(\Delta y)$ field goes up (Fig. 6, d). Thus, at relatively large numerical field aperture $NA_i = 0.2$ we have large speckles, $\rho_c \approx 1.68\,\mu$m, and we get $\Delta\rho_c \approx 0.09\,\mu$m, whereas at $NA_i = 0.8$ small speckles are obtained, $\rho_c \approx 0.42\,\mu$m, and the spread of $\Delta\rho_c \approx 0.02\,\mu$m with the same size B_y of the analyzed field.

4. Study of cross-correlation of a quasi-monochromatic wave field with a wide angular spectrum at various intervals $\Delta\varphi$ of random initial phases

In this paper, a numerical experiment was performed to study the formation of speckle structures in the cross-section of a circular aperture wave field with a gradual increase of random initial phases $\Delta\varphi$ of plane waves, from the totality

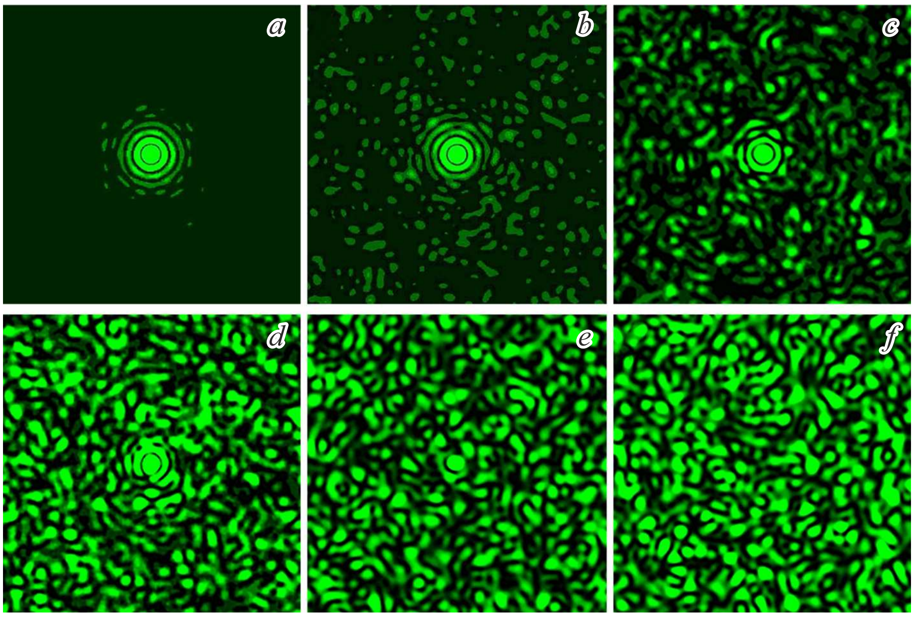

Figure 8. Fragments of modelled speckle patterns of wavefield in cross-section for round aperture; interval of random initial phases $\Delta\varphi$ and brightness of the speckle pattern: a — 0.4π, $+95\,\%$; b — 0.8π, $+95\,\%$; c — 1.2π, $+95\,\%$; d — 1.4π, $+95\,\%$; e — 1.8π, $+60\,\%$; f — 2π, $+30\,\%$; $\lambda_0 = 0.55\,\mu$m, $\Delta k \approx 0$, round shape of the angular spectrum aperture with $NA_i = 0.5$, size of speckle pattern fragments — $20 \times 20\,\mu$m.

of which the wave field is formed. Fragments of the formed speckle structures at various intervals of $\Delta\varphi$ random initial phases are shown in Fig. 8.

As can be seen in the Figure, the mature speckle field is formed in the interval of random initial phases distribution $\Delta\varphi = 2\pi$ rad (Fig. 8, f), and in case of small interval $\Delta\varphi$ the interference zero order maximum is clearly observed (Fig. 8, a–c). With a further increase in the interval$\Delta\varphi$, a gradual decrease of maximum intensity in the center of the pattern is observed (Fig. 8, d, e), as well as transition to a mature speckle pattern at the interval $\Delta\varphi = 2\pi$ rad. Figure 9 shows the normalized cross-correlation functions of the simulated wave fields along the coordinate Δy, respectively, for fragments of speckle patterns, which are shown in Figure 8. As can be seen from the graphs, the cross-correlation lengths of the field ρ_c (coordinates of the first minimum $\Gamma(\Delta y)$) practically coincide for all intervals of random initial phases of plane waves $\Delta\varphi$.

The results of numerical modelling show that the transverse coherence length of the quasi-monochromatic wave field obtained by superposition of plane waves with different random phases $\Delta\varphi$ does not depend on the phase variation interval — phase dispersion, but depends on the width and shape of the wave field angular spectrum.

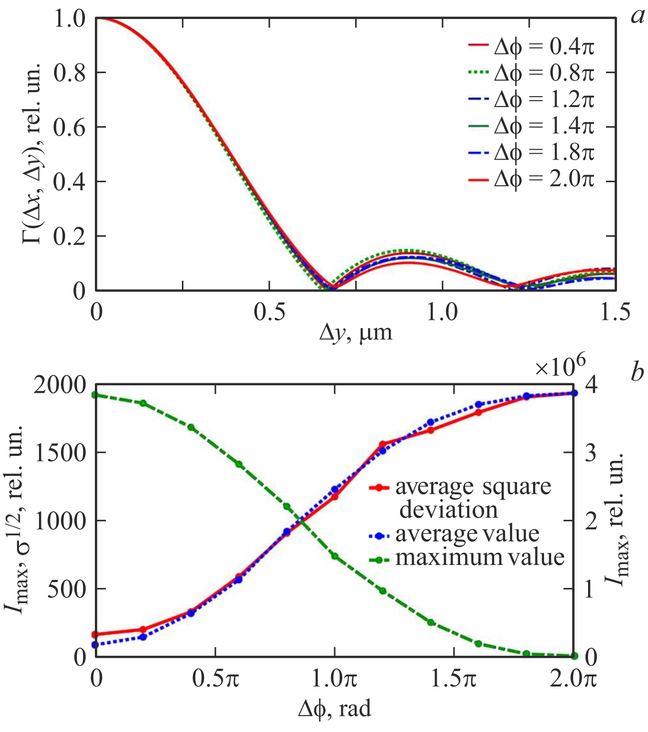

Figure 9. Correlation degree of the complex wave field amplitude in cross-section (a); statistical characteristics of the wave field intensity distribution in cross-section (b); $\Delta\varphi$ — interval of random initial phases of the field harmonic components, $\lambda_0 = 0.55\,\mu$m, $\Delta k \approx 0$, round-shaped aperture of the angular spectrum $NA_i = 0.5$.

Conclusion

The paper describes the testing of the proposed method for studying the spatial correlation of a stochastic wave field having wide angular spectrum based on a correlation analysis of the spatial distribution realization for the complex amplitude of this wave field, formed (computed) using numerical modeling. The transverse correlation properties of a monochromatic field found based on correlation analysis of the simulated distributions of the complex amplitude of this field compared with the properties obtained on the basis of analytical formulas showed their very good consistency. Using the proposed method, it is demonstrated that the transverse spatial coherence — the shape of the coherence function and coherence length of the quasi-monochromatic wave field — is determined by the magnitude of the numerical aperture and the shape of its angular spectrum.

The study includes numerical modeling of distribution of the complex amplitude of a quasi-monochromatic wave field in its cross-section with an interval of random initial phases of the field harmonic components in the range from 0 to 2π radians. Studies have shown that the length of transverse correlation of field fluctuations depends on the width and shape of the wave field angular spectrum and does not change with variation of the difference interval of initial phases of the field spatial harmonic components.

Studies have demonstrated that numerical modeling is a tool that to a great extent corresponds to the real processes of speckle structures formation and their spatial spectra, and is a good addition to the real experiment in ongoing research [35,36]. This circumstance is an important factor, since when implementing a full-scale experiment, certain technical difficulties may arise during preparing and implementing the experiment. Studies have proven the effectiveness and prospects of numerical methods for the formation of optical speckle-modulated wave fields and diffraction structures and examination of spatial correlation properties of a stochastic wave field having wide angular spectrum for further development and use of the proposed numerical methods as an alternative and augmentation of a full-scale experiment.

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Conflict of interest

The authors declare that they have no conflict of interest.

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