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Design and simulation of a multichannel DOE for aberration analysis with increased diffraction efficiency

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The efficient methods for calculating complex transmission function of a phase diffractive optical element (DOE) with a given multichannel intensity distribution have been designed. Based on numerical modeling, testing of the multichannel phase DOE designed for detecting wavefront aberrations, has been carried out. Numerical modeling was used to estimate the diffraction efficiency and root-mean-square error characteristics of DOEs. The possibility of using the partial coding algorithm to design the DOE matched with 25 and 49 basis functions has been shown. The smallest error of 15 % is achieved with diffraction efficiency values of 52 %. The maximum diffraction efficiency of 85 % using the proposed algorithm can be achieved with the error value equals to 31 %.

Keywords: diffractive optical element, multichannel, wave aberrations, diffraction efficiency.

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Introduction

The computation of the phase diffractive optical element (DOE) proposed in this paper is relevant because of many reasons. First, to provide a possibility of using simple methods for their fabrication. It is known that the diffraction efficiency of the elements significantly depends on the accuracy of manufacturing the phase relief [1,2]. Secondly, there is a need to develop methods for calculating phase DOEs consistent with a linear combination of a finite modes number with specified weights. This is required to form a given amplitude-phase distribution in the beam cross-section based on an arbitrary variation of the mode composition and the weight contribution of each of the modes [3–5]. Thirdly, to ensure high energy efficiency based on the use of phase DOEs in optical circuits based on liquid crystal spatial light modulators and digital micromirror devices for experimental testing of numerical results.

Various methods of phase DOEs computation are known: geometric-optical approach based on the analytical solution of the eikonal equation and construction of a ray path from points on DOE surface to points in a given image [6,7]; direct search algorithms [8,9]; error diffusion [10], pseudorandom coding [11], composition method [12,13], differential evolution [14], and their various modifications and combinations [15]. At the same time, iterative methods can be considered the most versatile tool for computation of DOEs in various applications [16–23], although they do not guarantee convergence to a global minimum and require repeated use of the direct and reverse operators at each iteration, which leads to significant time and computational costs.

In the framework of this paper, the partial encoding method is considered, which is itself a non-iterative and fairly fast digital holography algorithm[24–27], designed for use with spatial light modulators. The method is designed to replace the amplitude-phase transmission function by the phase modulation function, taking into account the replacement of part of the counts according to a certain rule, depending on the amplitude value. This approach ensures that some of the amplitude information about the encoded aberration is preserved and allows getting increased diffraction efficiency.

1. Theoretical fundamentals

The partial encoding method [26], focused on applications with spatial light modulators, is expressed as follows:

$$\tilde{g}(x, y) = \begin{cases} \exp\{i \arg[g(x, y)]\}, & |g(x, y)| \geq \alpha, \\ \exp\{i \arg[g(x, y)] + i\mu\}, & |g(x, y)| < \alpha, \end{cases} \quad (1)$$

$$\mu = \begin{cases} \pi, & \mathrm{sgn}(S_{ij}) > 0, \\ 0, & \mathrm{sgn}(S_{ij}) < 0 \end{cases} \quad (2)$$

$$S_{ij} \in [-0.5; 0.5], \quad (3)$$

where $g(x, y)$ — initial amplitude-phase transmission function; α — parameter corresponding to the amplitude threshold value where a phase jump will be added to the point; μ — magnitude of the phase jump; S_{ij} — pseudo-random value the sign of which impacts the value of the phase jump; $\tilde{g}(x, y)$ — calculated phase transmission function.

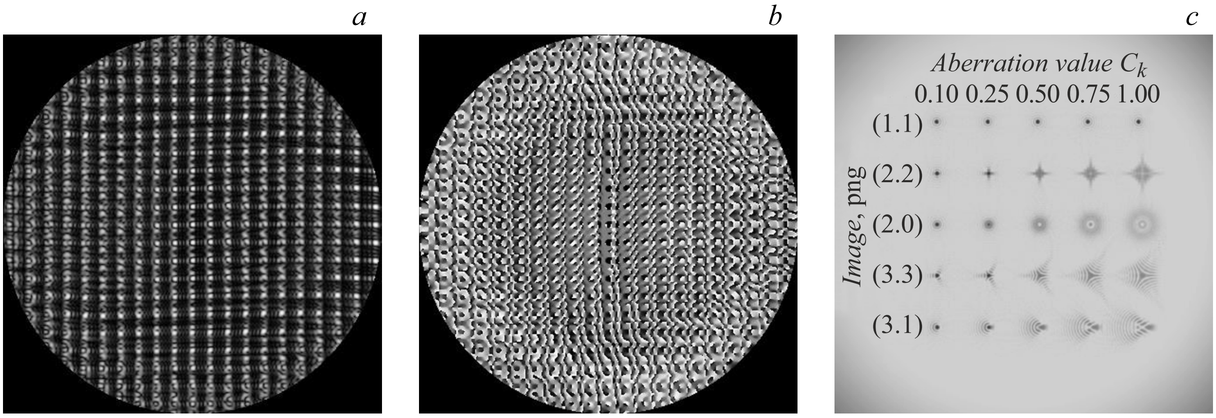

Figure 1. Amplitude (a), phase (b) of the amplitude-phase DOE (7) and its action; c — intensity in the focal plane (4).

To calculate the diffraction efficiency and the error of diffraction pattern formation using a multichannel DOE, it is necessary to simulate its effect on the analyzed aberrated wavefront. In the framework of this work, the focal plane is considered as the resulting field formation plane. The diffraction calculation is based on Fourier transformation:

$$F(u,\nu)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\exp[-i\kappa(ux+\nu y)]dxdy =\Im[f(x,y)], \quad (4)$$

where $\kappa = 2\pi/\lambda$ — wavenumber, λ — wavelength.

The diffraction efficiency is the ratio of the intensity $F(u,\nu)=\Im\left[\tilde{g}(x,y)\right]$ obtained in the focal plane of the calculated phase DOE to the intensity $F_0(u,\nu)=\Im\left[g(x,y)\right]$ obtained in the focal plane of the amplitude-phase DOE:

$$\varepsilon=\left(\iint_{\Omega}|F(u,\nu)|^2\mathrm{d}u\mathrm{d}\nu\right)\left(\iint_{\Omega}|F_0(u,\nu)|^2\mathrm{d}u\mathrm{d}\nu\right)^{-1}, \quad (5)$$

where Ω — near diffraction orders integration domain Diffraction efficiency is usually measured by the total integral intensity. However, given the specifics of multichannel DOEs [28–30], the intensity between the diffraction orders is either close to zero if the diffraction orders are separated from each other by a considerable distance; or is noise itself and does not participate in the transmission of a useful signal. Therefore, the computation of diffraction efficiency for a multichannel DOE consistent with aberrations is carried out only in the near diffraction orders integration domain.

Achieving high diffraction efficiency can lead to loss of useful information due to the replacement of a large number of DOE samples with an additional phase jump. Therefore, it is necessary to calculate and track the error in forming the intensity pattern in the focal plane of DOE:

$$\delta=\left(\iint_{\Omega}\left(|F(u,\nu)|^2-|F_0(u,\nu)|^2\right)^2\mathrm{d}u\mathrm{d}\nu\right)^{1/2} \times\left(\iint_{\Omega}|F_0(u,\nu)|^4\mathrm{d}u\mathrm{d}\nu\right)^{-1/2}. \quad (6)$$

2. Numerical modeling

Let's consider the amplitude-phase complex transmission function for a multi-channel DOE consistent with the wave aberrations Z_{pq} with the value C_k, expressed as

$$g(x,y)=\sum_{p=0}^{P}\sum_{q=p_0}^{p}\sum_{k=1}^{K}\exp[-i\kappa C_k Z_{pq}(x,y)] \times\exp[i(a_{kpq}x+b_{kpq}y)], \quad (7)$$

where indices (p,q) stand for the type of encoded aberration $Z_{p,q}$, which in this paper correspond to Zernike polynomials [30,31]; index k stands for the magnitude of aberration defined by coefficient C_k; α_{kpq}, β_{kpq} — spatial carrier frequencies directing the corresponding aberrated wavefronts into different diffraction orders. The presented Zernike polynomials can be easily correlated with classical aberrations. The classification of aberrations by order is given in accordance with the OSA standard [32]. To describe most of the aberrations, Zernike polynomials of the 4th degree are usually sufficient for smooth optics. The greatest contribution to the turbulent degradation of the image is made by lower-order aberrations — these are the slopes of the optical radiation wavefront, followed by the aberrations of defocusing and coma, astigmatism, the contribution of other aberrations is significantly less to the turbulent blurring of the image [33].

Let's calculate the amplitude-phase complex transmission function of the 25-channel DOE (7), which is consistent with five types of aberration of various magnitudes (5 types

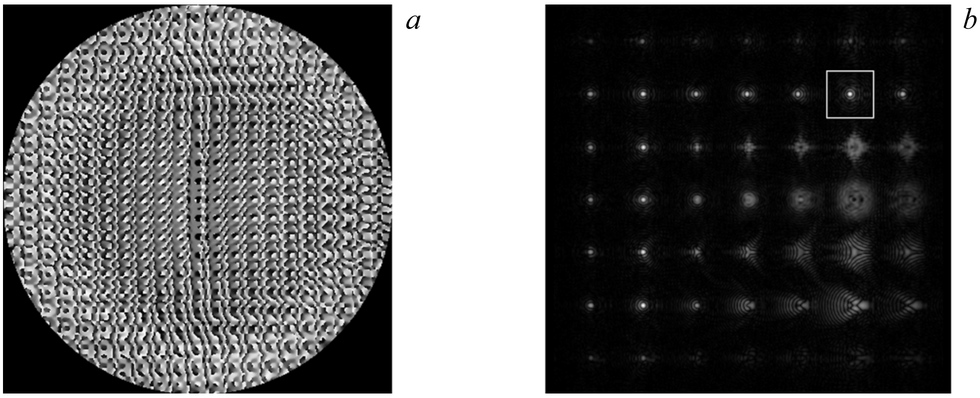

Figure 2. Phase (a) of the phase DOE calculated using algorithm (1)–(3) at $\alpha = 0$ and its action; b — intensity in focal plane (4).

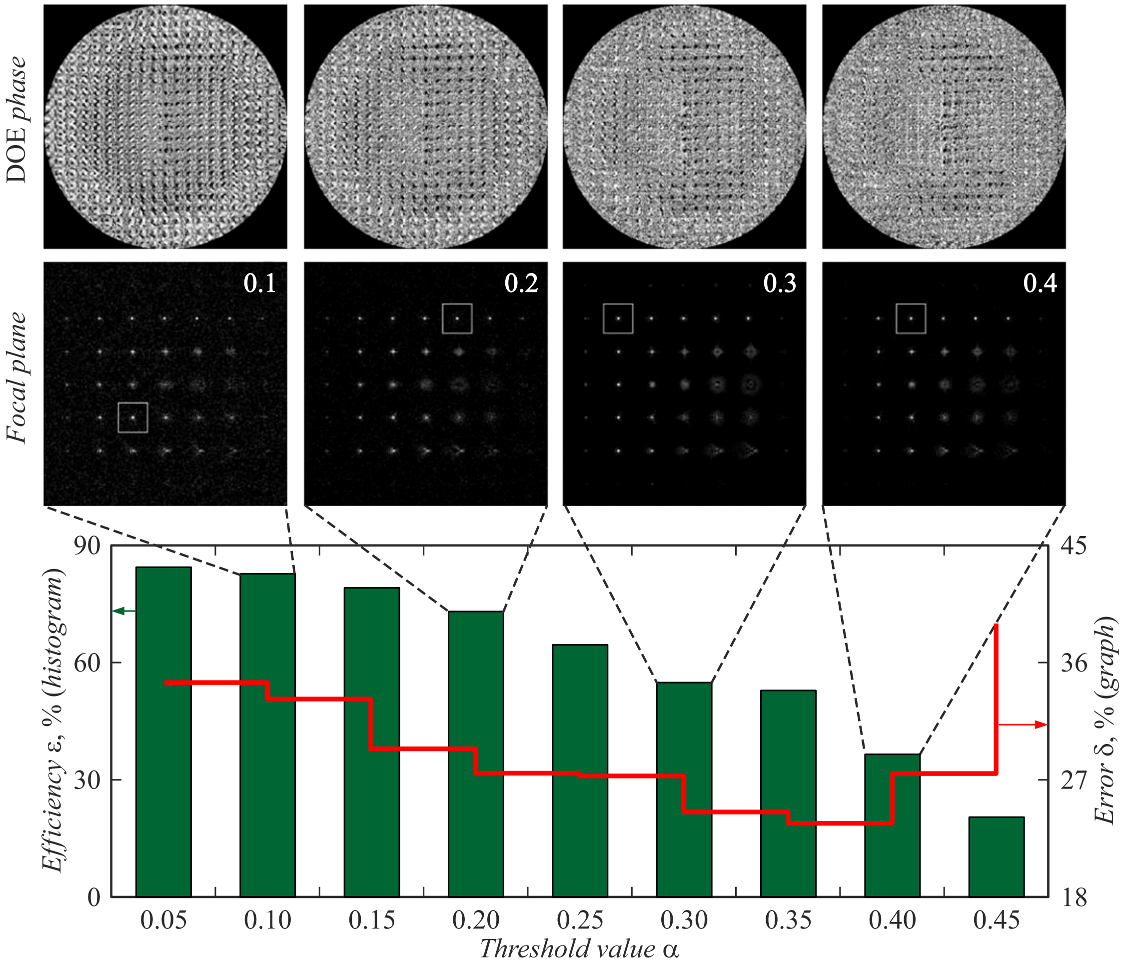

Figure 3. Phase of DOE calculated using (1)–(3) at various α, its action (intensity in the lens focal plane), consistency between the diffraction efficiency and error.

of aberration magnitudes in the range from 0.1 to 1). The single-index record of Zernike polynomials is the following relation: $t = (p(p+2)+q)/2$. The paper outlines Zernike functions corresponding to t for $p_0 \leq q \leq p$; where $p_0 = 0$, if p — even; $p_0 = 1$, if p — odd. The results of numerical simulation are shown in Fig. 1.

To calculate the phase DOE we may use algorithm (1)–(3), by varying the value of parameter α in the range from 0 to 0.5. The complex transmission function of the phase DOE for $\alpha = 0$ was calculated, which is equivalent to the „kinoforma" computation method which corresponds to replacing the calculated amplitude distribution in the

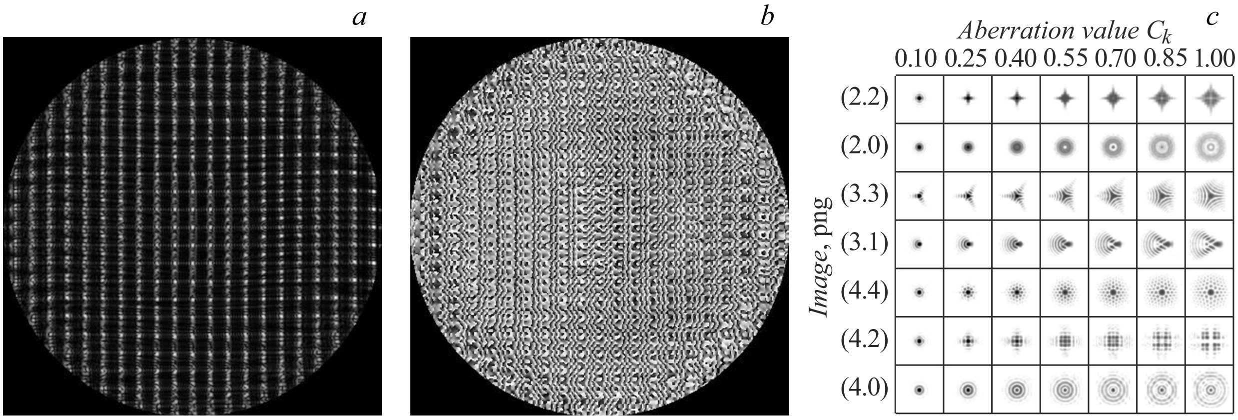

Figure 4. Amplitude (a), phase (b) of the amplitude-phase DOE (7) and its action; c — intensity in the focal plane (4).

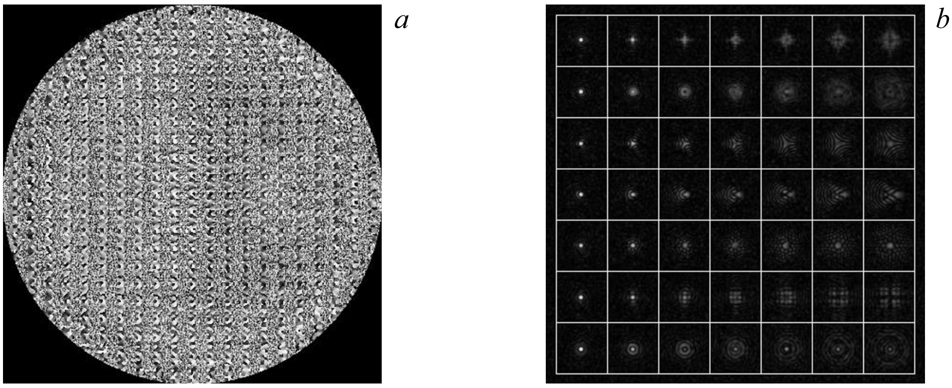

Figure 5. Phase (a) of the phase DOE calculated using algorithm (1)−(3) at $\alpha = 1/\pi$ and its action; b — intensity in focal plane (4).

hologram plane with the amplitude distribution of the illuminating beam. This term is associated with various kinoform, i.e. diffractive phase elements [34–37]. It was found that diffraction efficiency (5) makes $\varepsilon = 85.23$, however, the error (6) exceeds $\delta = 31.25$. Figure 2 shows the phase of the calculated DOE and its effect (intensity distribution in the focal plane; the frame highlights the area of integration of the diffraction order (p, q, k) with maximum intensity).

Within a number of numerical experiments the complex transmission function of a phase DOE was calculated at various $\alpha \neq 0$. Figure 3 shows DOE phases and their effects (intensity distribution in the focal plane of the lens); the corresponding efficiency values ε shown on the histogram; and the error shown on the graph. It was demonstrated that the highest diffraction efficiency in 85 % was reached at $\alpha = 0.05$, however the error makes 31 %. Further increase of parameter α to 0.35 leads to a decrease in diffraction efficiency and error. Minimal error was 15 % at $\alpha = 0.31$, while diffraction efficiency error was 52 %. A further increase in parameter α results in subsequent decline of diffraction efficiency to 20 % with a simultaneous increase in error to 42 %, which is associated with sampling problems (a larger number of pixels is required, exceeding the standard dimensions of spatial light modulator matrices).

Thus, optimal parameters for the partial coding method have been determined in relation to multi-channel elements consistent with several types of aberrations of various magnitudes. With the parameter value $\alpha = 1/\pi$, it is possible to calculate multi-channel DOE with minimal error and with increased diffraction efficiency.

Let's increase the number of DOE channels to (7) to 49, i.e. 7 types of aberrations with 7 different weights are encoded in the complex transmission function. The results of numerical modeling are shown in Fig. 4.

Let's calculate the phase DOE with an optimal ratio of diffraction efficiency and error. Fig. 5 illustrates the numerical modeling results.

Based on the results obtained, the diffraction efficiency and error were calculated. It was found that an error in 17 % is achieved at the values of diffraction efficiency near 50 %, which is fully corresponds to the optimal ratio.

Conclusion

The computation of a phase multichannel DOE intended for use in optical circuits based on liquid crystal spatial light modulators is carried out. To increase the diffraction efficiency of DOE, partial coding methods are considered, which are a more general case and cover not only the class of focusing DOE, but also various mode beam shapers, as well as spatial filters for decomposing the light field. Based on the considered partial coding algorithm, phase multichannel DOE has been developed, consistent with the 25 and 49 basic functions in the form of wavefront aberrations. It was found that the lowest error in 15% is reached at indicators of diffraction efficiency near 52%. This type of DOE can be used for adaptive compensation of wavefront distortions [38,39]. The maximum diffraction efficiency of 85,% can be achieved based on the developed algorithm with error values equal to 31%. In this case, the calculated DOE can be used as a consistent filter for the detection of pronounced types of aberrations by the presence of bright correlation peaks [40,41].

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Conflict of interest

The authors declare that they have no conflict of interest.

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