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Quantum fluctuations in mode-locked fiber lasers

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Calculation of parameters of quantum fluctuations of pulses generated in fiber lasers with mode synchronization or in nonlinear fiber ring resonators is presented. Quantum fluctuations are associated with fluctuations in the amplitudes and phases of longitudinal modes. Using standard mode oscillator quantization, expressions are recorded for sources of amplitude and phase fluctuations for each longitudinal mode. In this case, phase fluctuations are calculated by amplitude using Heisenberg relations. It was assumed that the fluctuations in the amplitudes of the modes obey the Poisson distribution, and the sources of fluctuations in the time-domain are obtained using the inverse discrete Fourier transform. For a pulse of $sech(t)^2$, fluctuations in quantum number, pulse maximum time, chirp and phase are calculated.

Keywords: Mode locking, quantum fluctuations, spectrum of longitudinal modes, $sech^2$--pulse, pulse parameters fluctuations.

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Introduction

Quantum fluctuations in lasers set the limiting characteristics of their radiation and largely determine the limits of their applicability [1]. For single-frequency lasers, quantum fluctuations determine the width of the laser radiation line and the level of intensity fluctuations. The calculation of these parameters based on the quantum laser model for these types of lasers is well known [1–3]. The equations for the photon creation and annihilation operators are usually solved by replacing the operators with matrices in a certain basis or using a density matrix [1]. At the same time, the number of quanta in the field is small, since the modes near the generation threshold are considered. For lasers generating pulses with a number of quanta of the order 10^8, the dimension of the matrices is extremely high, which makes it impossible to use numerical methods. In this case, various versions of perturbation theory are usually used: the field is represented as a classical part and a relatively small part responsible for quantum fluctuations, and linear equations are obtained for quantum operators, which, however, are also not very easy to solve [4].

Generation of ultrashort light pulses is possible using certain optical-physical circuits that lead to synchronization of longitudinal modes in a fiber laser. The most common is a circuit in which an annular fiber resonator has an amplification section, a section for controlling the total dispersion of the resonator, and a nonlinear device where, due to the nonlinear rotation of the polarization ellipse, the pulse is compressed in the time domain. Such optical-physical configuration was called a laser with additive modes synchronization [5–10]. In addition to such a configuration, short light pulses can be generated, for example, in a ring fiber resonator, which is excited by a laser with constant intensity [11]. In optical fiber, due to weak nonlinearity and dispersion, modulation instability is possible, resulting in modulation components, and the signal of constant intensity is divided into a sequence of soliton pulses. The ring resonator makes it possible to increase the effective distance that the pulses travel in the optical fiber. The modulation frequency for the parameters of a conventional single-mode SMF fiber is in THz- range, and the beat frequency of the longitudinal modes in a ring resonator with a length of several meters is tens and hundreds of MHz. That is, harmonic (or multiple) mode synchronization is observed when the harmonic of the beat signal coincides with the modulation frequency. The beat frequency of longitudinal modes is equal $2\pi/T$, were T — resonator bypass time. Mode synchronization occurs when conditions for the frequencies and phases of the longitudinal modes are met:

$$\nu_{n-1} + \nu_{n+1} = 2\nu_n, \quad \varphi_{n-1} + \varphi_{n+1} = 2\varphi_n,$$

where n — number of the mode. These conditions mean that the frequencies and phases of the modes are linear functions of n.

To meet the conditions, nonlinear control of the active medium dispersion and the presence of four-frequency interaction of modes $\nu_{n\pm1}$ and ν_n is necessary. In the process of the four-frequency interaction the number of quanta in n-th mode is diminished twofold, however, one quant appears in each of the modes numbered $n-1$ and $n+1$. Due to non-equidistance of the modes during four-frequency interaction, a field (combination tone) arises at a frequency near the mode with a frequency of ν_{n+1}. The interaction of fields leads to capture of the frequency of this mode by the frequency $2\nu_n - \nu_{n-1}$, which leads to the equidistance of the spectrum of modes and to the capture of the phase difference.

Thus, the total field in a laser with a ring fiber resonator can be expressed as a sum over the longitudinal modes:

$$E(z,t) = \sum_n A_n \exp(i\varphi_n) e^{[i\nu_n t - i\beta(\nu_n)z]},$$

where $\beta(\nu)$ — constant of main transverse mode propagation, A_n — actual modes amplitudes, φ_n — modes phases. The frequencies of the longitudinal modes for a ring fiber resonator can be determined from the condition $\beta(\nu_n)L = 2\pi n$, where L is the perimeter of the resonator. Next, we use the expansion

$$\beta(\nu_{n+1} - \nu_n) = \beta_1\Omega + \frac{\beta_2}{2}\Omega^2,$$

where Ω — difference in frequencies of adjacent modes, $\beta_1 = \frac{\partial\beta}{\partial\nu} = 1/\upsilon_g$, $\beta_2 = \frac{\partial^2\beta(\nu)}{\partial\nu^2}$, υ_g — group velocity. It can be seen that the spectrum of modes is not equidistant and the resonator modes do not coincide with the harmonics of the field. The multi-mode frequency may be calculated as

$$\Omega = \frac{\sqrt{8\beta_2\pi n/L + \upsilon_g^2} - \upsilon_g}{4\beta_2}.$$

It should be noted that $2\pi n/L$ is a wavenumber of resonator modes wit the same length, but without fiber dispersion. However, if the harmonics fall within the resonance width of the modes $\frac{\upsilon_g(1-R)}{L\sqrt{R}}$, then the pulse spectrum practically coincides with the spectrum of the modes. The effect of optical fiber dispersion can be taken into account by calculating the dispersion parameters and the group velocity through the effective refractive index, for example, based on the results [12]. Fig. 1 illustrates the pulse of $sech$ with a duration of 0.5 ns (Fig. 1, *a*) and spectrum of pulses sequence with a repetition period of 10 ns (Fig. 1, *b*), that we'll consider coinciding with the longitudinal modes spectrum. The amplitudes of the modes are normalized so that the total number of photons per pulse is $N_p = \Sigma_{n=-\infty}^{\infty}|A_n|^2 = 10^6$.

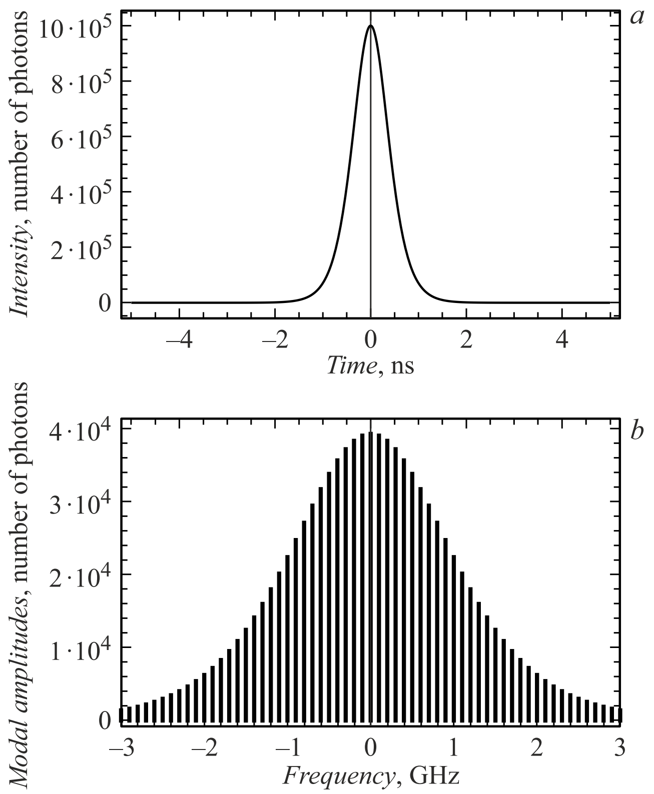

Figure 1. *a* — pulse like $sech^2(t/\tau)$ at $\tau = 0.5$ ns, *b* — pulse sequence spectrum Fig. 1, *a* with repetition period of 10 ns.

Quantum fluctuations of modes

Each resonator's mode is itself a field oscillator with a spatial dependence $\exp^{(i\beta(\omega)z)}$ and frequency ν_n, which is quantized in traditional way [1,2]. The field may be normalized so that $|A_n|^2$ is equal to the number of quanta, i.e. $A_n^2 = N_n$. Quantum fluctuations in the number of quanta in a mode obey Poisson statistics [1]:

$$P(n) = \frac{\langle N\rangle^n}{n!}e^{-\langle N\rangle},\ \langle N\rangle = N_n,\ \langle(N - \langle N\rangle)^2\rangle = N_n.$$

Quantum fluctuations in the number of quanta and quantum fluctuations in phases meet the condition $\Delta N\Delta\varphi \geq 1$, which follows from the conditions for the uncertainty of the energy and time of measurement $\Delta E\Delta t \geq \hbar$ [2]. If fluctuations sources in the modes are uncorrelated, then we can write an expression for the quantum fluctuations of the field:

$$\Delta E = \sum_n \left[\Delta A_n e^{i\varphi_n} + A_n e^{i\varphi_n}\frac{i}{\Delta N_n}\right]\exp[i\nu_n t - \beta(\nu_n)z]. \tag{1}$$

Then expression may be rearranged to the following form:

$$\Delta E = \sum_n A_n e^{i\varphi_n}\left[\frac{\Delta A_n}{A_n} + \frac{i}{\Delta N_n}\right]. \tag{2}$$

According to [13], the dependence of fluctuations on time can be expressed as a convolution

$$\Delta E(t) = \int_0^\infty d\xi E(t-\xi)Noise(\xi), \tag{3}$$

where $Noise(t)$ — quantum fluctuations sources function which is a Fourier transformation of $\frac{\Delta A_n}{A_n} + i\frac{\Delta\varphi_n}{A_n}$. Since $\Delta N_n = \Delta(A_n^2) = 2A_n\Delta A_n$, then, replacing A_n by $\sqrt{N_n}$, we obtain $\frac{\Delta A_n}{A_n} = \frac{\Delta N_n}{2\sqrt{N_n}}$. For a large number of quanta in the pulse, and therefore in each mode, phase fluctuations can be neglected. Since fluctuations of the mode amplitudes obey Poisson statistics, we can assume that the random numbers of quanta in each n- mode lie within $\pm3\sqrt{N_n}$ of the average value N_n. Fig. 2 shows the pulse corresponding to the average number of quanta in modes, as well as difference of fields$\Delta E_\pm$, corresponding to three standard deviations, i.e. pulses with spectra N_n, $3\sqrt{N_n}$, $-3\sqrt{N_n}$ for $N_p = 10^6$. The range of field changes is indicated by a gray fill. All realizations of the field's random value shall lie in the gray area. Therefore, we may assume that δE is equal to the averaged value $2\Delta E_p$.

It should be noted that peripheral modes with a small average number of quanta make a significant contribution to quantum fluctuations due to the root in the denominator. However, their contribution is largely masked by the contributions of the central modes, in which the relative fluctuations of the mode intensities are quite small, and the contribution to the pulse energy is significant.

It is clearly seen that ΔE repeats the shape of the pulse to a certain extent, since values ΔN_n for Poisson statistics fit into dispersion, which is proportional to $\sqrt{N_n}$.

It should be noted that when calculating field fluctuations, the four-frequency interactions of individual modes should also be taken into account. Each mode decreases in amplitude due to interaction, while the two neighboring modes increase by about half the change in intensity of the average mode each. This results in leveling the modes fluctuations and to decrease of ΔE approximately by two times. A rigorous analysis of fluctuations, taking into account the four-frequency interaction, requires solving a system of equations, the dimension of which in this case is — of the order of the ratio of the pulse repetition period to the pulse duration.

If we write down the pulse as

$$E(z,t) = \frac{n|c|^{1/2}}{2} sech\left(\frac{n^2|c|}{2}(t - t_0 - 2pz)\right) \times e^{i\theta + i\frac{n^2|c|}{4}z - ip^2 z + ip(t-t_0)},$$

where $n = \int |E(z,t)|^2 dt$, p — pulse per 1 photon, t_0 — pulse arrival time, θ — phase, c — fiber nonlinearity coefficient $\Delta E = f_1\Delta n + f_2\Delta p + f_3\Delta t_0 + f_4\Delta\theta$, where

$$f_1 = \frac{\partial E}{\partial n}, \quad f_2 = \frac{\partial E}{\partial p},$$

$$f_3 = \frac{\partial E}{\partial t_0}, \quad f_4 = \frac{\partial E}{\partial \theta}.$$

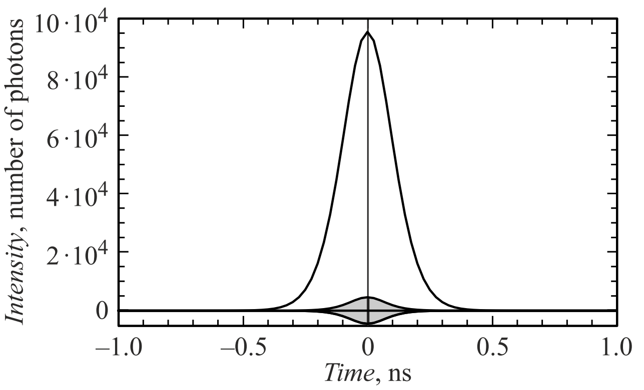

Figure 2. Pulse corresponding to the modes spectrum $\{N_n\}$ (Fig. 1, *b*) and reverse discrete Fourier-transformations $\pm 3\sqrt{N_n}$.

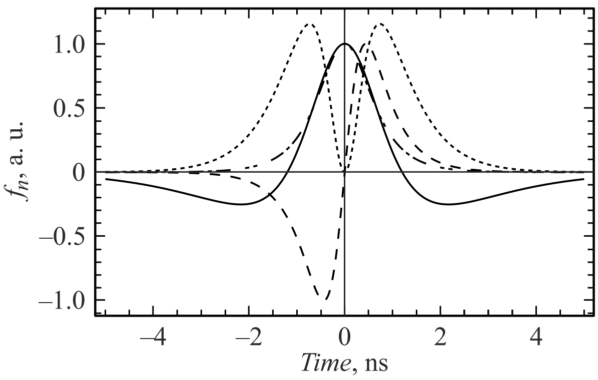

Figure 3. Functions f_1 (continuous curve), f_2 (dashed curve, long dashes), f_3 (dashed curve, short dashes), f_4 (dashed curve, dashes of different length).

Functions $f_1, \ldots, f_4$ are shown in Fig. 3. It is easy to check that these functions are orthogonal, i.e. $\int_{-\infty}^{\infty} dt f_i^*(t) f_j(t) = N_i \delta_{i,j}$, $i, j = 1\ldots4$. Using orthogonality, we obtain:

$$\Delta n = \int_{-\infty}^{\infty} dt \Delta E(t) f_1^*(t) \left[\int_{-\infty}^{\infty} dt f_1^*(t) f_1(t)\right]^{-1},$$

$$\Delta p = \int_{-\infty}^{\infty} dt \Delta E(t) f_2^*(t) \left[\int_{-\infty}^{\infty} dt f_2^*(t) f_2(t)\right]^{-1},$$

$$\Delta t_0 = \int_{-\infty}^{\infty} dt \Delta E(t) f_3^*(t) \left[\int_{-\infty}^{\infty} dt f_3^*(t) f_3(t)\right]^{-1},$$

$$\Delta\Theta = \int_{-\infty}^{\infty} dt \Delta E(t) f_4^*(t) \left[\int_{-\infty}^{\infty} dt f_4^*(t) f_4(t)\right]^{-1}. \quad (4)$$

Normalized integrals are equal

$$N_1 = \int dt f_1^*(t) f_1(t) = \frac{12 + \pi^2}{18A},$$

$$N_2 = \int dt f_2^*(t) f_2(t) = \frac{2A^3}{3\tau},$$

$$N_3 = \int dt f_3^*(t) f_3(t) = \frac{A(12 + \pi^2)}{18\tau},$$

$$N_4 = \int dt f_4^*(t) f_4(t) = 2A\tau.$$

Comparing expressions (2) and (3), we obtain expressions for fluctuations in the pulse parameters [7]:

$$\int dt f_i^* f_j = N_i \delta_{i,j}, N_i = \int dt |f_i|^2.$$

Therefore,

$$\Delta|N| = N_1^{-1} \int dt f_1^* (\Delta E_+ - \Delta E_-).$$

After sampling $n_n(i) = f_n(-T + 2T(i-1)/400)$, $n = 1, 2, 3, 4$, $i = 1 \ldots 400$, the fluctuations of the pulse parameters are calculated using the formulae:

$$\Delta n = 0.025 \sum_{i=1}^{401} n_{1i} 2F p_i.$$

The calculation using these formulas gives the following values of fluctuations in the pulse parameters: $\Delta A = 0.186A$, $\Delta p = -13.6\tau/A^3$, $\Delta t_0 = -0.015\tau$, $\Delta\theta = -0.00225/(A\tau)$. It can be seen that fluctuations in the number of quanta are quite noticeable, $\Delta n/n = 2\Delta A/A = 0.372$, fluctuations in the pulse (frequency) with the number of quanta in 10^6 are small, fluctuations in the pulse arrival time (of the order of 1 ns), and phase fluctuations are inversely proportional to the pulse duration and pulse amplitude, and are also small.

Let's consider the equivalents of uncertainty ratios: $\Delta n \Delta\theta = 0.0004185/\tau < 0.655$, $\Delta p \Delta t_0 = 0.816/\tau^2 > 0.25$. Here, the values of constants after the inequality signs correspond to the results [4]. The difference in uncertainty ratios indicates that, perhaps, with these parameters, the pulse is in a „compressed" (nonclassical) state. However, the full proof of „'s non-classical behavior" requires a more detailed study, which is planned to be carried out in the future.

Conclusion

The paper highlights the analysis of behavior of fluctuations of quantum pulses generated in the fiber lasers with mode synchronization or in the nonlinear fiber ring resonators. Since the pulse sequence can be represented as the result of interference of fields of longitudinal modes, the amplitude of which is determined by the shape of the pulse, quantum fluctuations are associated with fluctuations in the amplitudes and phases of longitudinal modes. Using the standard quantization of mode oscillators, it was possible to write expressions for the sources of amplitude and phase fluctuations for each longitudinal mode. In this case, the phase fluctuations are calculated from the amplitude using Heisenberg relations: energy-time (phase). It was assumed that fluctuations of the mode amplitudes obeyed the Poisson distribution. The sources of fluctuations in the time domain are obtained using the inverse discrete Fourier transformation. For a pulse $sech(t)^2$, the fluctuations of the pulse parameters (number of quanta, maximum time, chirp, and phase) have been calculated. These expressions are quite general and can be applied to any laser system that generates pulses of this shape. This calculation method can also be applied to systems that generate pulses in the form of a hyperbolic pulse and pulses with a different profile.

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Conflict of interest

The authors declare that they have no conflict of interest.

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