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Dynamic microcavities in collision of unipolar rectangular pulses of self-induced transparency in a dense gas medium

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In contrast to conventional multi-cycle pulses resonantly exciting quantum transitions in a medium, unipolar light pulses with a non-zero electric area can be used for ultrafast excitation of quantum systems. The interaction of such pulses with a resonant medium can give rise to many interesting effects that have been actively studied in recent years. These include the possibility, recently predicted by the authors, of creating dynamic microresonators that arise when such pulses collide in a medium. In this paper, based on the numerical solution of the Maxwell-Bloch system of equations, we study the dynamics of such resonators in a dense three-level medium during a collision of unipolar rectangular pulses acting like 2π-pulses of self-induced transparency. In contrast to earlier studies, we consider the case of an „asymmetric" collision, when the pulses enter the medium at different moments of time and collide not in its center. The possibility of forming structures of various shapes in different regions of the medium after each subsequent collision is shown.

Keywords: extremely short pulses, attosecond pulses, dynamic microresonators, optical switching, optical memory.

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Introduction

The generation of electromagnetic pulses of attosecond duration made it possible to study and control the movement of electrons in atoms, molecules and solids [1–3], which led to the emergence of attosecond physics [4–7] and the Nobel Prize award for work in this field in 2023 [8]. Typically, the received pulses contain several half-waves of a field of opposite polarity. Pulses of extremely short duration in a given spectral range — unidirectional (semi-cyclic) pulses from a single half-wave of the field are needed to study ultrafast processes occurring in quantum systems. They have a non-zero electric area, defined as the integral of the electric field strength over time $\mathbf{E}(\mathbf{r}, t)$ at a given point in space [9–12]:

$$\mathbf{S}_E = \int \mathbf{E}(\mathbf{r}, t) dt. \tag{1}$$

Such pulses, if their duration is less than the period of electron rotation in the Bohr orbit, can effectively transfer a mechanical moment to the electron [13–18]. This may make it possible to study the dynamics of atomic systems at ultra-short time intervals, less than the Bohr period [13–18]. Such pulses can be used for ultrafast excitation of quantum systems, studying the dynamics of bound electrons in atoms, molecules, and nanoscale structures [19–22], ultra-high-resolution holography [23], creating ultrafast attosecond switches [24], for generation of electron-positron pairs [25], for ultrafast petahertz electronics systems [26] and other applications.

The issues of the existence and generation of such impulses have been well studied to date, the results of recent studies are summarized in recent reviews in Ref. [27–29] and the chapter of the monograph of the Russian Academy of Sciences [30]. For example, such pulses can be obtained by rapid deceleration of electrons in thin targets [31,32], in nonlinear processes in plasma [33], in nested quantum wells [34], in quantum dots [35], in environments with magnetic hysteresis [36], superradiation of stopped polarization [37–39] and by other methods. The possibility of controlling the time shape of such pulses is also considered — obtaining unidirectional pulses of unusual shapes (rectangular and triangular) in the optical and terahertz ranges [37–41] and in the form of solitons [42]. The interaction of unusually shaped pulses is poorly understood today, but they can also be used to control quantum systems —atoms [43] and quantum qubits [44,45].

It should be noted that the physics of the interaction of unidirectional pulses with quantum systems on such small time scales (half a cycle of field oscillations) differs significantly from the case of multi-cycle pulses. Many traditional optical phenomena become impossible or follow completely different scenarios, which makes the interaction of such pulses with the environment an important urgent task [27–30]. Recent studies (see reviews and monograph [27–30]) has led to the study of a number of new phenomena that may have promising applications in modern ultrafast optics and attosecond physics.

One of these phenomena is the possibility, which has been actively studied recently, of creating high-quotient dynamic microresonators (DM) that occur when unidirectional pulses collide with a medium (see [46–52] and the review in Ref. [53]). DM are formed under conditions of coherent interaction of unidirectional pulses with the medium, i.e. when the pulse durations and delays between them are less than the time of the phase memory of the medium (relaxation time of the polarization of the medium) T_2.

When pulses collide in a medium, regions of the order of (or less than) the wavelength of the resonant transition can occur in it, for which the population difference has an almost constant value. It abruptly changes outside this region to another constant value or to a periodic „Bragg" population difference lattice. Such a structure is a „dynamic microresonator" (DM) [46–52], the parameters of which can be controlled by subsequent collisions between pulses and in other ways (which leads to the use of the term „dynamic"). The Q factor of such structures at a high concentration of working atoms of the medium can reach 1000 and higher [52]. It should be noted that such „quasi-resonators" can occur during the nonlinear self-action of a single pulse in a medium, which leads to a deceleration of the pulse in the medium and its subsequent complete stop [54]. These circumstances make such structures an interesting object of study with promising applications for creating memory cells based on atomic coherence [55], ultrafast attosecond switches [48] and other applications discussed in the review [53].

DM was studied in Ref. [46,50] during the collision of unidirectional pulses of unusual shape in a two- and three-level medium. The formation of „Bragg" microresonators localized in space during the collision of single-cycle 2π-pulses of self-induced transparency (SIT) was studied in Ref. [48]. The possibility of DM formation was shown in Ref. [49] even in the case of non-overlapping semi-cyclic pulses in a three-level environment. A simple analytical approach based on an approximate solution of the Schrodinger equation in the weak field approximation was proposed in Ref. [51], predicting the formation of DM at each resonant transition of a multilevel medium. This approach was expanded in detail in subsequent paper in Ref. [52], which studied the behavior of DM in both strong and weak fields at different polarities of colliding pulses and concentrations of two-level atoms. The results of early studies of the dynamics of DM in case of the collision of semi-cyclic pulses in a medium are summarized in the review in Ref. [53] and the cited literature.

The above studies [46–48,50–53] considered the case of „symmetric" pulse collision, when the pulses enter the medium simultaneously and collide in its center. In this case, the DM is always localized in the center of the medium, in the area of pulse overlap, and is symmetrical relative to it. The results of numerical calculations in Ref. [52], which examined the dynamics of DM during a symmetrical collision of rectangular SIT pulses in a two-level environment, showed that the shape of the induced DM and their localization in space remained qualitatively similar to each other after each subsequent collision. Only the characteristics of the induced structures changed — the depth of the lattices, their spatial frequency. The search for new possibilities for controlling the parameters of these structures is considered in this paper.

This paper studies the behavior of DM is studied in the case of a „asymmetric" collision of unidirectional pulses of unusual time shape with duration in the attosecond range in a dense three-level medium based on the numerical solution of a system of equations for the density matrix of a three-level medium, together with the wave equation. The impulses enter the medium towards each other at different points in time and collide not at the center of the medium. This makes it possible to obtain a DM of a different shape than in the case of a symmetrical collision. The parameters implemented in atomic hydrogen were used as parameters of the three-level medium.

An unusual behavior of DM has been found — DM can occur in another area inside the medium after each subsequent collision between pulses. Moreover, the shape of each subsequent DM may be very different from the previous one. These results, in contrast to those previously obtained in the case of symmetric collisions, make it possible to more broadly control the parameters of induced DM, vary their shape, characteristics, and create them in different areas of the environment. These results show the possibility of using unidirectional pulses of an unusual time shape (rectangular), whose interaction with the medium has been poorly studied to date, for ultrafast control of the properties of quantum systems.

The equations of the model, the considered system and the justification of the applicability of the assumptions used

To study the dynamics of the DM under the action of unipolar pulses, it is necessary to numerically solve the system of material equations for the medium together with the wave equation for the electric field strength, which describes the evolution of the electric field in the medium without using the traditionally employed slowly varying amplitude and rotating wave approximations. The material equations use a system of equations for the density matrix, the medium is modeled in a three-level approximation. This system of equations has the following form [56]:

$$\frac{\partial}{\partial t}\rho_{21} = -\frac{\rho_{21}}{T_{21}} - i\omega_{12}\rho_{21} - i\,\frac{d_{12}}{\hbar}\,E(\rho_{22}-\rho_{11}) - i\,\frac{d_{13}}{\hbar}\,E\rho_{23} + i\,\frac{d_{23}}{\hbar}\,E\rho_{31}, \qquad (2)$$

$$\frac{\partial}{\partial t}\rho_{32} = -\frac{\rho_{32}}{T_{32}} - i\omega_{32}\rho_{32} - i\frac{d_{23}}{\hbar}E(\rho_{33}-\rho_{22})$$
$$-i\frac{d_{12}}{\hbar}E\rho_{31} + i\frac{d_{13}}{\hbar}E\rho_{21}, \quad (3)$$

$$\frac{\partial}{\partial t}\rho_{31} = -\frac{\rho_{31}}{T_{31}} - i\omega_{31}\rho_{31} - i\frac{d_{13}}{\hbar}E(\rho_{33}-\rho_{11})$$
$$-i\frac{d_{12}}{\hbar}E\rho_{32} + i\frac{d_{23}}{\hbar}E\rho_{21}, \quad (4)$$

$$\frac{\partial}{\partial t}\rho_{11} = \frac{\rho_{22}}{T_{22}} + \frac{\rho_{33}}{T_{33}} + i\frac{d_{12}}{\hbar}E(\rho_{21}-\rho_{21}^*)$$
$$-i\frac{d_{13}}{\hbar}E(\rho_{13}-\rho_{13}^*), \quad (5)$$

$$\frac{\partial}{\partial t}\rho_{22} = -\frac{\rho_{22}}{T_{22}} - i\frac{d_{12}}{\hbar}E(\rho_{21}-\rho_{21}^*) - i\frac{d_{23}}{\hbar}E(\rho_{23}-\rho_{23}^*), \quad (6)$$

$$\frac{\partial}{\partial t}\rho_{33} = -\frac{\rho_{33}}{T_{33}} + i\frac{d_{13}}{\hbar}E(\rho_{13}-\rho_{13}^*) + i\frac{d_{23}}{\hbar}E(\rho_{23}-\rho_{23}^*), \quad (7)$$

$$P(z,t) = 2N_0 d_{12}\mathrm{Re}\,\rho_{12}(z,t) + 2N_0 d_{13}\mathrm{Re}\,\rho_{13}(z,t)$$
$$+2N_0 d_{23}\mathrm{Re}\,\rho_{23}(z,t), \quad (8)$$

$$\frac{\partial^2 E(z,t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E(z,t)}{\partial t^2} = 4\frac{\pi}{c^2}\frac{\partial^2 P(z,t)}{\partial t^2}. \quad (9)$$

The solved system of equations (2)–(9) contains the following parameters: z — longitudinal coordinate, N_0 — particle concentration, P — medium polarization, E — electric field strength, ρ_{21}, ρ_{32}, ρ_{31} — off-diagonal elements of the density matrix determining the dynamics of the polarization of the medium, d_{12}, d_{13}, d_{23} — transition dipole moments, $\hbar$ — reduced Planck constant, ω_{12}, ω_{32}, ω_{31} — resonant transition frequencies, variables ρ_{11}, ρ_{22}, ρ_{33} — populations of 1st, 2nd and 3rd environmental states accordingly, T_{ik} — relaxation times. The relaxation times can have values of tens and hundreds of nanoseconds in gases and cryogenically cooled quantum dots [57]. These values are much longer than the processes considered in this paper, so the final values of relaxation times below are neglected.

The integration domain of length $L = 12\lambda_0$ was considered in the numerical calculations. The studied three-level medium located between points $z_1 = 2\lambda_0$ and $z_2 = 10\lambda_0$. A vacuum was located between the medium and the boundaries of the integration region. Rectangular pulses were modeled by a hyper-Gaussian function and were emitted into the medium from the left and right boundaries of the integration region towards each other in the form

$$E(z=0,t) = E_{01}e^{-\frac{(t-\Delta_1)^{20}}{\tau^{20}}}, \quad (10)$$

$$E(z=L,t) = E_{02}e^{-\frac{(t-\Delta_2)^{20}}{\tau^{20}}}. \quad (11)$$

The delays Δ_1 and Δ_2 regulated the moments of pulse entry into the medium. The point at which the pulses collided can be varied by selecting the delay values. It is obvious that $\Delta_1 = \Delta_2$ in the case of a symmetric collision, when the pulses simultaneously enter the medium. In our consideration, the pulse collision points will alternate between two positions z_{I} and z_{II}, the distance between which is determined by the delay between the pulses related to their propagation velocity: $|z_{\mathrm{I}} - z_{\mathrm{II}}| = (|\Delta_1 - \Delta_2|)/c_i$. Zero boundary conditions were selected at the boundary of the integration domain that „reflected“ pulses back into the medium, which made it possible to set a sequence of pulses.

Parameters used in numerical calculations

Frequency (wavelength λ_0) of transition $1 \to 2$	$\omega_{12} = 1.55 \cdot 10^{16}$ rad/s ($\lambda_{12} = \lambda_0 = 121.6$ nm)
Dipole moment of transition $1 \to 2$	$d_{12} = 3.27$ D
Frequency (wavelength) of transition $1 \to 3$	$\omega_{13} = 1.84 \cdot 10^{16}$ rad/s ($\lambda_{13} = 102.6$ nm)
Dipole moment of transition $1 \to 3$	$d_{13} = 1.31$ D
Frequency (wavelength) of transition $2 \to 3$	$\omega_{23} = 2.87 \cdot 10^{15}$ rad/s ($\lambda_{23} = 656.6$ nm)
Dipole moment of transition $2 \to 3$	$d_{23} = 12.6$ D
Concentration of atoms	$N_0 = 10^{20}$ cm^{-3}
Field amplitude	$E_{01} = 210000$ ESU $E_{02} = 210000$ ESU
Parameter τ	$\tau = 200$ as
Delay Δ_1	$\Delta_1 = 2.5\tau$
Delay Δ_2	$\Delta_2 = 5.5\tau$

We consider a three-level model of the medium, the parameters of which (frequencies and dipole moments of transitions) correspond to the three lower levels in the hydrogen atom (without taking into account the fine and hyperfine splitting of levels). These parameters are shown in the table and are taken from Ref. [58].

We used the concentration value $N_0 = 10^{20}$ cm^{-3}, at which a high Q-factor value of the induced DM is achieved. It should be noted that similar concentrations of hydrogen atoms are achieved when placed in a matrix at very low temperatures [59,60]. However, the conclusions obtained below are universal and can be found in any medium with a high phase memory time T_2 [53]. Also, the results of numerical calculations provided in Ref. [18] have shown that the occurrence of a quasi-resonator and the phenomenon of light stopping are preserved when taking into account the effects of a local field in a dense medium. This circumstance makes it possible to neglect the effects of the local field in such tasks.

The three-level model discussed above takes into account only the three lower energy levels of the medium and does not take into account the ionization of the medium. The justification for the applicability of low-level models in such tasks is provided, for example, in Ref. [49,53,61]. Indeed, the analytical approach proposed in Ref. [50,52] is based on an approximate solution of the Schrodinger equation, it predicts the occurrence of DM at each resonant transition of a multilevel medium due to the interference of electrical pulse areas. Population lattices still occur in strong fields, shown by the results of the numerical solution of the Schrodinger time problem, even when taking into account the ionization of the medium, which can be minimized by selecting the delay value between pulses [62].

From a physical point of view, the preservation of the effects of DM formation in multilevel media is explained by the following simple arguments [49,53,61]. An extremely short unidirectional pulse acting as an impact excitation, passing through the medium, leaves behind oscillations of atomic coherence (off-diagonal elements of the density matrix that determine the behavior of the polarization of the medium) at each resonant transition of the medium, which exist during the time of the phase memory of the medium T_2. Each such subsequent pulse will coherently control these oscillations, which will lead to the formation of a Bragg-like lattice of population differences and the occurrence of DM at each resonant transition of the medium, which corresponds to the theoretical predictions [49,53,61].

We also note that in practice, pulses with a shape close to unidirectional are most often obtained, containing a long trailing edge of the opposite polarity [43]. In the general case, such a front can affect the dynamics of the system, however, the results of numerical calculations show that if this front is long enough and weak, then its influence is not significant [38], therefore, pulses without a trailing edge are considered below. It should be noted that the propagation of unidirectional pulses over considerable distances without significant loss of unidirectionality can be realized in coaxial waveguides [28–30] and is described by the one-dimensional wave equation [63].

After describing the equations of the model of the system under consideration and substantiating the approximations used, we proceed to analyze the results of computational modeling.

Results of computational modeling, case of collision of rectangular SIT pulses with the same initial polarity

The first series of numerical calculations used pulses of the same initial polarity, $E_{01} = E_{02}$. The pulse parameters (their duration, amplitude) are selected so that the pulses act like 2π-pulses of McCall and Khan SIT at the main transition 1-2 [64]: the first half of the pulse transfers the medium from the ground state to the excited state, the second half of the pulse returns the medium to the ground state. From a practical point of view, the creation of create high-Q DM [52] requires high concentration of working atoms, $N_0 \sim 10^{20}\,\mathrm{cm}^{-3}$, as shown in the table. In most early studies, the dynamics of DM was studied in the case when the concentration of medium particles was low, since the shape of the pulses changes in case of propagation at high concentrations, which leads to blurring of the structures of DM.

Thus, a high density of the medium and a constant pulse shape during propagation are necessary for obtaining high-Q structures. SIT pulses can be used to solve this problem, since they can propagate in an environment without changing shape. And although the pulses we are considering are not solitons in the strict sense (for unidirectional SIT solitons, see, for example, [65,66]), they act like SIT pulses and can propagate in a medium without significant shape changes. Therefore, we consider the dynamics of DM under the action of rectangular SIT pulses in a dense medium in the numerical calculations below.

It should be noted that the considered system is complex and contains a rich dynamic of possible solutions for different parameters. Below, we will limit ourselves to considering the dynamics of the system with the parameters given in the table, in the case of an asymmetric collision of rectangular SIT pulses in an atomic gas. Consideration of the remaining possible parameters is beyond the scope of this study.

The results of numerical calculations are shown in Fig. 1-4, which illustrate the dynamics of polarization and population differences at all resonant transitions of the medium.

The first pulse collision occurs at time $t = 5.9$ fs at a point in the medium $z_{\mathrm{I}} = 6.74\lambda_0$, the second pulse collision occurs at time $t = 10.62$ fs at point $z_{\mathrm{II}} = 6.74\lambda_0$, etc. at alternating points in the medium, the distance between which depends on the delay between the interacting pulses. The propagation directions for the first four pulses are marked with arrows in Fig. 1, *a*. The instantaneous values of the population difference distribution after the first and second passage of the pulses are shown in Fig. 2, *a* and 2, *b*, respectively, the collision points are marked with vertical lines in the figures.

It can be seen from Fig. 1 and 2, *a* that after the first collision outside the pulse overlap region, the population difference at each transition, although it has the form of a Bragg-like lattice, nevertheless has an almost constant value ($n = 0.8 \sim 1$). A localized DM of a non-standard shape occurs in the overlap area. The DM disappears after the second collision, as can be seen in Fig. 2, *b*, but an area of constant population difference appears already at the point of the second collision. It is also worth noting that after the second collision and beyond, there is a significant difference in the behavior of the inversion of the populations of the levels 1-2 and 1-3, which suggests the possibility of creation of different structures at different transitions of the medium.

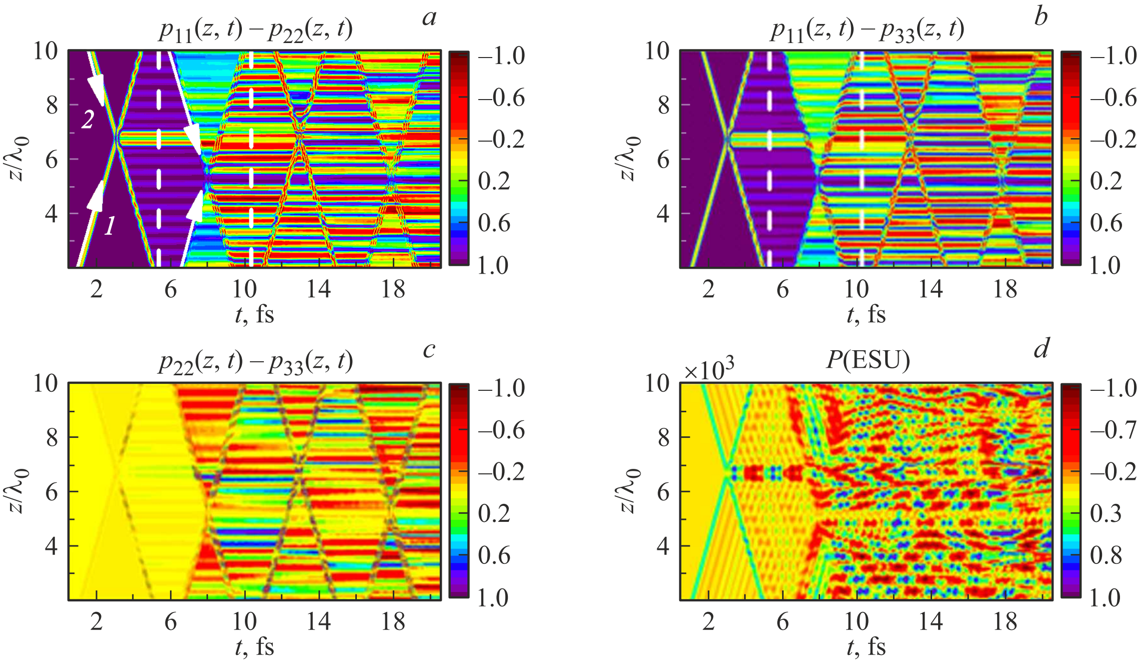

Figure 1. Spatial and temporal dynamics of population differences: (a) $\rho_{11} - \rho_{22}$, (b) $\rho_{11} - \rho_{33}$, (c) $\rho_{22} - \rho_{33}$ and polarization of (d) P(ESU) of a three-level environment.

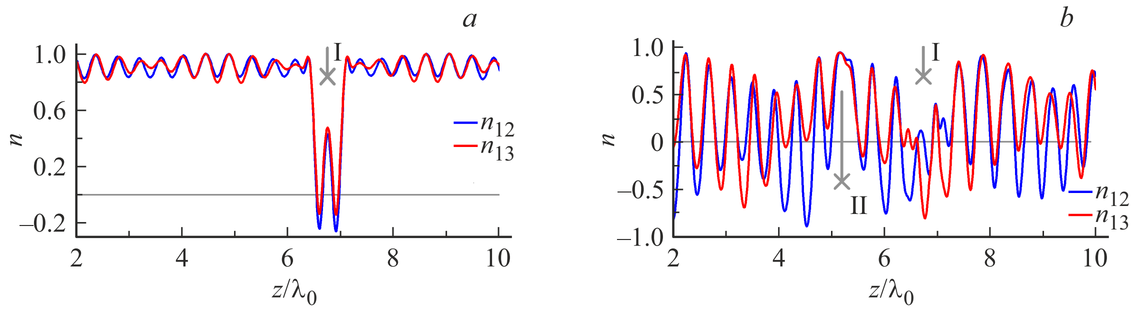

Figure 2. Instantaneous distribution of population differences after (a) the first and (b) the second passage of the pulses; the cross sections are marked with strokes in Fig. 1, a and 1, b. Blue: $\rho_{11} - \rho_{22}$, red: $\rho_{11} - \rho_{33}$. The calculation parameters are listed in the table.

Results of computational modeling, case of collision of rectangular pulses with opposite initial polarity

The behavior of DM was studied in this section when the colliding pulses had different initial polarities, $E_{01} = -E_{02}$. All other parameters coincide with the case above; the results of modeling are shown in Fig. 3.

The instantaneous values of the population difference distribution after the first and second passage of the pulses are shown in Fig. 4, a and 4, b, respectively, the collision points are marked with vertical lines in the figures.

As can be seen in Figs. 3 and 4, a (in contrast to the case of pulses of the same polarity), after the first collision, the population differences outside the pulse overlap region are expressed differently for transitions 1-2 and 1-3 and have a more significant amplitude ($n = 0 \sim 0.6$). A DM appears in the overlap area, which, as can be seen in Fig. 4, b, loses its structure after the second collision. The new DM that appears in the area of the second collision also does not have the correct structure, as does the population difference grid that appears on the sides.

It is worth noting that, as can be seen in Fig. 3 and 4, b, the shape of the lattices on the sides becomes more „chaotic“ with each subsequent collision; already starting from the second collision, it is impossible to talk about the presence of any period of structure. Such a sharp distortion of the structures was not observed for pulses of the same polarity.

Discussion of results and conclusions

As the results of numerical calculations have shown, during an asymmetric collision of pulses in a medium, struc-

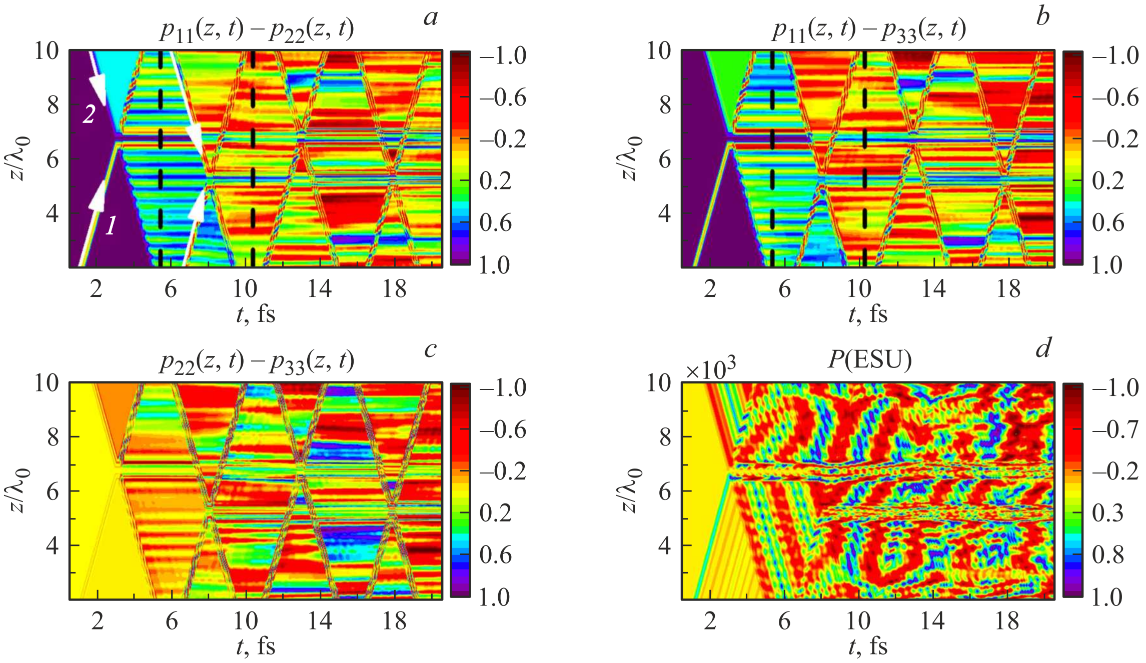

Figure 3. Spatial and temporal dynamics of population differences: (a) $\rho_{11}-\rho_{22}$, (b) $\rho_{11}-\rho_{33}$, (c) $\rho_{22}-\rho_{33}$ and polarization (d) P(ESU) of a three-level medium for the case of a collision of pulses of opposite polarity.

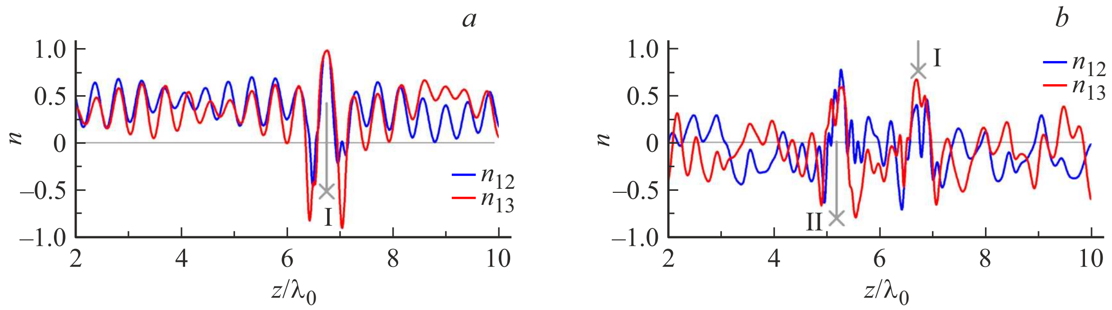

Figure 4. Instantaneous distribution of population differences after (a) the first and (b) of the second pulse passage; the cross sections are marked with strokes in Fig. 3, a and 3, b. Blue: $\rho_{11}-\rho_{22}$, red: $\rho_{11}-\rho_{33}$. The calculation parameters are listed in the table.

tures with an asymmetric profile appear — the parameters of the induced Bragg structures are different to the left and right of the collision region between the pulses. As can be seen from the figures, each subsequent pulse collision can occur at different points in the medium other than the point where the first pulse collision occurred. This leads to the fact that the resulting DM pulses are localized in different regions of the medium.

This feature has not been observed before in the case of symmetric pulse collisions. This unusual behavior of the system opens up new opportunities for controlling the parameters of the induced DM. It allows creating not only controllable profile structures, but also controlling their location inside the medium — enabling structures in one place, then disabling them and creating them in another place. It is difficult to propose a detailed physical explanation of all the features of the system behavior observed above because of the complexity and nonlinearity of the considered system. To date, analytical solutions have been obtained only in the approximation of a weak and given field [50–52].

Conclusion

Thus, the dynamics of DM in an asymmetric collision of unidirectional SIT-like 2π-pulses in a dense three-level medium is theoretically studied in this paper. The parameters of the medium were chosen to be the same as in the hydrogen atom. It is shown that, unlike the previously considered case of symmetric collision, the medium exhibits DM of an „asymmetric" form the parameters of the density matrix Bragg lattices (grating depth and shape) differ on the left and right sides of the region where the pulses overlap.

The localization of the induced DM changes with each pulse collision between two points.

The studied phenomena and the use of an asymmetric collision open up new possibilities in controlling the properties of dynamic resonators and show the possibilities of using unidirectional rectangular pulses of light to control the properties of matter.

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Conflict of interest

The authors declare that they have no conflict of interest.

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