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Measurement of two components of the spectrum of the radiation of a single-frequency laser diode

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A method for measuring two components, Lorentzian and Gaussian, of the spectral line of radiation from single-frequency laser diodes is presented using a pair of scanning ring fiber interferometers with different lengths of ring resonators. Experimental results are presented that confirm the effectiveness of the approach.

Keywords: laser diode, single-frequency radiation, laser linewidth.

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Single-frequency laser diodes (LDs) have a wide range of applications, which include coherent communications, interferometer sensors, etc. The linewidth is of great importance in the design and application of single-frequency LDs.

The viewpoints on the optical line shape of single-frequency LDs have evolved since their inception, at which point the spectrum was considered to be formed by fundamental processes (spontaneous emission of photons) and to have a natural width with a Lorentzian shape [1]. However, it soon became clear (largely due to the advent of single-frequency LDs with an external cavity) that the shape of the spectral line differs from the Lorentzian one: its width is significantly larger and depends on the measurement time [2]. This broadening is induced by noise associated with the drift of resonator parameters and the driver current [3].

Further studies have revealed that noise of this type (flicker noise) shapes a near-Gaussian laser line profile [3,4]. Therefore, the optical line shape of single-frequency LDs is specified by the convolution of two components (Lorentzian and Gaussian), which yields a spectrum shape known as the Voigt profile.

A considerable number of studies focused on measuring the spectral width of single-frequency LDs have already been published. The methods discussed there are based on photoelectric mixing of optical waves [5] (i.e., beating of radiation of two lasers, the one under study and the reference one, or radiation of the studied laser and its delayed/frequency-shifted part). The beat signal is then analyzed by an electrical spectrum analyzer to obtain spectral power density $S(\nu)$ of phase noise, which allows one to determine essentially the natural LD line width.

The width of both components is often determined by measuring the noise spectrum of laser frequency $S_{\Delta\nu}(f)$, again with the use of wave beating and devices that convert optical phase noise into intensity variations [6,7]. Spectrum $S_{\Delta\nu}(f)$ is divided into high- and low-frequency parts using the so-called β-separation line, which allows one to determine width $\Delta\nu_L$ of the Lorentzian component by the spectrum level in the high-frequency part: $\Delta\nu_L = \pi S_{\Delta\nu}(f)$, while width $\Delta\nu_G$ of the Gaussian component is determined in fairly cumbersome numerical calculations [7].

A scanning ring fiber interferometer (SRFI) [8,9] is a significantly simpler device for the examination of single-frequency radiation. Its output signal contains information about the line width of single-frequency radiation in the duration of interference signals (ISs) in the form of short-term drops in intensity. Owing to its large length, a ring fiber interferometer has a high spectral resolution; however, the spectrum is not divided into two components, and only the width of their convolution is measured.

In the present study, an approach to measuring two components of single-frequency radiation with a pair of SRFIs with different ring resonator lengths is proposed. This approach relies, first, on the SRFI model, wherein the IS width is determined by both the device parameters and the autocorrelation function of the studied radiation, and, second, on the fact that, according to the theorem of inverse Fourier transform of a convolution, the correlation function of a convolution of spectra is the product of two corresponding correlation functions.

Thus, two devices with different lengths of ring resonators ($L_1 \neq L_2$) allow one to determine the autocorrelation function values at two times $\tau_{1,2} = L_{1,2}n/c$ by measuring the IS width and write two equations with two unknown frequencies $\Delta\nu_L$ and $\Delta\nu_G$, which may be found by solving the system of equations.

The SRFI optical circuit is shown in Fig. 1. Being a multi-beam interferometer, this device may be regarded as a fiber version of a Fabry–Pérot interferometer. An asymmetrical 2×2 fiber coupler, which ensures multiple circulation of waves in the ring resonator, plays the part of mirrors with high reflection coefficients. The ring resonator features a built-in fiber phase modulator based on piezoelectric ceramics and a polarization controller; the device is also

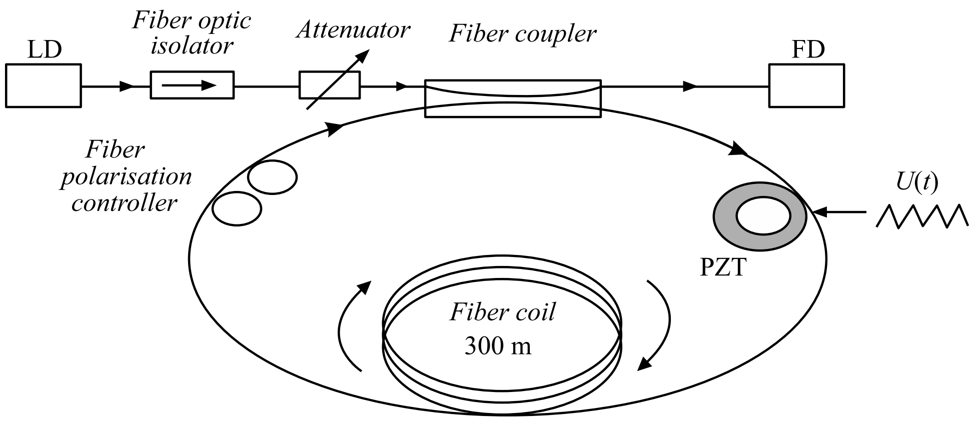

Figure 1. SRFI optical circuit.

fitted with a photodetector. An attenuator is mounted at the interferometer input; digital processing of the output signal is performed in real time.

The complex amplitude of the electric field of a wave at the output is represented by an expression that reflects the infinitely repeating process of separation and merger of waves travelling within the ring resonator and containing information about the radiation spectrum in phase fluctuations of the wave field:

$$E(t) = E_0 e^{i\tilde{\varphi}(t)} \left\{ \sqrt{k_1} - k_2\sqrt{g}\, e^{i\left(\Delta\varphi(t)+\Delta\tilde{\varphi}(t_1)\right)} \times \left[1 + \sqrt{k_1 g}\, e^{i\left(\Delta\varphi(t)+\Delta\tilde{\varphi}(t_2)\right)} + \left(\sqrt{k_1 g}\right)^2 e^{i\left(2\Delta\varphi(t)+\Delta\tilde{\varphi}(t_2)+\Delta\tilde{\varphi}(t_3)\right)} + \ldots \right]\right\}, \quad (1)$$

where E_0 and $\tilde{\varphi}(t)$ is the wave amplitude and phase at the input, t is the current time, k_1 and $k_2 e^{i\frac{\pi}{2}}$ are the amplitude transfer coefficients of the fiber coupler, $k_1 \gg k_2$, g are optical losses in the fiber ring resonator, $\Delta\varphi(t) = 2\pi\tau\nu/c + \varphi_{mod}(t)$, $\tau = Ln/c$ is the time of a single passage of light through the ring resonator, c is the speed of light, ν is the carrier frequency of radiation, n is the refraction index of the light guide, $\varphi_{mod}(t)$ is the periodic triangular phase modulation signal, $\Delta\tilde{\varphi}(\tau_p) = \tilde{\varphi}\big(t - (p-1)\tau\big) - \tilde{\varphi}(t - p\tau)$, $p = 1, 2, \ldots$, $\tilde{\varphi}(t - p\tau)$ is the wave phase at the input that was before current time t, and it is assumed that there are no fluctuations of the wave amplitude at the input.

In expression (1), exponents of the form $e^{i\Delta\tilde{\varphi}(p)}$ should be averaged, since they represent phase fluctuations within separate time intervals as independent random processes and have the same distribution law; therefore, the result of averaging is the same for all of them, and each averaged exponent may be regarded as a normalized autocorrelation function of radiation $R(\tau) = \langle e^{i\Delta\tilde{\varphi}(p)} \rangle$.

Taking this into account, we move on to the calculation of radiation intensity at the output. Following simple transformations, we obtain

$$I(t) = \frac{k_1 + g(k_1+k_2)^2 R(\tau)^2 - 2\sqrt{k_1 g}(k_1+k_2) R(\tau)\cos\Delta\varphi(t)}{1 + k_1 g R(\tau)^2 - 2\sqrt{k_1 g} R(\tau)\cos\Delta\varphi(t)}. \quad (2)$$

As was already noted, the correlation function of a convolution of spectra is the product of correlation functions of these spectra. Writing the Lorentzian and Gaussian spectra in the form $S_L(\nu) = \frac{1}{2\pi}\Delta\nu_L^2/(\nu^2 + \Delta\nu_L^2)$ and $S_G(\nu) = \exp\big(-(2\pi\nu)^2/2\sigma^2\big)/\sigma\sqrt{2\pi}$, $\sigma = \pi\Delta\nu_G/\sqrt{2\ln 2}$ and applying the inverse Fourier transform to them, we find the autocorrelation functions of these spectra: $r_L(\tau) = \exp(-0.5\pi\Delta\nu_L\tau)$ and $r_G(\tau) = \exp(-2\pi^2\Delta\nu_G^2\tau^2/8\ln 2)$. Therefore, the autocorrelation function of the convolution has exponential form $R(\tau) = \exp[-h(\tau)]$, where the exponential factor is

$$h(\tau) = 0.5\pi\Delta\nu_L\tau + 2\pi^2\Delta\nu_G^2\tau^2/8\ln 2. \quad (3)$$

Expressions (2), (3) represent a model of the SRFI, which allows one to determine the IS width by setting the SRFI parameters and the spectral width of two components $\Delta\nu_L$, $\Delta\nu_G$. However, the model is used here to solve the inverse problem: to determine frequencies $\Delta\nu_L$ and $\Delta\nu_G$ based on the IS width at the output of two devices (Δw_1 and Δw_2).

To do this, the SRFI parameters are fixed, and a series of h_i^* values are specified. Two sets $\{\Delta w_{1,i}\}$ and $\{\Delta w_{2,i}\}$ are then calculated using (2) with $R(\tau)$ replaced by the values of $R_i = \exp(-h_i^*)$, and inverse functions $h_1^* = f_1^{-1}(\Delta w)$ and $h_2^* = f_2^{-1}(\Delta w)$ are determined using these sets by the least squares method with approximation by second-degree polynomials.

These functions allow one to determine two values of exponential factor $h(\tau)$ corresponding to times τ_1 and τ_2 based on measured width Δw_1 and Δw_2: $h(\tau_1) = f_1^{-1}(\Delta w_1)$ and $h(\tau_2) = f_2^{-1}(\Delta w_2)$. This, in turn, enables one to write the following equations:

$$h(\tau_1) = 0.5\pi\Delta\nu_L\tau_1 + 2\pi^2\Delta\nu_G^2\tau_1^2/8\ln 2,$$

$$h(\tau_2) = 0.5\pi\Delta\nu_L\tau_2 + 2\pi^2\Delta\nu_G^2\tau_2^2/8\ln 2, \quad (4)$$

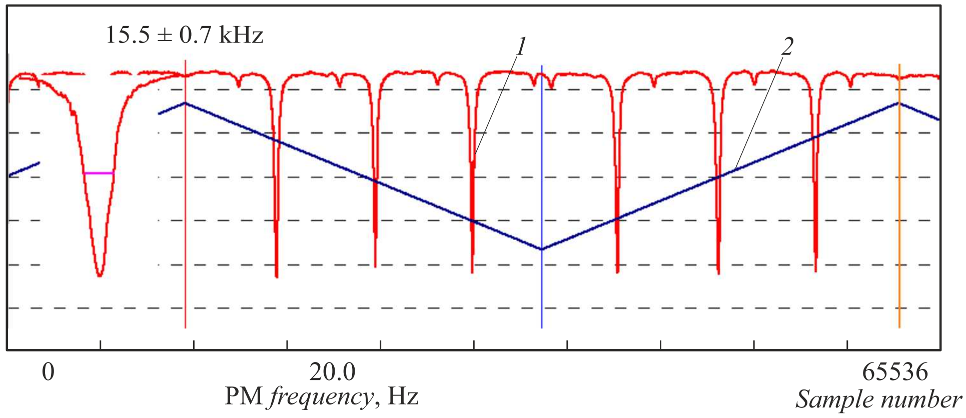

Figure 2. Screenshot of the monitor with the interference pattern (*1*), the phase modulation signal (*2*), and the result of measuring the IS width. One IS is enlarged to demonstrate its shape and width.

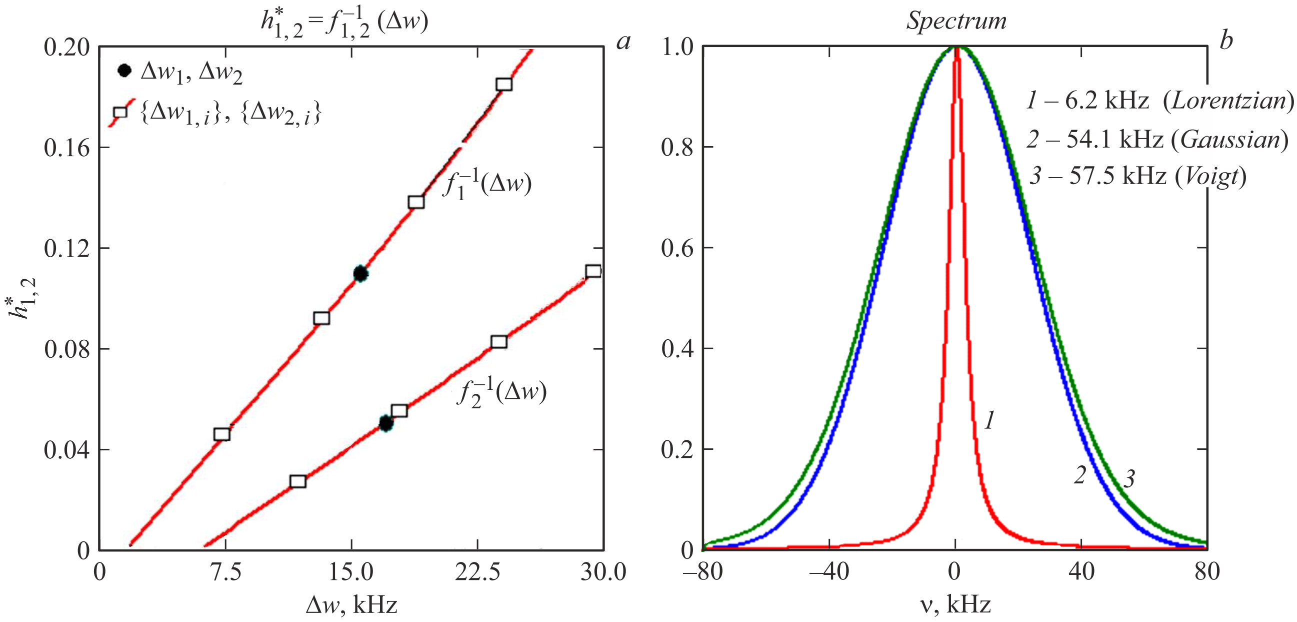

Figure 3. *a* — Calculation of $h(\tau_1)$ and $h(\tau_2)$; *b* — profiles of Lorentzian and Gaussian spectra and their convolution.

which yield desired frequencies $\Delta\nu_L$ and $\Delta\nu_G$.

The proposed approach was verified experimentally with the use of two SRFIs with their ring resonator lengths being equal to $L_1 = 500$ m and $L_2 = 300$ m. The other parameters were identical, except for the transfer coefficients of fiber coupler k_1 (0.054 and 0.12 dB) and k_2 (22.6 and 24.6 dB). The IS width was determined from the interference patterns as $\Delta w = D\Delta t/T$, where $D = c/Ln$ is the free spectral range, Δt is the half-amplitude IS duration, and T is the time interval between ISs.

A single-frequency LD with a wavelength of approximately $1.55\,\mu$m produced by JSC „Nolatech“ was used as a radiation source. Since the optical line shape of this LD was unstable on the scale of the free spectral range of both SRFIs, the LD needed to be modified by coupling it at the output to a device that served as an external resonator. Two 2×2 fiber couplers were used for this purpose: the first one diverted radiation to the photodetector, and the second one, placed behind the first, acted as a mirror (by shorting the output ports to each other via fusion splicing); the total length of the folded external cavity was 1.3 m. The LD spectrum stability was improved significantly: a stable signal structure was observed at the output of each SRFI (Fig. 2). The LD pumping current from an accumulator battery could vary within a narrow range of 120–140 mA (due to the lack of necessary thermal stabilization), but this did not lead to a noticeable change in IS width; the output power was 1.2–1.4 mW. The IS width at the output of two SRFIs determined as a result of measurements was $\Delta w_1 = 15.5$ kHz and $\Delta w_2 = 17.0$ kHz with an error of ± 0.7 kHz (with averaging over 5 s).

Figure 3, a serves to illustrate further calculations. It shows the plots of functions $f_1^{-1}(\Delta w)$ and $f_2^{-1}(\Delta w)$, which were used to determine the exponential factors: $h(\tau_1) = 0.111$ and $h(\tau_2) = 0.051$. The last stage of calculations (finding a solution to system of equations (4)) made it possible to determine the width of two spectral components: $\Delta\nu_L = 6.2$ kHz and $\Delta\nu_G = 54.1$ kHz. The profiles of these components and their convolution are shown in Fig. 3, b.

We note in conclusion that a fairly simple approach to measuring the spectral width of two components of radiation of single-frequency LDs was proposed, and experimental results verifying the effectiveness of this approach were presented.

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] C. Henry, IEEE J. Quantum Electron., **18**, 259 (1982). DOI: 10.1109/JQE.1982.1071522
[2] K. Kikuchi, T. Okoshi, Electron. Lett., **21**, 1011(1985). DOI: 10.1049/el:19850717
[3] L.B. Mercer, J. Lightwave Technol., **9**, 485 (1991). DOI: 10.1109/50.76663
[4] G.M. Stéphan, T.T. Tam, S. Blin, P. Besnard, M. Tetu, Phys. Rev. A, **71**, 043809 (2005). DOI: 10.1103/PhysRevA.71.043809
[5] M. Nazarathy, W.V. Sorin, D. Baney, N.A. Newton, J. Lightwave Technol., **7**, 1083 (1989). DOI: 10.1109/50.29635
[6] G. Di Domenico, S. Schilt, P. Thomann, Appl. Opt., **49**, 4801 (2010). DOI: 10.1364/AO.49.004801
[7] N. Bucalovic, V. Dolgovsiy, C. Schori, P. Thomann, G. Di Domenico, S. Schilt, Appl. Opt., **51**, 4582 (2012). DOI: 10.1364/AO.51.004582
[8] V.K. Sakharov, E.G. Gerasimov, *Ustroistvo dlya izmereniya spektral'nykh kharakteristik opticheskogo izlucheniya*, RF Patent 2408853 (applied on June 4, 2009; published on October 1, 2011) (in Russian).
[9] A.I. Trikshev, A.S. Kurkov, V.B. Tsvetkov, Yu.N. Pyrkov, V.N. Paramonov, Quantum Electron., **41** (7), 656 (2011). DOI: 10.1070/QE2011v041n07ABEH014623.

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