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Scattering and emission of exchanged spin waves in a magnetic structure with competing exchange interactions

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In this work, within the framework of the lattice model, the conditions for the emergence of a ground state of the spin helicoid type, caused by the competition of two exchange interactions of magnetic ions of the first two coordination spheres, are studied. For a homogeneous ground state, the coefficients of scattering and generation of exchanged spin waves by the interface between such a structure and a uniaxial ferromagnet are obtained. The possibility of the emergence of a new type of volume-surface waves in the structure under consideration is shown.

Keywords: Exchange helix (spin helicoid), long-range order of exchange interaction, evanescent spin wave, scattering and generation of exchange spin waves.

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1. Introduction

Currently, the use of terahertz devices in magnonics seems to be a near-term prospect [1,2]. High-energy magnons belonging to the exchange wavelength range (exchange spin waves, ESW) correspond to the exchange spin modes in antiferromagnets. The practical application of ESW makes it possible to reduce the size and heat losses in magnonic waveguides [1]. At the same time, magnetic structures with nonreciprocal properties are of interest for creating, in particular, magnon valves. These properties can manifest themselves in structures with an heterogeneous ground state with a certain chirality of the spin helicoid [2], and in case of the propagation of exchange dipole waves [3].

The ground state in the form of a spin helicoid is realized, in particular, in structures with the Dzyaloshinski interaction. The scattering and generation of ESW by a relativistic spiral were studied in Ref. [4,5]. The competition of short-range and long-range exchange interactions may be another mechanism of its formation. Magnetic materials in which such structures exist are discussed in detail in Ref. [5]. Their characteristic spatial period has an order of magnitude of 100−200 of lattice constants, so that it can be considered long-period. Boundary conditions (BC) for waves in heterogeneous structures were obtained in the continuum approximation in Ref. [6] to solve the problems of ESW scattering. Recent studies [7–9] have shown that ESW can be both volumetric and evanescent in limited magnetic structures, which also differ from volumetric ESW by precession chirality. For this reason both nonreciprocal properties and a greater variety of possible wave types can be expected in more complex magnetic structures with competing exchange interactions.

The propagation, scattering, and generation of ESW in such long-period structures is theoretically studied in this paper.

2. Ground state in magnetic structures with competing exchange interactions

Let us consider an unlimited magnetic structures with competing exchange interactions in which the dynamic variables are functions only of the coordinate z. We assume that the exchange interaction between the nearest atoms is ferromagnetic, and it is antiferromagnetic between the atoms of the second coordination sphere. The Hamiltonian of such a structure can be written as:

$$W_H = \frac{A_H}{16} \sum_n \big(\mathbf{S}_n \mathbf{S}_{n+2} - (4+\Delta)\mathbf{S}_n \mathbf{S}_{n+1}\big), \qquad (1)$$

where $\mathbf{S}_n$ is the spin of the nth lattice node, $A_H > 0$ is an exchange constant, Δ is a dimensionless parameter, the meaning of which will become clear later.

The first term in (1) describes the exchange interaction with the spins of the second coordination sphere and is antiferromagnetic in nature. The second term corresponds to the ferromagnetic exchange interaction of neighboring atoms, and its minimum is provided by their collinear mutual orientation. Thus, competition occurs between the two types of exchange, which can lead to an exchange spiral.

The spin dynamics of the considered structure is described by the Landau-Lifshitz equation:

$$\hbar\dot{\mathbf{S}} = \frac{1}{S_H}\left[\mathbf{S} \times \frac{\partial W_H}{\partial S}\right], \quad (2)$$

where S_H is the spin value, $\hbar$ is the Planck constant.

Let us write the dynamics equation as follows after substituting (1) into (2):

$$\hbar S_H \dot{\mathbf{S}}_n = \frac{A_H}{16}\Big[\mathbf{S}_n \times \big((\mathbf{S}_{n+2} + \mathbf{S}_{n-2} - 2\mathbf{S}_n) - (4+\Delta)(\mathbf{S}_{n+1} + \mathbf{S}_{n-1} - 2\mathbf{S}_n)\big)\Big]. \quad (3)$$

Hence follows the equation defining the ground state:

$$(\mathbf{S}^{(0)}_{n+2} + \mathbf{S}^{(0)}_{n-2} - 2\mathbf{S}^{(0)}_n) - (1+\Delta)(\mathbf{S}^{(0)}_{n+1} + \mathbf{S}^{(0)}_{n-1} - 2\mathbf{S}^{(0)}_n) = 0. \quad (4)$$

The solution (5) in the form of a spin helicoid ($z_n = nd_H$) has the form:

$$S^{(0)}_{n\pm} = S^{(0)}_{nx} \pm iS^{(0)}_{ny} = S_H e^{\pm iK_H z_n} \quad (5)$$

and it yields the possible values of the wave number after substituting in (3):

$$K_H = \frac{2}{d_H}\arcsin(\sqrt{-\Delta}), \quad \Delta \le 0,$$

$$K_H = 0, \qquad \Delta \ge 0. \quad (6)$$

Thus, the long-range order of the exchange interaction distorts the collinear structure under the condition $\Delta < 0$. $K_H d_H \ll 1$ in real long-period structures. If $N \gg 1$ is the period of the structure in lattice constants, then $|\Delta| \approx \left(\frac{\pi}{N}\right)^2 \ll 1$. The estimate $|\Delta| \sim 10^{-3}$ is obtained for $N = 100$. The ground state is homogeneous when $\Delta \ge 0$.

3. Types of waves in uniformly magnetized magnetic structures with competing exchange interactions and uniaxial ferromagnets

Let's assume $\mathbf{S}^{(0)}_{Hn} = S_N \mathbf{e}_x$ considering the ground state to be homogeneous ($\Delta \ge 0$). We will look for small perturbations in the form of ESW described by equation (3) in the first approximation ($\mathbf{s}_H \sim e^{-i\omega t}$):

$$-i\mathscr{E}\mathbf{s}_{Hn}S_H = \left[\mathbf{S}^{(0)}_{Hn} \times \boldsymbol{\chi}_{Hn}\right], \quad (8)$$

where

$$\boldsymbol{\chi}_{Hn} = \frac{A_H}{16}\Big((\mathbf{s}_{H(n+2)} + \mathbf{s}_{H(n-2)} - 2\mathbf{s}_{Hn}) - 4(1+\Delta)(\mathbf{s}_{H(n+1)} + \mathbf{s}_{H(n-1)} - 2\mathbf{s}_{Hn})\Big), \quad (9)$$

and $\mathscr{E} = \hbar\omega$ is the magnonic energy.

Then for the cyclic components of the dynamic spin:

$$s^{(l/r)}_{Hn} = s_{Hny} \pm is_{Hnz},$$

$$\chi^{(l/r)}_{Hn} = \frac{A_H}{16}\Big((s^{(l/r)}_{H(n+2)} + s^{(l/r)}_{H(n-2)} - 2s^{(l/r)}_{Hn}) - 4(1+\Delta)(s^{(l/r)}_{H(n+1)} + s^{(l/r)}_{H(n-1)} - 2s^{(l/r)}_{Hn})\Big) \quad (10)$$

the system follows from (8):

$$\chi^{(l/r)}_{Hn} \pm \mathscr{E}s^{(l/r)}_{Hn} = 0 \quad (11)$$

and after substituting $s^{(l/r)}_{Hn} = D^{(l/r)}_H e^{ik_H n d_H}$ we obtain:

$$\left(\sin^2\frac{k_H d_H}{2}\left(\sin^2\frac{k_H d_H}{2} + \Delta\right) \pm \frac{\mathscr{E}}{A_H}\right)D^{(l/r)}_H = 0. \quad (12)$$

A straight clockwise-polarized wave ($D^l_H = 0$, $D^{(r)}_H \ne 0$) corresponds to two wavenumber values:

$$k^{(r)}_{H\pm}d_H = 2\arcsin\sqrt{\pm\sqrt{\frac{\Delta^2}{4} + \frac{\mathscr{E}}{A_H}} - \frac{\Delta}{2}}. \quad (13)$$

The boundary of the first band $k^{(r)}_{H\pm}d_H \cup [0\ldots 2\pi]$ corresponds to the energy interval $\mathscr{E} \cup \{0, (1+\Delta)A_H\}$, where $k^{(r)}_{H+}$ is real, and $k^{(r)}_{H-}$ is purely imaginary.

Wave numbers of left-polarized waves ($D^{(r)}_H = 0$, $D^{(l)}_H \ne 0$)

$$k^{(l)}_{H\pm}d_H = 2\arcsin\sqrt{\pm\sqrt{\frac{\Delta^2}{4} - \frac{\mathscr{E}}{A_H}} - \frac{\Delta}{2}} \quad (14)$$

are purely imaginary in the range of magnon energies $\mathscr{E} \cup [0, \frac{\Delta^2}{4}A_H]$. $k^{(l)}_{H\pm}$ have real and imaginary parts outside this range and are mutually conjugate, corresponding to volume-surface waves.

The energy dependences of the real and imaginary parts of the wave numbers are shown in Figure 1.

Thus, taking into account the long-range order of the exchange interaction leads to an increase of the order of the dispersion equation and a doubling of the number of waves. Its four solutions form a complete system within the framework of the linearized Landau-Lifshitz equation.

The lattice Hamiltonian in uniaxial ferromagnet with „easy axis“ anisotropy type (ox):

$$W_F = -\frac{1}{4}\sum_n (A_F \mathbf{S}_n\mathbf{S}_{n+1} + 2B_F S^2_{nx}) \quad (15)$$

(all constants are positive). The ferromagnet is uniformly magnetized in the direction of the axis x in the ground state. The linearized equation (3) for ferromagnet has the form:

$$-i\mathscr{E}S_F\mathbf{s}_n = [\mathbf{S}^{(0)}_n \times \boldsymbol{\chi}_{Fn}], \quad (16)$$

where

$$\boldsymbol{\chi}_{Fn} = -\frac{A_F}{4}(\mathbf{s}_{n+1} + \mathbf{s}_{n-1} - 2\mathbf{s}_n) + B_F(s_{ny}\mathbf{y} + s_{nz}\mathbf{z}). \quad (17)$$

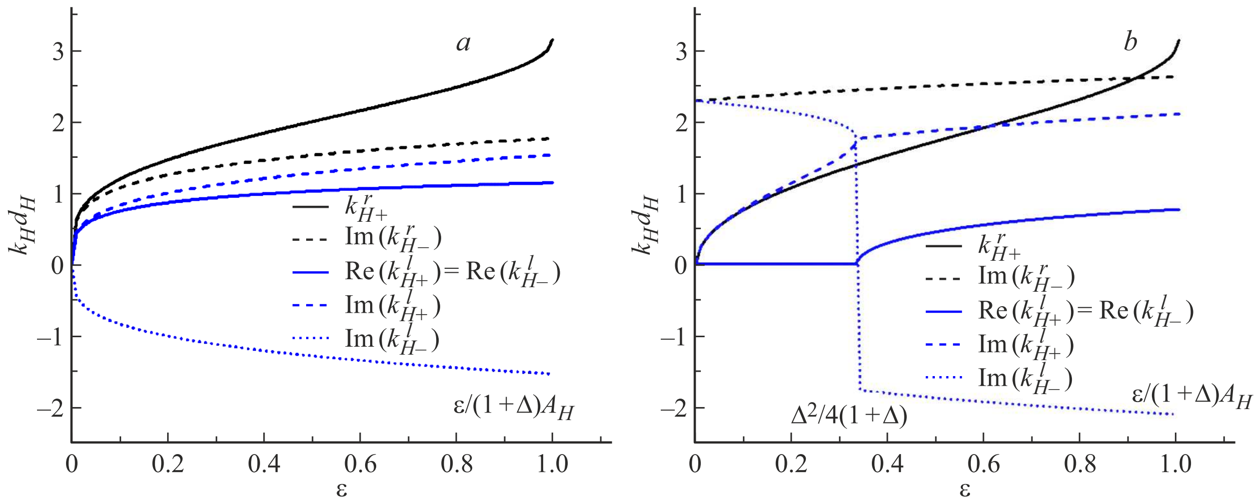

Figure 1. Dependence of the provided wave numbers $k_{H\pm}^{(r/l)} d_H$ on the magnonic energy for $\Delta = 0$ (a) and $\Delta = 2$ (b).

The solution of (16) is represented as:

$$s_{Fn}^{(l/r)} = D_{Fn}^{(l/r)} e^{i(k_F n d_F - \omega t)} \quad (18)$$

and after substitution we obtain:

$$\left(A_F \sin^2 \frac{k_F d_F}{2} + B_F \pm \mathscr{E}\right) D_{Fn}^{(l/r)} = 0 \quad (19)$$

The clockwise-polarized wave in the ferromagnet ($s_F^{(r)} \neq 0,\ s_F^{(l)} = 0$) has a real wavenumber, and the counterclockwise-polarized wave is purely imaginary:

$$k_F^{(r)} d_F = 2 \arcsin\left(\sqrt{\frac{\mathscr{E} - B_F}{A_F}}\right),$$

$$k_F^{(l)} d_F = 2i \operatorname{arsh}\left(\sqrt{\frac{\mathscr{E} + B_F}{A_F}}\right). \quad (20)$$

Having established the types of waves in a magnetic structure with competing exchange interactions and ferromagnet, we proceed to solving the problem of scattering and generation of electromagnetic waves by their interface.

4. ESW scattering by an isolated boundary ferromagnet-magnetic structure with competing exchange interactions

Let's consider the normal incidence of the ESW from ferromagnet ($z < 0$) on a magnetic structure with competing exchange interactions ($z > 0$). This choice of model is attributable to the presence of the fixing light axis x in the ferromagnet structure. Due to the ferromagnet coupling at the boundary, the ground state in the magnetic structures with competing exchange interactions is fixed along the axis x.

Since all waves have circular polarization in the considered model, only traveling waves are excited in case of FM coupling between ferromagnet and magnetic structure with competing exchange interactions in the ferromagnet above the activation energy — incident wave (with an amplitude taken as one) $k_F^{(r)}$ and reflected wave ($-k_F^{(r)}$), and while a travelling wave ($k_{H+}^{(r)}$) and an evanescent wave ($k_{H-}^{(r)}$) are excited in the magnetic structure with competing exchange. Waves with numbers $k_{H+}^{(l)}$ and $-k_{H-}^{(l)}$ are physical in case of AFM coupling in the magnetic structure with competing exchange interactions.

In each case, the amplitudes of three waves are to be determined, which requires three boundary conditions. Let's write down the Hamiltonian of the entire system, taking into account the exchange interaction of ferromagnetic type:

$$W = \frac{A_H}{16} \sum_{n \le 0} \left(\mathbf{S}_n \mathbf{S}_{n+2} - (4 + \Delta) \mathbf{S}_n \mathbf{S}_{n+1}\right)$$

$$- \frac{1}{4} \sum_{n \ge 0} (A_F \mathbf{S}_n \mathbf{S}_{n+1} + 2 B_F S_{nx}^2) - \sigma J \mathbf{S}_{F0} \mathbf{S}_{H0}, \quad (21)$$

where J is the constant of the interlayer exchange interaction, $\sigma \pm 1$ is for the ferromagnetic/antiferromagnetic interlayer interaction.

The boundary conditions are the dynamics equations for the boundary spins 0 and 1 for magnetic structures with competing exchange interactions and 0 for ferromagnet in the lattice model [6]:

$$i\mathscr{E}\mathbf{s}_{F0} + \left[\mathbf{x} \times \left(-\frac{A_F}{4}(\mathbf{s}_{F1} - \mathbf{s}_{F0}) + B_F\left((\mathbf{s}_{F0}\mathbf{y})\mathbf{y} + (\mathbf{s}_{F0}\mathbf{z})\mathbf{z}\right)\right.\right.$$

$$\left.\left. - \sigma J\left(\mathbf{s}_{H0} - \frac{s_H}{s_F}\mathbf{s}_{F0}\right)\right)\right] = 0,$$

$$i\mathscr{E}\mathbf{s}_{H0} + \left[\sigma \mathbf{x} \times \left(\frac{A_H}{16}\left((\mathbf{s}_{H2} - \mathbf{s}_{H0}) - 4(1+\Delta)(\mathbf{s}_{H1} - \mathbf{s}_{H0})\right)\right.\right.$$

$$\left.\left. - \sigma J\left(\mathbf{s}_{F0} - \frac{s_F}{s_H}\mathbf{s}_{H0}\right)\right)\right] = 0,$$

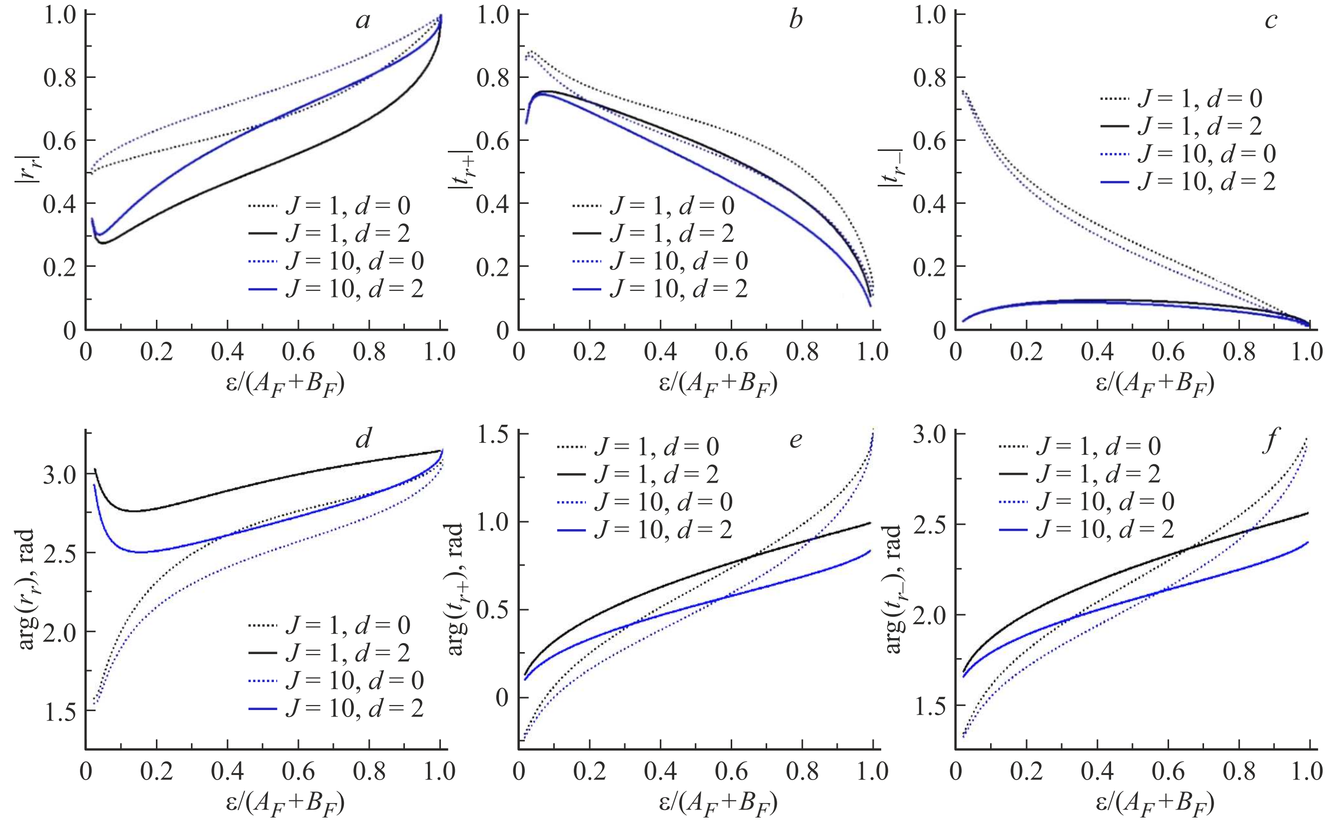

Figure 2. Coefficients (a–c) and phases (d–f) of scattering of clockwise-polarized waves for the values of the interlayer interaction and exchange constants in the first Brillouin zone indicated on the tab. a, d is the reflected wave; b, e is the transmitted volume wave; c, f is the transmitted surface wave. Dependencies are constructed for the following model values of constants $A_F = A_H = 1$ (relative units), $S_F = S_H = 1$ (relative units), $B_F = 0.01$.

$$i\mathscr{E}\mathbf{s}_{H1} + \left[\sigma\mathbf{x}\times\left(\frac{A_H}{16}\left((\mathbf{s}_{H3}-\mathbf{s}_{H1}) - 4(1+\Delta)(\mathbf{s}_{H2}-\mathbf{s}_{H0}) - 2\mathbf{s}_{H1}\right)\right)\right] = 0. \quad (22)$$

On the one hand, their structure differs from the equations for internal spins. On the other hand, their solution also represents a wave.

The equations (22) have the following form after linearization and transition to cyclic variables:

$$(B_F \pm \mathscr{E})s_{F0}^{(l/r)} - \frac{A_F}{4}\left(s_{F1}^{(l/r)} - s_{F0}^{(l/r)}\right) - \sigma J\left(s_{H0}^{(l/r)} - \frac{s_H}{s_F}s_{F0}^{(l/r)}\right) = 0,$$

$$\pm\sigma\mathscr{E}s_{H0}^{(l/r)} + \frac{A_H}{16}\left((s_{H2}^{(l/r)} - s_{H0}^{(l/r)}) - 4(1+\Delta)\times(s_{H1}^{(l/r)} - s_{H0}^{(l/r)})\right) - \sigma J\left(s_{F0}^{(l/r)} - \frac{s_F}{s_H}s_{H0}^{(l/r)}\right) = 0,$$

$$\pm\sigma\mathscr{E}s_{H1}^{(l/r)} + \frac{A_H}{16}\left((s_{H3}^{(l/r)} - s_{H1}^{(l/r)}) - 4(1+\Delta)\times(s_{H2}^{(l/r)} + s_{H0}^{(l/r)} - 2s_{H1}^{(l/r)})\right) = 0. \quad (23)$$

It is necessary to take into account that the incident wave has a clockwise-polarized, and let us represent the dynamic spin components in each medium as:

$$s_{Fn}^{(r)} = 1\cdot e^{ik_F^{(r)}nd_F} + r_r e^{-ik_F^{(r)}nd_F} \quad (n = 0, -1, -2, \ldots),$$

$$s_{Hn}^{(r)} = t_{r+}e^{ik_{H+}^{(r)}nd_H} + t_{r-}e^{ik_{H-}^{(r)}nd_H} \quad (n = 0, 1, 2, \ldots). \quad (24)$$

By substituting (24) into (23), we obtain a system for the amplitudes for $\sigma = +1$. Counterclockwise-polarized waves are not excited by the clockwise-polarized field of the original wave due to the opposite chirality [9,10]. The dependences of the scattering coefficients obtained from (23) and their phases on the magnonic energy are shown in Figure 2.

5. Generation of ESW by an isolated boundary of the magnetic structures with competing exchange interactions section

Let us consider a model in which the ground state is homogeneous ($\Delta > 0$) and introduce a fixing uniaxial anisotropy field $\frac{B_H}{2}S_{nx}^2$ and an external variable pumping

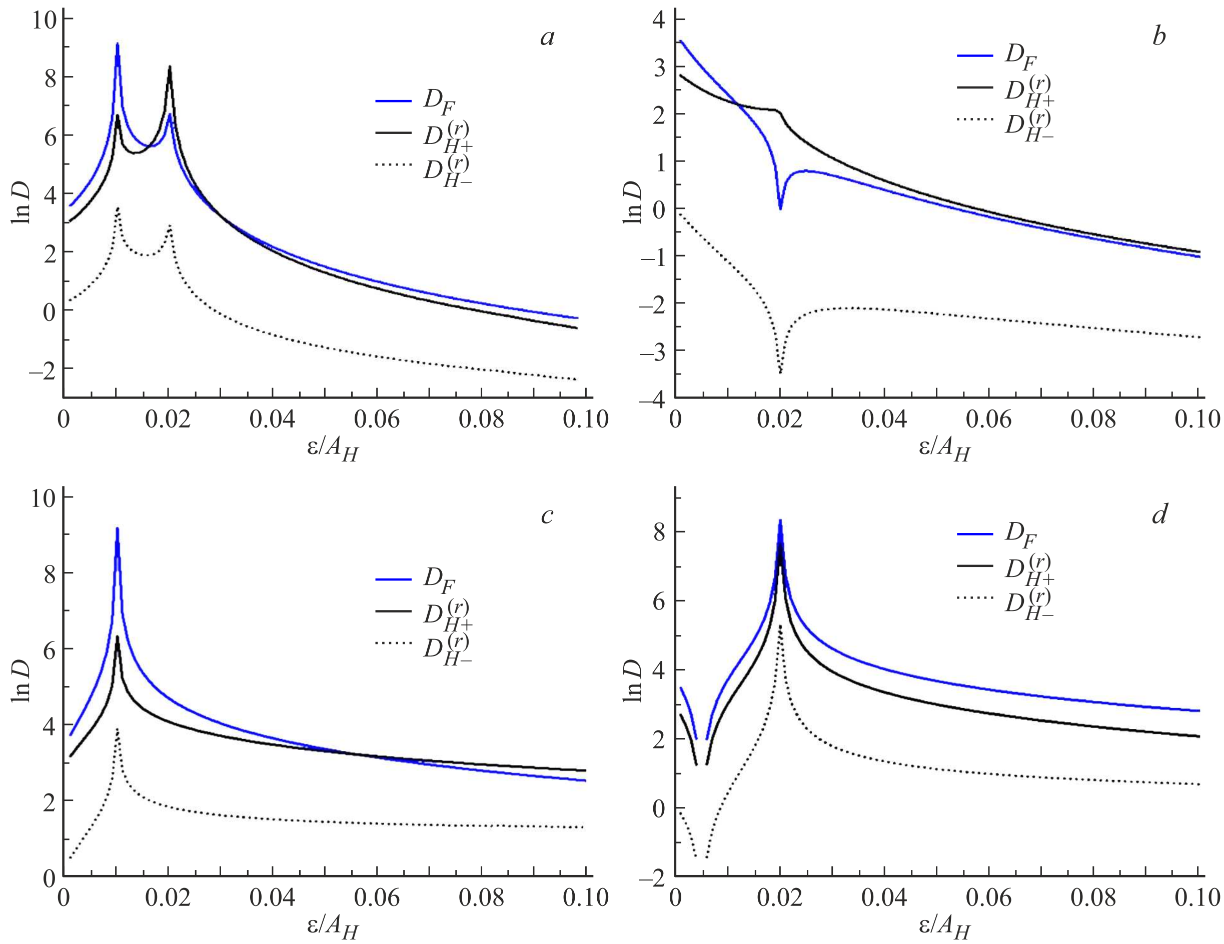

Figure 3. Logarithms of the relative amplitudes of wave generation for the cases indicated in the table with a single amplitude of the circular pumping field. The blue curve corresponds to the wave in FM, the black solid curve corresponds to the volume wave, dotted curve corresponds to surface waves in magnetic structures with competing exchange interactions. The values of the constants $\Delta = 2$, $J = A_F = A_H = 1$ (relative units), $S_F = S_H = 1$ (relative units), $B_F = 0.01$, $B_H = 0.02$. Hilbert attenuation constants $\alpha_F = \alpha_H = 0.01$.

field $\mathbf{h} \cdot \mathbf{S}_n$ into the Hamiltonian (21). The equations of dynamics in an unlimited magnetic structure with competing exchange interactions in this case, taking into account Hilbert attenuation:

$$-i\mathscr{E}\mathbf{s}_n = [\sigma \mathbf{x} \times (\boldsymbol{\chi}_{Hn} - \mathbf{h} - i\mathscr{E}\alpha_H \mathbf{s}_n)] \tag{25}$$

they will have the following form in cyclic variables

$$\chi_{Hn}^{(l/r)} + (B_H + \mathscr{E}(\pm\sigma - i\alpha_H))s_{Hn}^{(l/r)} = h_{l/r}, \tag{26}$$

where $\chi_{Hn}^{(l/r)}$ are determined by the formula (10).

The solution (26) contains a general solution of the homogeneous equation $s_n^{(U)}$ in the form of a superposition of waves (24) and a partial solution of the heterogeneous equation in each medium corresponding to homogeneous oscillations $s_{(h)Hn}^{(l/r)} = \text{const}$:

$$s_{(h)H}^{(l/r)} = \frac{h_{l/r}}{B_H + \mathscr{E}(\pm\sigma - i\alpha_H)}. \tag{27}$$

Wave numbers (13)–(14), taking into account uniaxial anisotropy and attenuation, will be written as:

$$k_{H\pm}^{(r)} d_H = 2 \arcsin \sqrt{\pm\sqrt{\frac{\Delta^2}{4} + \frac{(\sigma + i\alpha_H)\mathscr{E} - B_H}{A_H}} - \frac{\Delta}{2}},$$

$$k_{H\pm}^{(l)} d_H = 2 \arcsin \sqrt{\pm\sqrt{\frac{\Delta^2}{4} - \frac{(\sigma - i\alpha_H)\mathscr{E} + B_H}{A_H}} - \frac{\Delta}{2}}. \tag{28}$$

Let us consider four cases corresponding to different polarizations of the pumping field and the ground state of the magnetic structure with competing exchange interactions, for which the possible types of waves are indicated in the table.

The boundary conditions similar to (23) will have the following form by virtue of (27):

$$-\frac{A_F}{4}(s_{F1}^{(l/r)} - s_{F0}^{(l/r)}) - \sigma J(s_{H0}^{(l/r)} - \frac{s_H}{s_F} s_{F0}^{(l/r)}) = 0,$$

Polarization of the pumping field and the ground state of the magnetic structures with competing exchange interactions

	Polarization of pumping field	Polarization of ground state of magnetic structures with competing exchange interactions, σ	Waves in FM	Waves in magnetic structure with competing exchange interactions
a	Clockwise ($h_- = 1$, $h_+ = 0$)	$+1$	$-k_F^{(r)}$	$k_{H+}^{(r)}$, $-k_{H-}^{(r)}$
b	Clockwise ($h_- = 1$, $h_+ = 0$)	-1	$-k_F^{(r)}$	$k_{H+}^{(r)}$, $-k_{H-}^{(r)}$
c	Counterclockwise ($h_+ = 1$, $h_- = 0$)	$+1$	$-k_F^{(l)}$	$k_{H+}^{(l)}$, $-k_{H-}^{(l)}$
d	Counterclockwise ($h_+ = 1$, $h_- = 0$)	-1	$-k_F^{(l)}$	$k_{H+}^{(l)}$, $-k_{H-}^{(l)}$

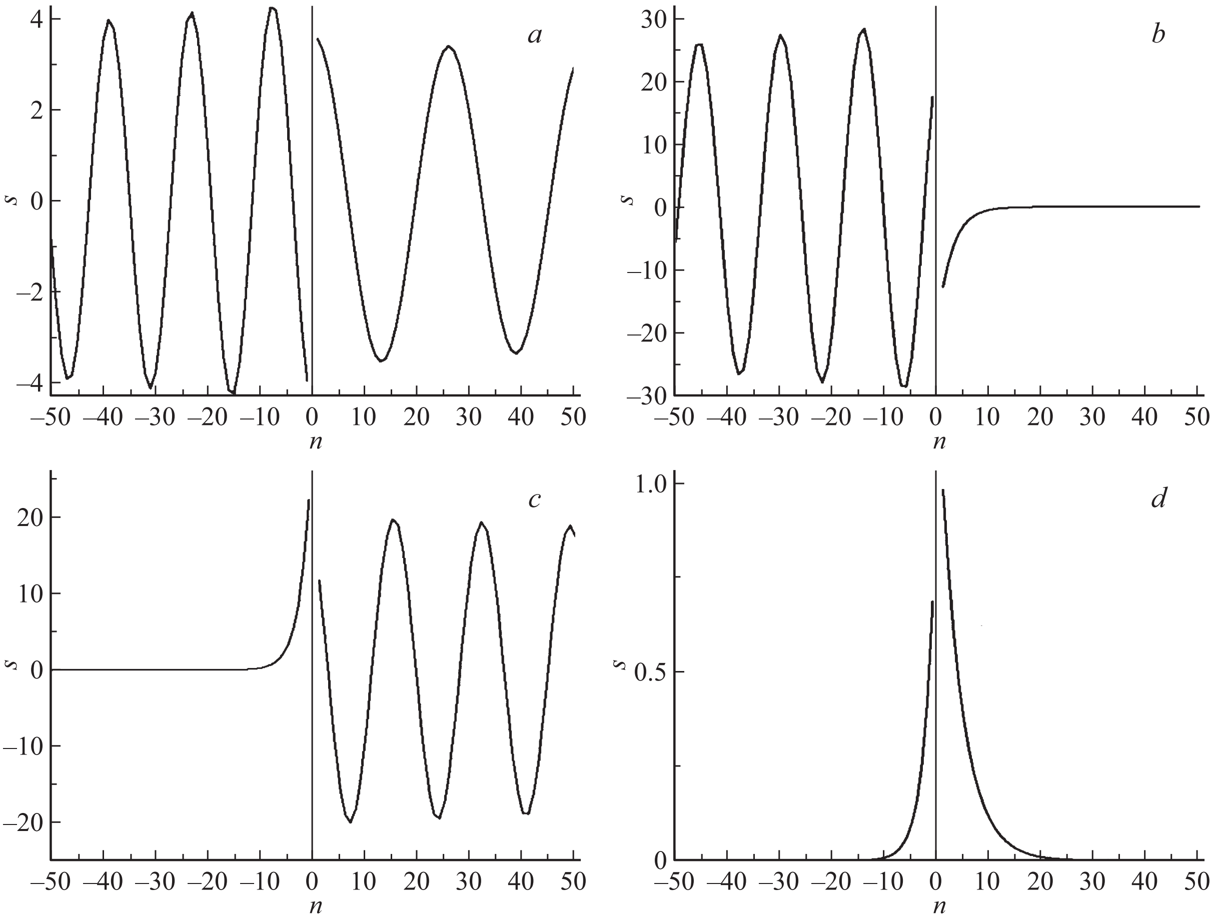

Figure 4. The distribution of the dynamic spin field in terms of the amplitude of the pumping field relative to the boundary along the normal to it for the cases of equilibrium orientation in the ferromagnet and in the magnetic structure with competing exchange interactions shown in the table.

$$\frac{A_H}{16}\Big((s_{H2}^{(l/r)} - s_{H0}^{(l/r)}) - 4(1+\Delta)(s_{H1}^{(l/r)} - s_{H0}^{(l/r)})\Big) - \sigma J(s_{F0}^{(l/r)} - \frac{s_F}{s_H} s_{H0}^{(l/r)}) = 0,$$

$$\frac{A_H}{16}\Big((s_{H3}^{(l/r)} - s_{H1}^{(l/r)}) - 4(1+\Delta)(s_{H2}^{(l/r)} + s_{H0}^{(l/r)} - 2s_{H1}^{(l/r)})\Big) = 0, \qquad (29)$$

where the following should be substituted:

$$s_{Fn}^{(r)} = D_F^{(r)} e^{-ik_F^{(r)} n d_F} + \frac{h_r}{B_F - \mathscr{E}(1 + i\alpha_F)},$$

$$s_{Fn}^{(l)} = D_F^{(l)} e^{-ik_F^{(l)} n d_F} + \frac{h_l}{B_F + \mathscr{E}(1 - i\alpha_F)}, \quad (n = 0, -1, -2, \ldots),$$

$$s_{Hn}^{(r)} = D_{H+}^{(r)} e^{ik_{H+}^{(r)} n d_H} + D_{H-}^{(r)} e^{ik_{H-}^{(r)} n d_H} + \frac{h_r}{B_H - \mathscr{E}(\sigma + i\alpha_H)},$$

$$s_{Hn}^{(l)} = D_{H+}^{(l)} e^{ik_{H+}^{(l)} n d_H} + D_{H-}^{(l)} e^{-ik_{H-}^{(l)} n d_H} + \frac{h_l}{B_H + \mathscr{E}(\sigma - i\alpha_H)}, \quad (n = 0, 1, 2, \ldots). \qquad (30)$$

The solution (29), when decompositions (30) are substituted into it, yields the desired generation amplitudes shown in Figure 3, which are determined by the value of the coupling constant J and the difference in the susceptibilities of the interfacing media. The calculated distribution of the dynamic spin field for the cases of equilibrium orientation in the ferromagnet and in the magnetic structure with competing exchange interactions is shown in Figure 4.

6. Conclusion

The magnon spectrum for a ferromagnet is obtained in this study taking into account the long-range order of the exchange interaction. It has been established that volume-surface waves occur in such structures in addition to evanescent waves, which differ in the direction of precession and have chirality opposite to volume and evanescent waves. The scattering and generation of ESW at the boundary of such a structure with a ferromagnet are considered in the framework of the lattice model, since the continuum approximation is not applicable for structures with a long-range interaction order.

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Conflict of interest

The author declares that he has no conflict of interest.

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