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Modes and threshold condition of a gradient waveguide with inhomogeneous amplification and absorption.

© A.A. Gladky, N.N. Rosanov

Ioffe Institute, St. Petersburg, Russia

e-mail: gladkij.aa@edu.spbstu.ru

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An analysis of the modes of a linear gradient optical waveguide with a quadratic radial dependence of absorption and amplification has been conducted. In this case, the profiles of the refractive index and absorption/amplification differ. The parameters at which the generation threshold is achieved have been calculated, meaning that the absorption of radiation is compensated by the amplification, which ensures the propagation of a radiation beam with a constant amplitude.

Keywords: gradient waveguide, amplification, threshold condition, gaussian beams.

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Introduction

Diffraction widening of radiation accompanied with reduction in its intensity may be compensated by a heterogeneous profile of the medium refractive index — a case of gradient optical waveguides [1]. However, in actual media the radiation is absorbed, which causes gradual loss of intensity. The profiles of the refractive index and absorption differ in the general case, usually the absorption may be considered to be spatially homogeneous. Therefore, it becomes necessary to compensate for such absorption by amplification, which is substantially heterogeneous and dominates in the axial area of the waveguide. This is the objective of this message. Note that this task is relevant in some problems of non-linear optics, for example, for solitons of self-induced transparency [2–4].

Theoretical description

Distribution of a beam of monochromatic radiation with electric intensity $\tilde{E}$ and frequency ω in the linear medium with dielectric permittivity ε, which depends on the distance to the axis of waveguide r, is described with a Helmholtz equation

$$\Delta\tilde{E} + \frac{\omega^2}{c^2}\varepsilon(r)\tilde{E} = 0, \qquad (1)$$

where c — light speed.

Let us proceed from the equation for amplitude $\tilde{E}$ to equation for envelope [5–7], substituting $\tilde{E} = E\exp(ik_0z)$ to equation (1):

$$2ik_0\frac{\partial E}{\partial z} + \Delta_\perp E + k_0^2\frac{\delta\varepsilon(r)}{\varepsilon_0}E = 0, \qquad (2)$$

where E — slowly changing envelope, $\varepsilon(r) = \varepsilon_0 + \delta\varepsilon(r)$, $k_0^2 = (\omega/c)^2\varepsilon_0$, $\Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, x and y — transverse Cartesian coordinates, the radiation spreads along axis z. We are interested in the axisymmetric modes without angular dependence. For them in the cylindrical coordinate system

$$E = A(r)\exp(i\delta kz).$$

In the medium with absorption the additive to wave number δk in the general case is complex. We will obtain the ordinary differential equation for $A(r)$:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dA}{dr}\right) - 2k_0\delta kA + k_0^2\frac{\delta\varepsilon}{\varepsilon_0}A = 0. \qquad (3)$$

We will search for a solution in the form of a Gaussian beam $A(r) = A_0\exp(-\gamma r^2)$ [1]. In the considered case the value γ is complex, $\mathrm{Im}\gamma$ is responsible for the curvature of the wave front. After substituting in (3) we will obtain

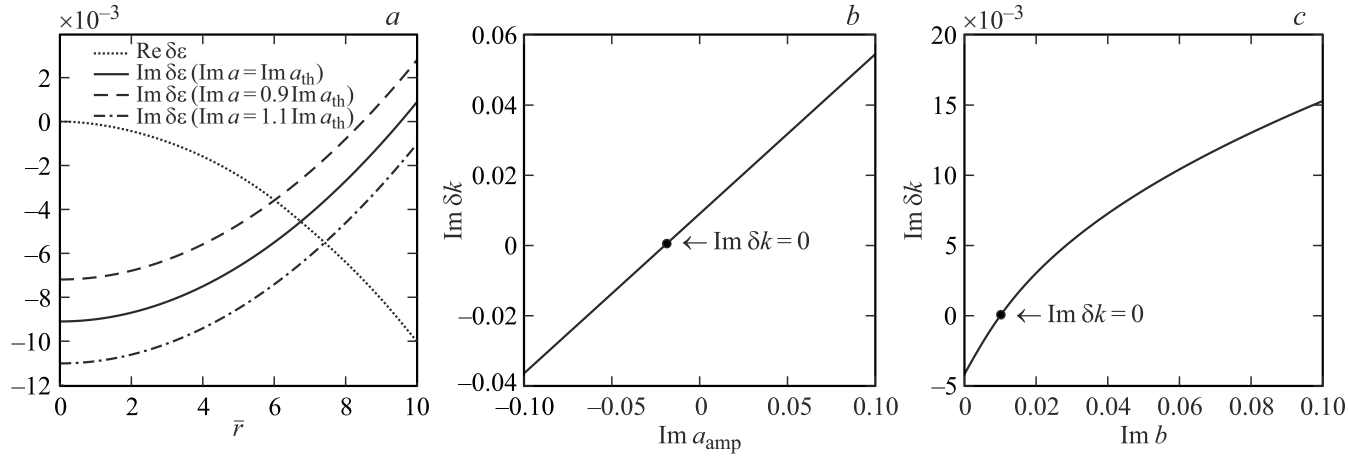
$$4\gamma(1-\gamma r^2) + 2k_0\delta k = k_0^2\frac{\delta\varepsilon}{\varepsilon_0}. \qquad (4)$$

From (4) you can see a suitable view of $\delta\varepsilon$: $\delta\varepsilon(r) = a + b(\frac{r}{r_0})^2$, specified constants a and b — are complex. Equating terms at different degrees r, we will obtain two complex equations for complex variables δk and γ:

$$4\gamma^2 = -\frac{k_0^2}{\varepsilon_0}\frac{b}{r_0^2},$$

$$4\gamma + 2k_0\delta k = \frac{k_0^2}{\varepsilon_0}a. \qquad (5)$$

The threshold condition is met at $\mathrm{Im}\delta k = 0$, this limits the parameters a and b. From the first equation in (5) we find one value γ (we need $\mathrm{Re}\gamma > 0$). Then from the second equation in (5) we also find one value δk. Nonzero value $\mathrm{Im}\delta k$ maintains the exponential multiplier $\exp(-\mathrm{Im}\delta kz)$, at



(a) Profiles of real (dotted line) and imaginary parts of additive $\delta\varepsilon$ to dielectric permittivity ε depending on dimensionless distance from axis z. The solid line corresponds to the threshold case $\mathrm{Im}a = (\mathrm{Im}a)_{\mathrm{th}}$, the dashed line — to case $\mathrm{Im}a = 0.9(\mathrm{Im}a)_{\mathrm{th}}$, the dot and dash line — $\mathrm{Im}a = 1.1(\mathrm{Im}a)_{\mathrm{th}}$. ($b$) Change in value $\mathrm{Im}\delta k$ depending on $\mathrm{Im}a_{amp}$ at $\mathrm{Im}b = 0.01$ and $\mathrm{Im}a_{\mathrm{loss}} = 0.01$. The threshold value is achieved at $\mathrm{Im}a_{\mathrm{amp}} \approx -0.0191$. ($c$) Change in value $\mathrm{Im}\delta k$ depending on $\mathrm{Im}b$ at $\mathrm{Im}a_{\mathrm{amp}} = -0.0191$ and $\mathrm{Im}a_{\mathrm{loss}} = 0.01$. The threshold value is achieved at $\mathrm{Im}b \approx 0.01$.

$\mathrm{Im}\delta k > 0$ the radiation fades, and at $\mathrm{Im}\delta k < 0$, it amplifies on the contrary. Therefore, we obtain the following ratios:

$$\mathrm{Re}\gamma = \frac{\omega}{\sqrt{2}r_0 c}\sqrt{|b| - \mathrm{Re}b},$$

$$\mathrm{Im}\gamma = -\frac{\omega}{\sqrt{2}r_0 c}\frac{\mathrm{Im}b}{\sqrt{|b| - \mathrm{Re}b}},$$

$$\mathrm{Re}\delta k = -\frac{1}{\sqrt{\varepsilon_0}}\left[\frac{1}{\sqrt{2}r_0}\sqrt{|b| - \mathrm{Re}b} - \frac{\omega}{2c}\mathrm{Re}a\right],$$

$$\mathrm{Im}a = -\frac{\sqrt{2}c}{r_0\omega}\frac{\mathrm{Im}b}{\sqrt{|b| - \mathrm{Re}b}}. \quad (6)$$

We assume the profile of the real refractive index (values $\mathrm{Re}a$ and $\mathrm{Re}b$) to be set. And values $\mathrm{Im}a$ (proportionately to the amplification at the waveguide axis) and $\mathrm{Im}b$ (determines the radial profile of amplification/absorption) vary. The threshold condition is achieved at the ratio of two last parameters specified in the last equation (6). Using this ratio, one can find the required value of amplification on the waveguide axis depending on the steepness of the amplification/absorption profile. Without loss of generality, it may be assumed that $\mathrm{Re}a = 0$, and for radiation localization $\mathrm{Re}b < 0$ must be assumed. Initially the medium is absorbing. Parameter $\mathrm{Im}a$ may be assumed as $\mathrm{Im}a = \mathrm{Im}a_{\mathrm{loss}} + \mathrm{Im}a_{\mathrm{amp}}$, where $\mathrm{Im}a_{\mathrm{loss}}$ determines the initial absorption of the medium and is a known value, and $\mathrm{Im}a_{\mathrm{amp}}$ is one of the amplification parameters. If parameters $\mathrm{Im}a_{\mathrm{amp}}$ and $\mathrm{Im}b$ are chosen properly, the above propagation of radiation is achieved, namely:

$$\mathrm{Im}a_{\mathrm{amp}} = -\mathrm{Im}a_{\mathrm{loss}} - \frac{\sqrt{2}c}{r_0\omega}\frac{\mathrm{Im}b}{\sqrt{|b| - \mathrm{Re}b}}. \quad (7)$$

Real parts a and b set the waveguide parameters:

$$\mathrm{Re}\varepsilon = \varepsilon_0 + \mathrm{Re}a + \mathrm{Re}b\left(\frac{r}{r_0}\right)^2 = \varepsilon_0 - |\mathrm{Re}b|\left(\frac{r}{r_0}\right)^2. \quad (8)$$

Imaginary parts a and b determine the nature of the change in the medium absorption or amplification depending on the distance to the main axis of radiation propagation z:

$$\mathrm{Im}\varepsilon = \mathrm{Im}a + \mathrm{Im}b\left(\frac{r}{r_0}\right)^2. \quad (9)$$

At $r = 0$ in the medium with amplification $\mathrm{Im}a < 0$, when $\mathrm{Im}\varepsilon$ changes the sign, the transition to radiation absorption at the periphery takes place. Let us specify parameter $\mathrm{Im}a$, meeting the threshold condition (7), as $(\mathrm{Im}a)_{\mathrm{th}}$. This corresponds to the case when amplification on axis z compensates diffraction and absorption of radiation at the periphery (fig. a). If value $|\mathrm{Im}a|$ is lower than threshold value $|(\mathrm{Im}a)_{\mathrm{th}}|$, amplification will not be sufficient to compensate for the processes causing damping of the radiation amplitude. If $|\mathrm{Im}a|$ exceeds $|(\mathrm{Im}a)_{\mathrm{th}}|$, an opposite situation will occur, when the amplification dominates the absorption. Fig. b, c reflects the change in value $\mathrm{Im}\delta k$ depending on the amplification parameters.

Conclusion

The presented analytical review shows the possibility of compensation for the radiation absorption in the gradient waveguide by amplification. A threshold condition is found, which specifies the ratio for the parameters of dielectric permittivity, when the radiation spreads without a change in the amplitude. The described effects may manifest themselves in the gradient optic fiber doped with active centers with the specified concentration profile.

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Conflict of interest

The authors declare that they have no conflict of interest.

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