# Spectrum of positrons produced due to interaction of gamma-ray quanta with X-ray pulse of curvature radiation

© A.V. Khalyapin[1], S.V. Bobashev[2], D.P. Barsukov[2]

[1] HSE University, St. Petersburg, Russia
[2] Ioffe Institute, St. Petersburg, Russia
E-mail: bars.astro@mail.ioffe.ru

*Received May 3, 2024*
*Revised August 7, 2024*
*Accepted October 30, 2024*

The electron-positron pair production due to the interaction of gamma-ray quanta with short X-ray pulse of curvature radiation is considered. The positron spectrum and pair production probability has found.

**Keywords:** X-ray pulse, gamma-quanta, coherent radiation.

DOI: 10.61011/TPL.2024.12.60340.6600k

An electron beam moving in circular orbits at ultrarelativistic velocities generates highly collimated and, in the general case, polarized magnetobremsstrahlung radiation. Although the formation region of a pulse is macroscopic in length ($\sim 0.1\,\mathrm{mm}-10\,\mathrm{m}$), its radiation frequency may well lie within the X-ray or gamma range. The pulse length turns out to be comparable to its (inverse) frequency [1]. If a single bunch of particles enters the pulse formation region, its radiation becomes (at least partially) coherent. Such pulses may be produced at specialized synchrotron radiation sources, including possibly the „SKIF" synchrotron radiation source [2] that is currently under construction. These pulses may also be generated by radio pulsars.

The process of production of an electron–positron pair from one gamma quantum in vacuum contradicts directly the laws of conservation of energy and momentum. However, a strong electromagnetic field may establish conditions under which the production of an electron–positron pair from one gamma quantum becomes consistent with these laws. This process was considered, for example, in the case of a plane monochromatic wave [3–5] and a long almost monochromatic pulse [6].

In the present study, we discuss the process of production of an electron–positron pair from one gamma quantum in the field of an X-ray pulse. The X-ray pulse is regarded as a plane electromagnetic wave with its profile coinciding with the profile of a magnetobremsstrahlung pulse emitted by an ultrarelativistic particle moving along a circle of radius $\rho$. Its vector potential $\mathbf{A}(\mathbf{x}, t)$ may then be written in the following approximate form [7]:

$$\mathbf{A}(\mathbf{x}, t) = \mathbf{A}_0 \frac{\eta}{1+\eta^2}, \quad t - z = \frac{\eta}{\omega}\Big(1 + \frac{\eta^2}{3}\Big), \tag{1}$$

where $\mathbf{x} = r\mathbf{e}_z$ is the observation point, amplitude $\mathbf{A}_0 = -2q\gamma/r\mathbf{e}_x$, $q$ is the particle charge, and $\omega = 2\gamma^3/\rho$ is the X-ray pulse frequency. Vector potential (1) far from the source (i.e., at large $r$) may be written in the approximate form $\mathbf{A} = A(t-z)\cdot\mathbf{e}_x$ (a plane linearly polarized electromagnetic wave propagating along axis $z$). The width of this pulse is $\delta t_{50} \approx 42.1/\omega$, and its spectrum takes the form

$$\frac{d\mathscr{E}}{dw} = \frac{A_0^2}{3\pi^2}\left(\frac{w}{\omega}\right)^2 K_{2/3}^2\left(\frac{2}{3}\frac{w}{\omega}\right), \tag{2}$$

where $w$ is frequency and $\mathscr{E}$ is energy. Its maximum is reached at frequency $w_{\max} \approx 0.83\omega$; therefore, coefficient $\omega$ may be regarded as the characteristic pulse frequency and, since $\delta t_{50} w_{\max} \approx 35$, it can be said that the pulse duration corresponds approximately to the characteristic (inverse) frequency of the pulse. Formula (2) matches the spectrum from expression (3.37) [8] at angle $\beta = 0$ (i.e., for radiation strictly co-directional with the particle motion).

The wave function of an electron in the field of a plane electromagnetic wave $A(x)$ may be written as [3,6]

$$\Psi_p(x) = \frac{1}{\sqrt{2E}}\left[1 + \frac{e\hat{k}\hat{A}}{2(kp)}\right]\exp(iS)u_p,$$

$$S = -(px) - e\int_0^\eta \left(\frac{pA(\varphi)}{kp} - \frac{e^2}{2}\frac{A^2(\varphi)}{kp}\right)d\varphi,$$

where $p = (E, \mathbf{p})$ is the electron 4-momentum, $u_p$ is the Dirac bispinor, and $k = (\omega, \mathbf{k})$ is the 4-momentum of an external wave such that $k^2 = 0$ and $kA = 0$; the normalizing volume is assumed to be equal to unity. The following notation is also introduced: scalar product of 4-vectors $(ab) = a^\mu b_\mu$ and convolution $\hat{a} = (a\gamma)$ of a $\gamma$-matrix with 4-vector $a$.

The amplitude of electron–positron pair production in the field of a short X-ray pulse takes the form

$$S_{fi} = -ie\int d^4x\overline{\Psi}_-(x)\hat{\mathscr{A}}(x, k')\Psi_+(x), \tag{3}$$

where wave function of a gamma quantum $\mathscr{A}(x, k') = \sqrt{2\omega'}\varepsilon\exp(ik'x)$; $\varepsilon$ and $k' = (\omega', \mathbf{k}')$ are the polarization 4-vector and the 4-momentum of a gamma

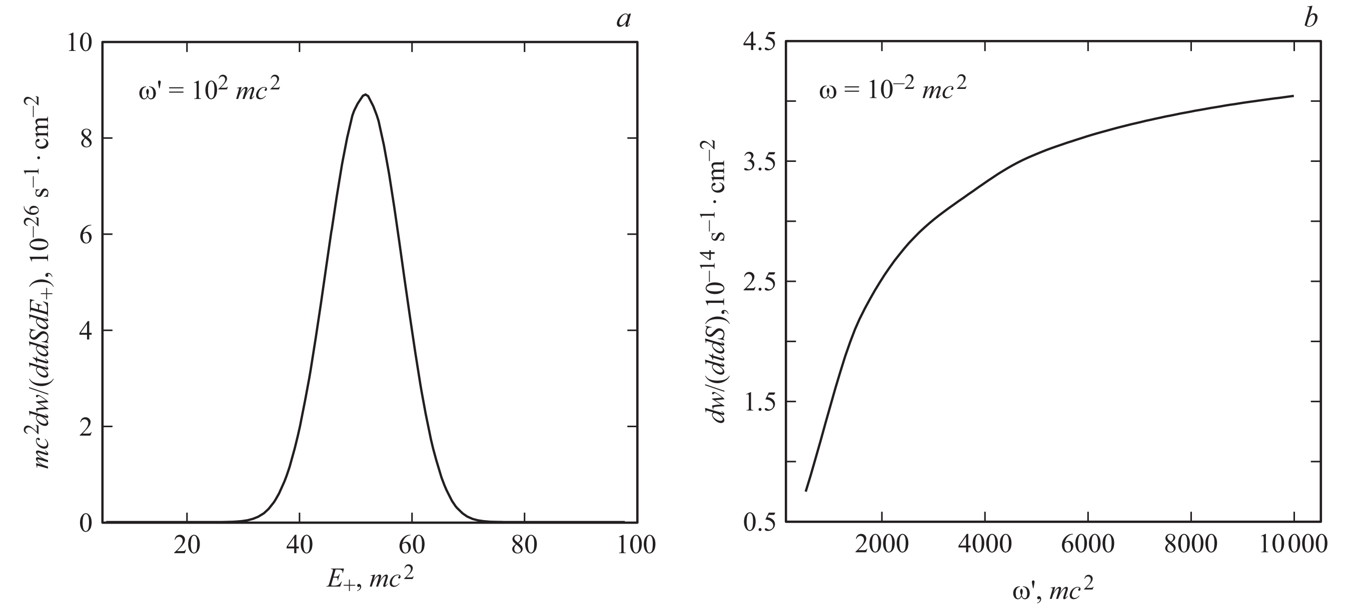


**Figure 1.** *a* — Spectrum of produced positrons at $\omega' = 10^2 mc^2$. *b* — Dependence of the probability of positron production on energy $\omega'$ of a gamma quantum. In both cases, $\omega = 10^{-2} mc^2$ and $qA_0 = 10^{-3} mc^2$.

quantum, respectively. Here and elsewhere, subscripts $+$ and $-$ indicate that a certain quantity corresponds to a positron or an electron, respectively.

Integration in $x$, $y$, $z$ and $\xi = \omega(t - z)$ in (3) yields delta functions and integrals $J_0$, $J_1$, and $J_2$, respectively:

$$S_{fi} = ie(2\pi)^3 \frac{T_{fi}}{2\omega\sqrt{2\omega' E_- E_+}}\, \delta(\Delta_z - \Delta_t)\delta(\Delta_x)\delta(\Delta_y),$$

$$T_{fi} = \bar{v}\hat{\epsilon} u J_0 + e\bar{v}\left(\frac{\hat{\epsilon}\hat{k}\hat{A}_0}{2\omega\varkappa_-} - \frac{\hat{A}_0\hat{k}\hat{\epsilon}}{2\omega\varkappa_+}\right) u J_1 + \frac{eA_0^2(k\epsilon)}{2\omega^2\varkappa_-\varkappa_+}\bar{v}\hat{k} u J_2,$$

where $\bar{v}$ is the bispinor corresponding to a positron, $\Delta^i = p^i_+ + p^i_- - k'^i$ ($p^i$ and $k'^i$ are momenta components) and $\varkappa_\pm = E_\pm - p^z_\pm$, and

$$J_0 = \int\limits_{-\infty}^{+\infty} (1 + \chi^2) e^{i\varphi(\chi)} d\chi, \quad J_1 = \int\limits_{-\infty}^{+\infty} \chi e^{i\varphi(\chi)} d\chi,$$

$$J_2 = \int\limits_{-\infty}^{+\infty} \frac{\chi^2}{1 + \chi^2} e^{i\varphi(\chi)} d\chi,$$

$$\varphi(\eta) = \frac{E^+ - E^- - \omega'}{\omega}\eta\left(1 + \frac{\eta^2}{3}\right) + \frac{eA_0}{2\omega}\left(\frac{p^x_-}{\varkappa_-} - \frac{p^x_+}{\varkappa_+}\right)\eta^2$$
$$- \frac{e^2A_0^2}{2\omega}\left(\frac{1}{\varkappa_-} + \frac{1}{\varkappa_+}\right)(\eta - \operatorname{arctg}\eta).$$

The squared modulus of scattering amplitude $T_{fi}$ summed over the polarizations of an electron and a positron and averaged over the polarization of a photon takes the form

$$|T_{fi}|^2 = 8|J_0|^2\big((p_-p_+) + 2m^2\big) - 8e^2A_0^2\mathrm{Re}(J_0J_2^*)$$
$$- 4A_0^2|J_1|^2\left(\frac{\varkappa_+}{\varkappa_-} + \frac{\varkappa_-}{\varkappa_+}\right)$$
$$+ 8\mathrm{Re}(J_0J_1^*)(\varkappa_+ + \varkappa_-)\left(\frac{A_0p_-}{\varkappa_-} - \frac{A_0p_+}{\varkappa_+}\right).$$

The differential probability of production of an electron–positron pair per unit time and unit area within element $d^3p_+d^3p_-$ of the momentum space of a positron and an electron at a unit photon flux is given by

$$dW_{fi} = |S_{fi}|^2\frac{d^3p_-d^3p_+}{(2\pi)^6} = e^2\delta(\Delta_z - \Delta_t)\delta(\Delta_x)\delta(\Delta_y)$$
$$\times |T_{fi}|^2\frac{d^3p_-d^3p_+}{8\omega^2\omega' E_-E_+}\, dSdt.$$

Integration over the positron directions yields the probability of production of a positron per unit time within the energy interval

$$\frac{dW_{fi}}{dtdSdE_+} = e^2\int |T_{fi}|^2\frac{|\mathbf{p}_+|d\Omega_+}{8\varkappa_-(p_+)\omega^2\omega'}.$$

In $T_{fi}$, a dependence of the energy and momentum of an electron on the momentum of a positron arises after removal of the integral over $d^3p_-$ by delta functions. The resulting spectra at amplitude $qA_0 = 10^{-3}mc^2$ and several values of $\omega'$ and $\omega$ are shown in panels *a* of Figs. 1–3. The presented spectra have a single-peak profile. It follows from Figs. 1, *a* and 2, *a* that the peak width

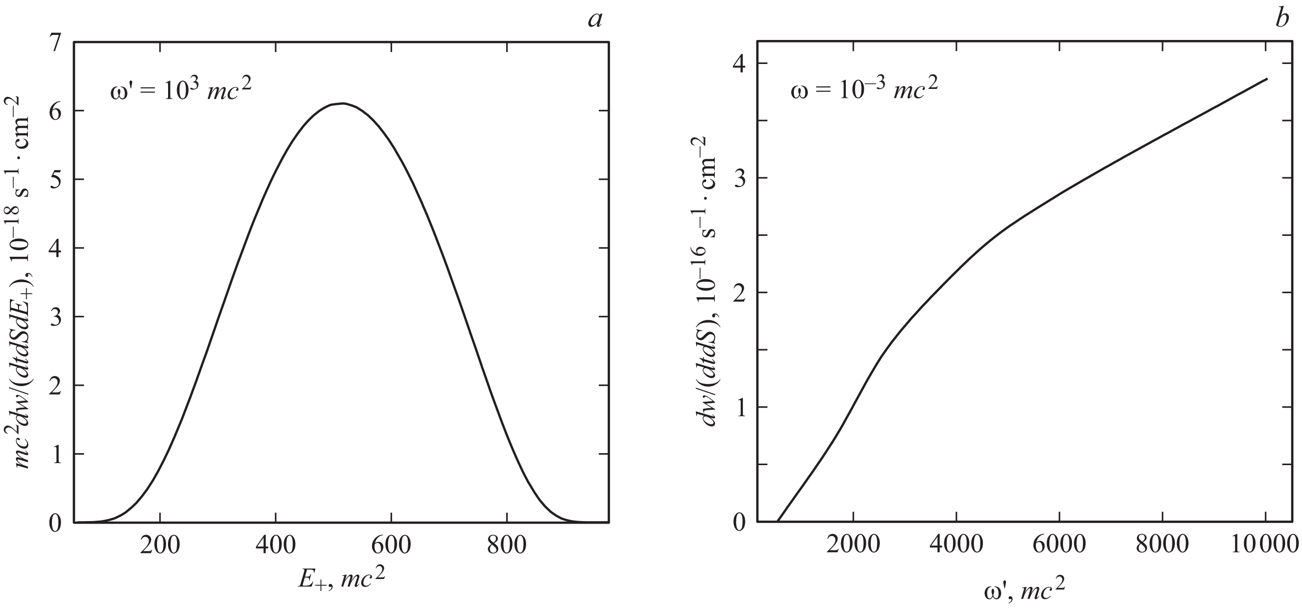


**Figure 2.** $a$ — Positron spectrum at $\omega' = 10^3 mc^2$. $b$ — Dependence of the probability of positron production on energy $\omega'$ of a gamma quantum. In both cases, $\omega = 10^{-3} mc^2$ and $qA_0 = 10^{-3} mc^2$.

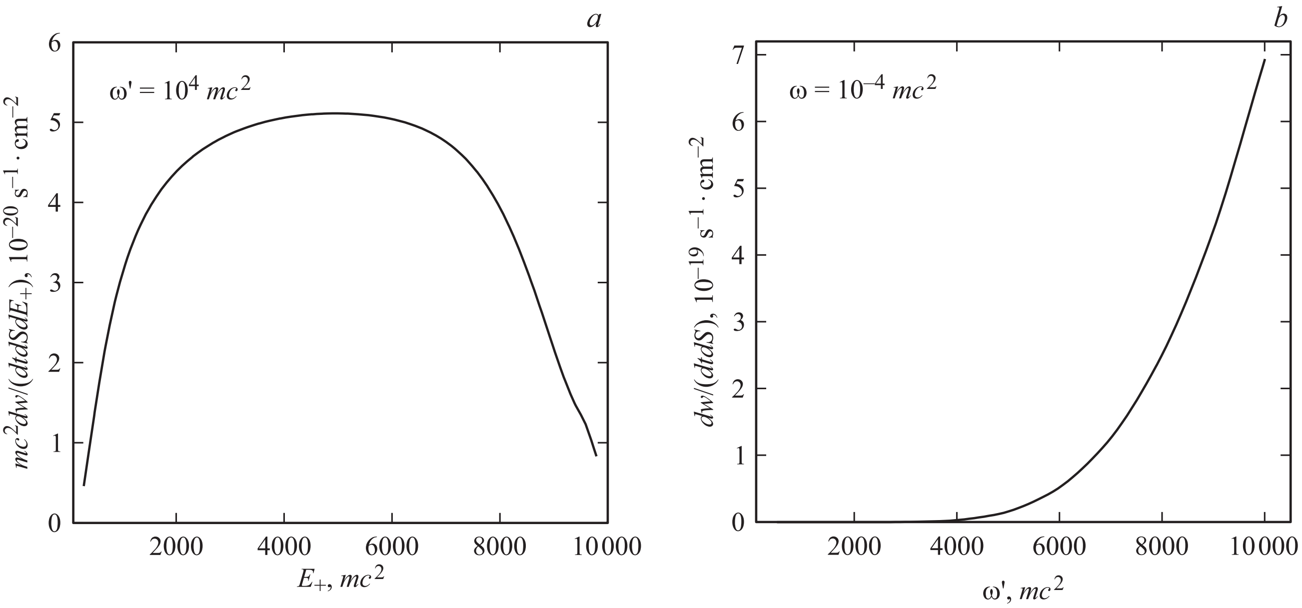


**Figure 3.** $a$ — Positron spectrum at $\omega' = 10^4 mc^2$. $b$ — Dependence of the probability of positron production on energy $\omega'$ of a gamma quantum. In both cases, $\omega = 10^{-4} mc^2$ and $qA_0 = 10^{-3} mc^2$.

at $\omega' = (10^2 - 10^3) mc^2$ is approximately equal to half the energy of a gamma quantum, while Fig. 3, $a$ demonstrates that the width at $\omega' = 10^4 mc^2$ is approximately equal to the total energy of a gamma quantum. The total probability of production of an electron–positron pair per unit time at various X-ray pulse frequencies $\omega$ and fixed amplitude $qA_0 = 10^{-3} mc^2$ is shown in panels $b$ of Figs. 1–3. The obtained results may help develop the methods for probing the properties of such pulses at the „SKIF" accelerator that is currently under construction [2].

### Conflict of interest

The authors declare that they have no conflict of interest.

## References

[1] K.V. Zhukovsky, Moscow Univ. Phys. Bull., **72** (2), 128 (2017). DOI: 10.3103/S0027134917020126.

[2] K.V. Zolotarev, A.I. Ancharov, Z.S. Vinokurov, B.G. Goldenberg, F.A. Darin, V.V. Kriventsov, G.N. Kulipanov,

K.E. Cooper, A.A. Legkodymov, G.A. Lyubas, A.D. Nikolenko, K.A. Ten, B.P. Tolochko, M.R. Sharafutdinov, A.N. Shmakov, E.B. Levichev, P.A. Piminov, A.N. Zhuravlev, Bull. Russ. Acad. Sci. Phys., **87** (5), 541 (2023). DOI: 10.3103/S1062873822701635.

[3] V.I. Ritus, A.I. Nikishov, in *Trudy FIAN*, Ed. by V.L. Ginzburg (Nauka, M., 1979), Vol. 111, p. 5 (in Russian).

[4] V.D. Serov, S.P. Roshchupkin, V.V. Dubov, Theor. Math. Phys., **216** (3), 1396 (2023). DOI: 10.1134/S0040577923090131.

[5] S.P. Roshchupkin, V.D. Serov, V.V. Dubov, Symmetry, **15** (10), 1901 (2023). DOI: 10.3390/sym15101901

[6] S.P. Roshchupkin, A.A. Lebed', *Effekty kvantovoi elektrodinamiki v sil'nykh impul'snykh lazernykh polyakh* (Nauk. Dumka, Kiev, 2013), p. 29 (in Russian).

[7] V.N. Baier, V.M. Katkov, V.S. Fadin, *Izluchenie relyativistskikh elektronov* (Atomizdat, M., 1973), p. 26 (in Russian).

[8] V.N. Baier, V.M. Katkov, V.S. Fadin, *Izluchenie relyativistskikh elektronov* (Atomizdat, M., 1973), p. 34 (in Russian).

*Translated by D.Safin*