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Dynamics of entangled states in the three-qubit Tavis-Cummings model

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We consider a model consisting of three identical qubits, one of which is in a free state and the other two are locked in an ideal resonator with a Kerr medium and interact resonantly with a selected mode of this resonator. The degree of coincidence is calculated for two genuine entangled W-type states and the GHZ-type state and the thermal state of the resonator field. The influence of the resonator thermal noise intensity and Kerr nonlinearity on the degree of coincidence of qubit states is investigated. It is shown that the thermal field of the resonator does not completely destroy the initial entangled states of the qubits, even for relatively high intensities of thermal noise of the resonator. It is also established that the Kerr nonlinearity of the resonator leads to stabilization of the initial entanglement of qubits.

Keywords: qubits, genuine entangled W- and GHZ-type states, thermal field, Kerr nonlinearity, degree of coincidence of qubit states.

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Introduction

Multi-qubit quantum entangled states play a key role in quantum computing [1]. Over the previous decades, special attention has been paid to the study of entangled states of two-qubit systems. Much of this interest was due to the fact that two-qubit gates, such as controlled „NOT" and one-qubit rotations, form the complete set of universal gates required for a quantum computer [2]. Alternatively, three-qubit gates, such as Toffoli gates [3], can be used in quantum computing. Three-qubit gates also play an important role in applications such as quantum error correction or simplification of quantum circuits [1]. Lately, there have been a large number of theoretical and experimental works (see references in [4]). For three-qubit systems, there are two non-equivalent classes of genuine entangled states: Greenberger-Horn-Zeilinger states (GHZ-states) and Werner states (W-states). Three-qubit states of both types have now been experimentally realised for various types of qubits (see references in [4]).

Electromagnetic fields of resonators are commonly used to generate, control, and manipulate entangled qubit states. To describe cubit dynamics theoretically, the Jaynes-Cummings model and its generalisations are used. Recently, much attention has been paid to the study of the dynamics of qubits interacting with the thermal fields of resonators (see references in [4]). This is because the resonators used in quantum information processes have finite temperatures (from nano- and millikelvin to room temperatures) [1]. The interaction of qubits with the thermal photons of the resonators can lead to Rabi oscillations of the qubit entanglement parameters, as well as to the sudden death of entanglement, which leads to additional errors.

In our study [4] we considered the entanglement dynamics of a three-qubit model in which two qubits are locked in a single-mode ideal resonator and interact with the single-mode thermal field of this resonator, and the third qubit is in a free state. In this case, the negativity of the qubit pairs was used as the criterion for qubit entanglement. However, in case of GHZ-states, the criterion of qubit entanglement, called the degree of coincidence or fidelity [5], is more informative. In case of W-states, such a criterion is important for considering schemes for generating three-qubit entangled states that cannot be divided into mixtures of two-qubit entangled states.

It has recently been shown that the Kerr medium in a microwave resonator can have a significant effect on the dynamics of superconducting qubits (transmons). Therefore, it has been shown in a number of theoretical papers that Kerr nonlinearity can contribute to stabilising the qubit entanglement in the resonator and eliminating the phenomenon of sudden death of entanglement (see references in [6]).

In this paper, we have generalised the results of [4] by including the Kerr nonlinearity of the resonator. Using the degree of coincidence as a criterion for qubit entanglement, we investigate the effect of the resonator thermal field intensity and the Kerr nonlinearity on the dynamics of qubit entanglement.

A model of three identical qubits and the calculation results

Consider a system consisting of three identical qubits Q_1, Q_2 and Q_3. Two qubits Q_2 and Q_3 interact resonantly

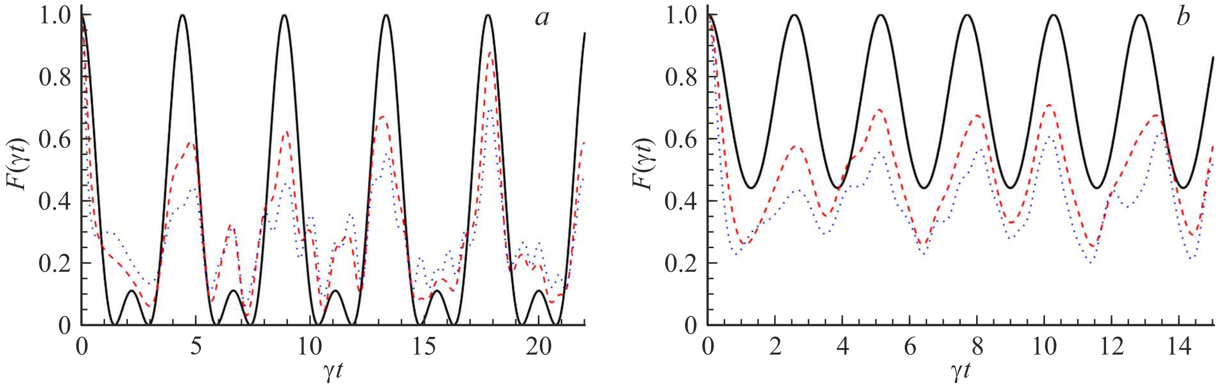

Figure 1. The degree of coincidence F from the reduced time γt for the initial entangled state $|W_1\rangle$ at $\theta = \arccos[1/\sqrt{3}]$ and $\varphi = \pi/4$ (a) and $|G\rangle$ at $\phi = \pi/4$ (b)in the absence of Kerr medium ($\chi = 0$). The average number of thermal photons $\bar{n} = 0.001$ (black solid line), $\bar{n} = 1$ (red dashed line), $\bar{n} = 2.5$ (blue dashed line).

with the quantised electromagnetic field of a resonator with Kerr medium. The Q_1 qubit can move freely outside the resonator. The coherence times for qubits in resonators are orders of magnitude larger than the times of one- and two-qubit operations, which allows us to neglect energy dissipation and other incoherent processes when analysing the entanglement dynamics. The interaction Hamiltonian of such a system can be written as

$$\hat{H}_I = \sum_{i=2}^{3} \hbar\gamma(\hat{\sigma}_i^+ \hat{a} + \hat{\sigma}_i^- \hat{a}^+) + \hbar\chi \hat{a}^{+2}\hat{a}^2,$$

where $\hat{\sigma}_i^+ = |+\rangle_i\langle -|$ and $\hat{\sigma}_i^- = |-\rangle_i\langle +|$ — are raising and lowering operators in the i-qubit, $\hat{a}$ and $\hat{a}^+$ — are photon annihilation and creation operators in the resonator mode, γ — the constant of the qubit-photon interaction and χ — the Kerr nonlinearity. The parameters of the Hamiltonian for qubits of different nature vary over very wide ranges. For example, the width of the energy gap for superconducting qubits is in the interval 1–20 GHz, and γ — in the interval of 0.1–10 GHz. At the same time, for electron spins, the width of the energy slit is in the range 1–10 GHz, and γ is of the order of 1 MHz in the case of quantum dots and 100 Hz for impurities. For superconducting qubits in a Kerr medium resonator, the Kerr nonlinearity of the order of 0.1 GHz [7] was achieved.

As the initial states of the qubits, we choose two different genuine entangled W-type states:

$$|W_1\rangle = \cos\theta|-,-,+\rangle + \sin\theta\sin\varphi|-,+,-\rangle + \sin\theta\cos\varphi|+,-,-\rangle,$$

$$|W_2\rangle = \cos\theta|+,+,-\rangle + \sin\theta\sin\varphi|+,-,+\rangle + \sin\theta\cos\varphi|-,+,+\rangle$$

or a genuine entangled GHZ-state of the form

$$|G\rangle = \cos\phi|+,+,+\rangle + \sin\phi|-,-,-\rangle.$$

Here θ, φ and ϕ — parameters that determine the initial degree of qubit entanglement ($0 < \theta, \varphi, \phi < \pi$).

As part of the existing experiment with superconducting qubits [8], the considered model can be implemented as follows. The three qubits are located in the first harmonic bunches of the Kerr medium resonator and can be tuned into resonance with the field mode or taken out of resonance by applying current pulses to the magnetic flux control lines. For example, to put the system into the initial $|W_1\rangle$ state, the following must be done: initially bring all qubits out of resonance. By applying an π-pulse, bring the Q_1 qubits into the excited state $|+\rangle_1$ and then bring them into resonance with the field mode. After time interval $\tau = \pi/2\gamma$, when the resonator moves to the $|1\rangle$ state, all qubits are brought into resonance. After time interval $\tau/\sqrt{3}$ the qubits are in a genuine entangled state of the form $|W_1\rangle$. Next the Q_1 qubit is brought out of resonance.

The state of the investigated model at an arbitrary time instant is determined by the full time matrix of the qubit and field density $\rho_{Q_1Q_2Q_3;F}(t)$. We have found the solution of the quantum Liouville equation for the full density matrix of the system „three qubits + mode field" in the representation of „dressed states" for pure initial states of the qubits and the mixed thermal state of the resonator field. In this paper, we have chosen the degree of coincidence (fidelity) [5] as a quantitative criterion for the qubit entanglement. To determine the above parameter, it is sufficient to compute the reduced three-qubit density matrix by averaging the full density matrix over the field variables:

$$\rho_{Q_1Q_2Q_3}(t) = Tr_F \rho_{Q_1Q_2Q_3;F}(t).$$

The degree of coincidence can be represented in the form

$$F(t) = \mathrm{tr}(\rho\rho'),$$

where $\rho = |\psi\rangle\langle\psi|$ — the initial density matrix of the qubits and ψ — their initial wave function, ρ' — the density matrix at time $t > 0$.

The results of numerical calculations of the degree of coincidence for the W- and GHZ initial states of the qubits

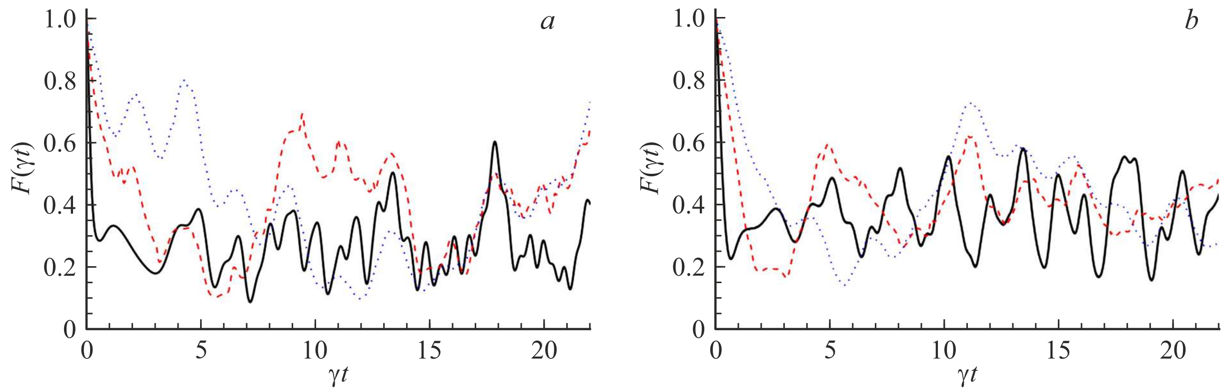

Figure 2. Degree of coincidence F from the reduced time γt for the initial entangled state$|W_1\rangle$ at $\theta = \arccos[1/\sqrt{3}]$ and $\varphi = \pi/4$ (a) and $|G\rangle$ at $\phi = \pi/4$ (b). The Kerr nonlinearity of $\chi = 0$ (black solid line) $\chi = 2\gamma$ (red dashed line), and $\chi = 5\gamma$ (blue dashed line). The average number of thermal photons $\bar{n} = 4$.

are shown in Figs. 1 and 2. Fig. 1 shows the dependence of the F parameter on the dimensionless time γt for the initial entangled states of the $|W_1\rangle$ (Fig. 1, a) and $|G\rangle$ (Fig.1, b) for different values of the average number of thermal photons in the absence of the Kerr medium. The figures show that the interaction of qubits with thermal photons destroys the initial entanglement. Moreover, as the intensity of the thermal field increases, the effect of reducing the qubit entanglement degree is more noticeable for the initial GHZ state. At the same time, complete destruction of the initial entanglement does not occur even for sufficiently high intensities of the thermal field.

Fig. 2 shows the dependence of the parameter F on the dimensionless time γt for the initial entangled states of the $|W_1\rangle$ (Fig. 2, a) and $|G\rangle$ qubits for different values of the Kerr nonlinearity and a fixed value of the average number of thermal photons. It can be seen that the inclusion of the Kerr nonlinearity results in a stabilisation of the initial qubit entanglement. The effect for both initial states is most noticeable at small times, and the increase in the degree of qubit entanglement is more significant for the W-type initial state. Note that for the $|W_2\rangle$ state, the behaviour of the coincidence F is similar to that of the above value for the $|W_1\rangle$ state.

Conclusion

In this paper we have investigated the dynamics of three identical qubits, one of which is free and the other two are locked in a resonator with a Kerr medium and interact resonantly with the thermal field mode of this resonator. The temporal behaviour of the degree of coincidence of the qubit states for the initial W- and GHZ-tangled states is found. It is shown that the thermal field of the resonator does not completely destroy the initial entanglement of qubits even for high intensities of the thermal noise of the resonator. It is also found that the Kerr nonlinearity of the resonator can act as a natural mechanism for stabilising the initial qubit entanglement.

Conflict of interest

The authors declare that they have no conflict of interest.

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