# Focus line and mass distribution in a mirror-type magnetic mass spectrograph

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It is shown that the shift of ion trajectories in the edge field of a magnetic mirror leads to a curvature of the line of foci and a distortion of the distribution of ion mass numbers on it.

Keywords: mass spectrograph, edge field, fringing field, magnetic mirror, line of foci.

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The possibility of instantaneous recording of the entire mass spectrum of ions of the analyzed substance in static mass spectrographs predetermines their advantage in analysis speed over mass analyzers with spectra recorded in the scan mode with sequential detection of single-mass ion components. A straight line of foci (LF) of singlemass beams, which allows one to install a significant number of receiving slits of a multi-collector detector or a spatially extended detector for instantaneous recording of the entire mass spectrum of the analyzed substance, is needed to implement this possibility. A magnetic mirror (MM) with a uniform magnetic field and orthogonal ion beam injection, which has first been used by Dempster as a mass spectrometer [1], is an example of an ideal ionoptical element with a straight LF, since the MM line of foci coincides with the effective boundary of the mirror field [2] in this ion injection geometry. However, since the mass spectrum detector needs to be placed in the edge magnetic field in this case, the indicated feature of LF positioning is a significant drawback. When an ion beam is introduced into an MM at an angle, the mentioned issue is made moot by the focusing action of the edge field, which establishes the conditions for focusing of single-mass ion beams in the second order of approximation [3-5]. However, the passage of ions in the MM edge field is accompanied by a transverse displacement [6–8] proportional to  $(k^2/r)$ , where k is one half of the gap between the magnet poles and r is the ion orbit radius in the uniform part of the MM magnetic field. Thus, being negligible at large values of r, this shift (let us call it "edge shift") increases with decreasing orbital radii and may ultimately lead to a noticeable deviation of single-mass foci in the region of light ions from the straight line that corresponds to the LF geometry in the SCOFF (sharp cut off) MM model constructed without regard to the indicated edge shift.

This present study is focused on the geometry of the MM line of foci and the distribution of single-mass foci of an ion

beam with the shift of ion trajectories in the MM edge field taken into account.

It is a continuation of our earlier work [9] on an MMbased static mass spectrograph.

Figure 1 shows the effective model of an MM-based mass spectrograph and the geometry of effective axial orbits of single-mass components of an ion beam introduced into the MM. In the MM effective field, these orbits assume the shape of circular arcs, and their turn angle is defined unambiguously by angle  $\varepsilon$  of introduction of the ion beam into the MM field:

$$\varphi + 2\varepsilon \equiv \pi, \tag{1}$$

while radii r [mm] of axial orbits at accelerating voltage U [eV] of the ion source and magnetic field strength B [G] are specified by mass number m [u] of ions:

$$r \approx 144\sqrt{Um}/B.$$
 (2)

Within this simplified model, single-mass ion groups separated in the MM field are focused at distance  $l_{2r} = l_2(r)$  from the effective MM boundary if the ion source is located at distance  $l_1$  from this boundary [3]:

$$l_2(r) = r\sin\varphi - l_1. \tag{3}$$

The linear dependence of exit arm  $l_2$  on radii r of the orbits of single-mass beams in (3) indicates that the LF is straight within the discussed model and distribution  $\rho(m)$  of the mass numbers of ions along the LF follows, in accordance with (2), dependence

$$\rho(m) = cR\sqrt{m/m_R - 1},\tag{4}$$

where c is a constant;  $\rho$  is the distance between the foci of single-mass ions with mass numbers m and a known "reference" value  $m_R$ ; and R is the turn radius of orbits of the latter.

The algorithm for determining the exit point of the optical axis (point  $A_2$ ) with axis shift in the MM edge field taken



Figure 1. Geometry of axial orbits of ion beams in a magnetic mirror within the SCOFF model (without regard to edge distortions).

into account is illustrated in Fig. 2. The edge matrices underlying this geometric construction take the following form in profile planes  $CA'_1$  and  $CA'_2$  [8]:

$$\tilde{\mathbf{T}}_{k1} = \begin{bmatrix} 1 & -2\gamma r & \delta/2 \\ t/r & 1 - 2t\gamma & \gamma/2 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\tilde{\mathbf{T}}_{k2} = \begin{bmatrix} 1 - 2t\gamma & -2\gamma r & -\delta/2 \\ t/r & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (5)$$

where  $t = \operatorname{tg} \varepsilon$ . The second element in the first row of the first matrix in (5) specifies longitudinal shift  $\omega$  (Fig. 2)

$$\omega = -2\gamma r = -2\delta t \tag{6}$$

from point  $A_1$  of intersection with the optical axis of the Herzog [2] effective MM boundary to point  $A'_1$ , which is then shifted by  $\delta$ 

$$\delta = J(k^2/r)/\cos^2\varepsilon,\tag{7}$$

where J is the integral of distribution function h(u) of the edge field along the normal to the physical boundary of pole pieces (see [7]),

$$J = \int_{u_a}^{u_b} (1 - h(u)) \left( \int_{u_a}^{u} h(\xi) d\xi \right) du, \tag{8}$$

to point  $A_1^*$ , which is the origin of a circular arc with radius r of the optical axis in the MM. The turn in an arc by angle  $\varphi$  to point  $A_2^*$  with subsequent transverse displacement  $\delta$  to point  $A'_2$  and shift  $\omega$  perpendicular to radius vector  $CA'_2$  to

point  $A_2$  specifies the direction of axis exit from the MM shifted by the edge field. This algorithm for determining the point and direction of exit of the optical axis of a singlemass ion beam from the MM effective field is grounded in the geometric significance of the elements of edge matrices (5) and provides an adequate description of the ion beam shift in the edge field in the entry and exit MM regions, ensuring the coincidence of effective and actual trajectories of ions in the uniform field region and at the MM exit.

The calculation of position of the angular (Gaussian) focus of single-mass ions with the shift of ion trajectories by the edge field and small quantities of order  $O(k^2/r^2)$  taken into account reveals (analytical calculations are omitted due to their cumbersome nature) that the LF for an MM with an ion beam introduced at an angle is a hyperbola, which may be written in the following parametric form:

$$y(r) = J\left[\cos\left(\psi - \frac{\varphi}{2}\right) - \cos\left(\psi + \frac{\varphi}{2}\right)\right]\frac{k^2}{r}\sec^2\varepsilon, \quad (9a)$$

$$x(r) = \frac{ar}{\sin\psi} - J\left[\sin\left(\psi - \frac{\varphi}{2}\right) - \sin\left(\psi + \frac{\varphi}{2}\right)\right]\frac{k^2}{r}\sec^2\varepsilon,$$
(9b)

where the parameter is radius r of orbits of single-mass ions in the uniform MM field, axis x is codirectional with the "ideal" straight LF, and axis y is perpendicular to axis x(Fig. 2); the y(r) values indicate the magnitude of deviation of the actual LF from the ideal one; and  $\psi$  is the tilt angle of the ideal LF set by angle  $\varepsilon$  of ion injection into the MM:

$$\operatorname{tg}\psi = \frac{\cos\varepsilon\sin\varepsilon}{1+\sin^2\varepsilon}.$$
 (10)



**Figure 2.** Geometric determination of the position of exit arm  $l_2$  of the optical axis of a single-mass ion beam in a mass spectrograph based on a magnetic mirror. Angles  $\varepsilon$  and  $\gamma$  correspond to negative values, and points *S* and *F* correspond to the positions of the exit slit of the source and the focus of the single-mass ion beam diverging from point *S* with the radius of its axis in the uniform MM field being equal to *r*.

It is easy to demonstrate (and unsurprising) that the asymptote of this hyperbola is a straight line of foci in the SCOFF MM model.

Using the above data and the same approximation (with small terms of the order no higher than  $O(k^2/r^2)$  taken into account), we find the following distribution m(r) of mass numbers of ions along the actual LF:

$$m(r) = \left(\frac{x^2(r) - 4c \operatorname{ctg} \psi J k^2 \operatorname{sec}^2 \varepsilon}{x^2(R) - 4c \operatorname{ctg} \psi J k^2 \operatorname{sec}^2 \varepsilon}\right) m_R, \qquad (11)$$

where  $m_R$  is the mass number of ions of the reference component. It is easy to derive "edge correction"  $\delta_m$  to distribution  $m^o(r)$ , which corresponds to dependence (4), from (11):

$$\delta_m = 2c \operatorname{ctg} \psi J \operatorname{sec}^2 \varepsilon \, k^2 \big( x^{-2}(r) - x^{-2}(R) \big), \qquad (12)$$

$$m(r) = (1 - \delta_m)m^o(r). \tag{13}$$

The results presented above demonstrate that the line of foci in mirror-type magnetic mass spectrographs is not ideally straight (with an increase in curvature in the region of lower mass numbers in the case of instantaneous recording of a wide range of mass spectrum lines). This factor needs to be taken into account in the process of design and application of mass spectrographs of this type. The influence of the MM edge field on ion trajectories also leads to distortion of the distribution of mass numbers of ions along the line of foci, and this distortion becomes more significant as the MM pole gap grows wider.

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### **Conflict of interest**

The authors declare that they have no conflict of interest.

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