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## Tunable standing-wave field in layered photonic structures

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Received March 5, 2024

Revised November 20, 2024

Accepted November 22, 2024

In this paper we consider the tuning of the configuration of the optical field in a dielectric layer bordering a homogeneous media or a photonic crystal when changing the wavelength. Analytical consideration of the problem is carried out. It is shown that the most optimal configuration for effective tuning of the position of optical standing wave antinodes is the use of a photonic crystal as a substrate. In this case, the phase of the optical field inside the thin film strongly depends on the radiation wavelength (3.8 rad/ $\mu\text{m}$ ) compared to the film without a photonic crystal (0.4 rad/ $\mu\text{m}$ ). The results obtained are useful for applications in such areas as excitonics and magnonics, as well as for the development of methods for calculating photonic structures.

**Keywords:** photonic crystal, optical near-field, layered structures.

DOI: 10.61011/PSS.2024.11.60113.43

The task of creation of a given spatial distribution of the optical field is practically important for many applications. In particular, the localization of electromagnetic field energy in the semiconductor layers of the structure is required for the creation of efficient solar cells [1,2]. Medical applications include the thermal destruction of infected cells, which also requires localization of the field in a specific spatial area [3,4]. Nonlinear optics is an extensive field of application [5,6]. Surface-enhanced Raman scattering is an example of a nonlinear optical effect requiring the concentration of the pump field in a small spatial region [7]. Thermomagnetic recording of information is an important area of application of the concentrated field, which requires local heating of the magnetic material in the area of magnetization reversal [8]. Another area of application requiring spatial localization of the optical field is the problem of optical magnetization reversal and generation of spin waves, which requires concentration of the field in magnetic materials in the required region of magnetization reversal. In this case, magnetization reversal is carried out, for example, by the inverse magneto-optical Faraday effect, the magnitude of which depends on the intensity of the pumping field [9–11].

Generation of a standing wave is one of the methods of creation of an inhomogeneous optical field. It may be useful to rearrange the positions of the antinodes of the standing wave for practical purposes, thus creating nanometer areas of concentration of electric field energy. In particular, it can be applied for the efficient binding of photons and excitons in semiconductor structures [12,13]. The optomagnetic generation of standing spin waves in a magnetic material is another possible application [14].

This paper describes the distribution of the optical field in a dielectric layer bordering a dielectric substrate, metal, or photonic crystal. A fully analytical approach is proposed, which can be used to obtain the required distribution.

Let us consider a dielectric layer with a refractive index  $n_2$  and thickness  $h$ . Let it border on one side with a medium with a refractive index  $n_1$ , and with a medium with a refractive index  $n_3$  on the other side. The light falls normally from the first medium (Figure 1, a).

It can be obtained from Fresnel's formulas [15] that the electric field in the layer is expressed by the formula:

$$E(z) = A \left\{ n_2 \cos(k_0 n_2 (z - h)) + i n_3 \sin(k_0 n_2 (z - h)) \right\}, \quad (1)$$

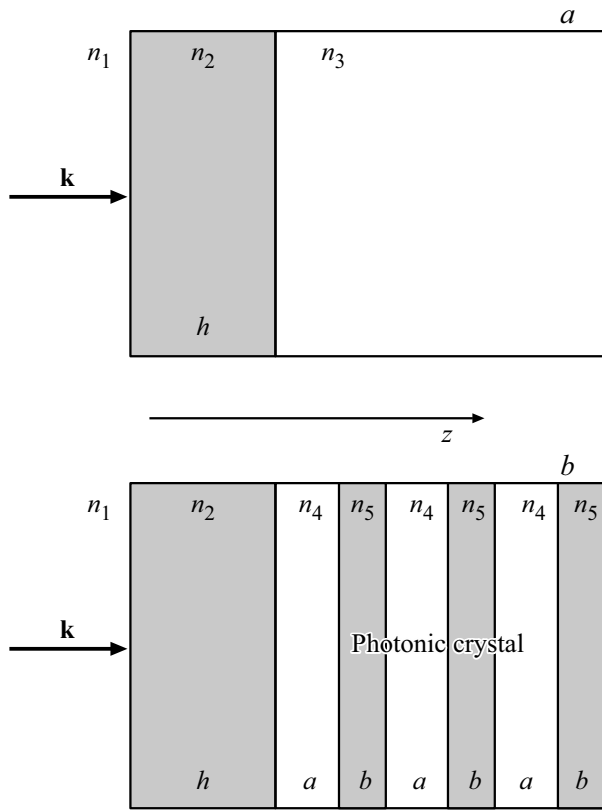
where  $k_0 = 2\pi/\lambda$  is vacuum wavenumber,  $\lambda$  is vacuum wavelength,

$$A = \frac{2n_1}{n_2(n_1 + n_3) \cos(k_0 n_2 h) - i(n_2^2 + n_1 n_3) \sin(k_0 n_2 h)}, \quad (2)$$

Here, the  $z$  axis is directed perpendicular to the boundary of the layers, the points  $z = 0$  and  $z = h$  correspond to the boundaries with the first and with the third media, respectively. If absorption is neglected, the field intensity distribution in the layer can be expressed as follows:

$$|E(z)|^2 = |A|^2 \left\{ n_3^2 + (n_2^2 - n_3^2) \cos^2(k_0 n_2 (z - h)) \right\}. \quad (3)$$

It can be seen from Eq. (3) that the optical field inside the layer does not become zero, and the field is always maximal at the boundary with the third medium. Only the distance between the antinodes of the optical field in the



**Figure 1.** The considered structure is a dielectric layer bordering (a) with a homogeneous medium, (b) with a photonic crystal.

layer changes when the wavelength changes. However, it may be important for practical applications to obtain the optical field distribution in the form of a standing wave, and it is desirable to make the position of nodes and antinodes easily tunable. This can be achieved if the refractive index of the third medium is imaginary:

$$n_3 = i\kappa_3. \tag{4}$$

In particular, this is true for metals, which are characterized by negative values of dielectric constant in the optical range. In this case, the field intensity distribution has the form:

$$|E(z)|^2 = |A|^2 \{n_2^2 + \kappa_3^2\} \cos^2(k_0 n_2(z - h) + \phi), \tag{5}$$

where

$$\phi = \text{atan}\left(\frac{\kappa_3}{n_2}\right). \tag{6}$$

The value  $\phi$ , determined according to Eq. (5), is called the phase of the field. It characterizes the standing wave field in the considered layer of finite thickness, taking into account the incidence of light from the first medium. The phase of the field turns out to be important for rearranging the configuration of the field distribution inside the layer, since the change of the wavelength only changes the

distance between the antinodes of the standing wave, and the change of the phase of the field changes their position.

The field distribution obtained in this way is slightly rearranged with a change of wavelength as calculations show (Figure 2, a). SiO<sub>2</sub> was used as the dielectric for calculations, gold was used as the metal, and the thickness of the dielectric layer was 500 nm. The permittivity values are taken from Ref. [16] and [17] for SiO<sub>2</sub> and gold, respectively, and we do not take into account the imaginary part of the permittivity of gold in order to consider the ideal case of a purely imaginary refractive index. The distance between the antinodes obviously changes, as the wavelength of the standing wave changes, but there is always almost a minimum field at the boundary with the third medium.

A photonic crystal as a third medium is another interesting case. Since Eqs. (1)–(2) are derived from Fresnel formulas, they remain valid for any inhomogeneous medium, including a photonic crystal. In this case, the refractive index is implied to be the inverse of the wave impedance of the transmitted wave, that is, the ratio of the complex amplitudes of the magnetic and electric fields of the transmitted wave at the boundary of the photonic crystal:

$$n_3 = \frac{H(z = h)}{E(z = h)}. \tag{7}$$

Let the photonic crystal be formed by alternating layers with refractive indices  $n_4$  and  $n_5$  and thicknesses  $a$  and  $b$ , respectively, while the considered dielectric layer  $n_2$  borders the layer  $n_4$  (Figure 1, b). The following can be obtained from the explicit form of the Bloch wave:

$$n_3 = i n_4 \times \frac{2n_5 \exp\{iK(a + b)\} - (n_4 + n_5) \cos(k_0 n_4 a + k_0 n_5 b) + (n_4 - n_5) \cos(k_0 n_4 a - k_0 n_5 b)}{(n_4 - n_5) \sin(k_0 n_4 a - k_0 n_5 b) - (n_4 + n_5) \sin(k_0 n_4 a + k_0 n_5 b)}, \tag{8}$$

where  $K$  is a Bloch wavenumber. Let's denote  $\alpha = \cos\{K(a + b)\}$ . Then the following equation is valid [18]:

$$\alpha = \cos(k_0 n_4 a) \cos(k_0 n_5 b) - \frac{1}{2} \left( \frac{n_4}{n_5} + \frac{n_5}{n_4} \right) \sin(k_0 n_4 a) \sin(k_0 n_5 b). \tag{9}$$

Eq. (9) provides two solutions for  $K$  corresponding to Bloch waves propagating in opposite directions along the  $z$  axis. It is necessary to select a solution corresponding to the transmitted wave.

The transmitted wave in the photonic band, where  $|\alpha| < 1$ , is a Bloch wave with a positive group velocity. Such waves have a positive value of the Bloch wavenumber in the first Brillouin zone in odd photonic bands, (while it should be  $\sin\{K(a + b)\} > 0$ ), and they have a negative value of the Bloch wavenumber in even bands ( $\sin\{K(a + b)\} < 0$ ). Since the photonic band does not exceed the range of

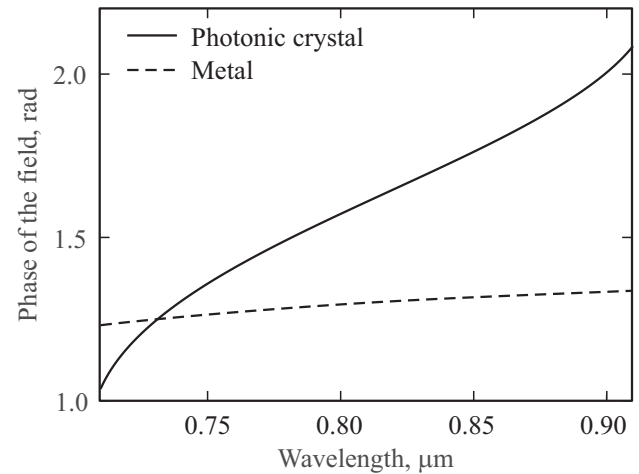
$k_{0(m-1)} < k_0 < k_{0m}$ , where  $k_{0m} = \pi m / (n_4 a + n_5 b)$  are the centers of the band gaps, the formula for the Bloch wavenumber of the transmitted wave can have the following form:

$$\exp\{iK(a+b)\} = \alpha + i \{ \text{sign}(\sin(k_0 n_4 a + k_0 n_5 b)) \} \sqrt{1 - \alpha^2}. \quad (10)$$

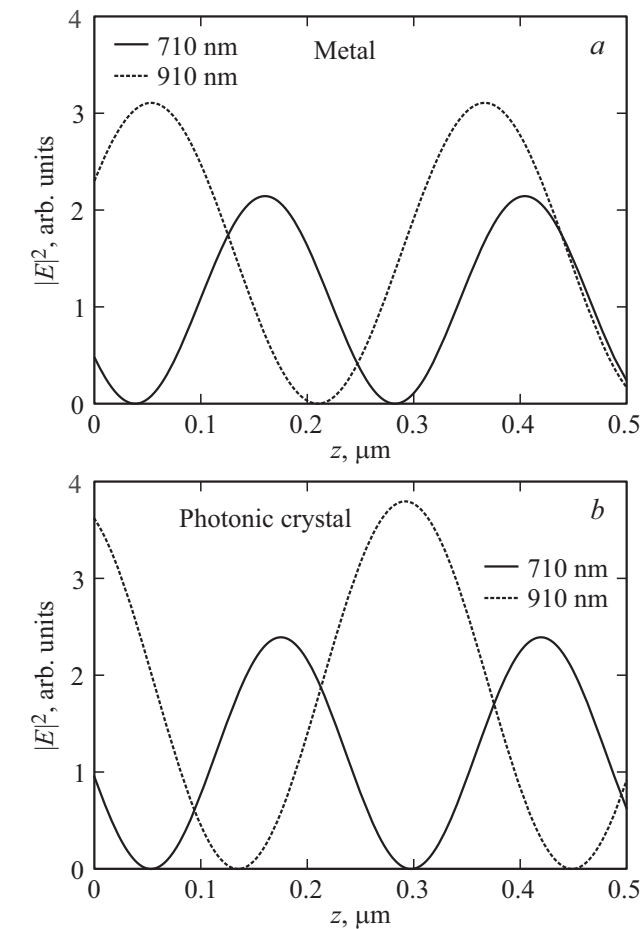
The transmitted wave in the band gap ( $|\alpha| > 1$ ) is a Bloch wave that decays in the positive direction of the  $z$  axis, that is, it has a positive imaginary part of the Bloch wavenumber. This is equivalent to the condition  $|\exp\{iK(a+b)\}| < 1$ , from which we obtain:

$$\exp\{iK(a+b)\} = \alpha - (\text{sign } \alpha) \sqrt{\alpha^2 - 1}. \quad (11)$$

Eqs. (8)–(11) fully determine the effective refractive index of a photonic crystal for calculating the reflectance from it using Fresnel formulas and, as a result, for calculating the optical field distribution according to Eq. (1). In case of the photonic band the effective refractive index of a photonic crystal is complex, therefore, generally speaking, the optical field in the dielectric layer (the second medium) does not vanish, while the phase of the field at the boundary with the



**Figure 3.** Dependence of the phase of the field  $\phi$ , defined by Eq. (5), on the wavelength for a photonic crystal (solid line) or gold (dashed line) as a substrate.



**Figure 2.** Field distribution in the dielectric layer bordering (a) with gold and (b) with a photonic crystal, at wavelengths of 710 nm (solid line) and 910 nm (dashed line).

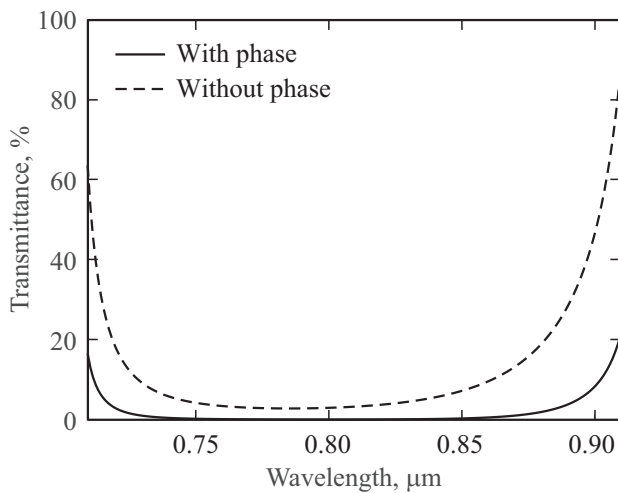
photonic crystal is arbitrary. In the case of the band gap, as can be seen from Eq. (11), the value  $\exp\{iK(a+b)\}$  is real, therefore the photonic crystal effective refractive index  $n_3$  is purely imaginary. In this case, the distribution of the optical field in the dielectric layer is described by the Eqs. (4)–(6). The optical field becomes zero at the nodes of the standing wave, and the phase of the field  $\phi$  at the boundary with the photonic crystal, defined by Eq. (5), significantly changes with a change of wavelength.

A photonic crystal consisting of  $\text{TiO}_2$  and  $\text{SiO}_2$  layers with thicknesses of 89 and 138 nm, respectively, was used for calculations. The refractive index for  $\text{TiO}_2$  is taken from Ref. [19]. The band gap ranges from 710 to 910 nm with these parameters. The results of calculation for the boundaries of the band gap are shown in Figure 2, b. It can be seen that the phase of the field  $\phi$  significantly varies.

Figure 3 shows the dependence of the phase of the field  $\phi$  on the wavelength for gold and for a photonic crystal as a substrate. It can be seen from the figure that there is a fairly strong dependence of the phase of the field at the wavelength in the band gap in the case of a photonic crystal: the change of the phase of the field is approximately  $3.8 \text{ rad}/\mu\text{m}$  for a photonic crystal, and  $0.4 \text{ rad}/\mu\text{m}$  for gold. Moreover, this dependence of the phase of the field on the wavelength can be changed by selecting the parameters of the photonic crystal. Therefore, photonic crystals are much preferable.

Negligible optical losses are another important advantage of photonic crystals. The absorption is always present in real metals, which leads to the fact that the refractive index is not purely imaginary, and therefore the optical field does not become zero at the nodes of the standing wave.

The phase of the field  $\phi$  allows evaluating the far-field optical properties of the structure, in the case when the third medium has an imaginary effective refractive index, and therefore, an imaginary impedance. Since the phase of the



**Figure 4.** Transmittance of a structure with a photonic crystal (solid line) and a hypothetical structure with a photonic crystal without taking into account the phase of the field (dashed line).

field determines the magnitude of the field at the boundary of the second and third media, the total transmittance can be estimated using the formula

$$T = \frac{n_{\text{out}}}{n_1} |E(h)|^2 \exp(-2 \operatorname{Im}(k_{3z})d)$$

$$= \frac{n_{\text{out}}}{n_1} |A|^2 \{n_2^2 + \kappa_3^2\} \cos^2(\phi) \exp(-2 \operatorname{Im}(k_{3z})d), \quad (12)$$

where  $d$  is the thickness of the third medium,  $n_{\text{out}}$  is the refractive index of the medium behind the structure into which the transmitted wave propagates,  $k_{3z} = K$  for a photonic crystal and  $k_{3z} = k_0 n_3 = i k_0 \kappa_3$  for a metal. At the same time, it is assumed that the third medium is thick enough, and backward waves in it can be ignored.

It can be seen from Eq. (12) that the dependence of the transmittance on the wavelength is attributable to the imaginary part of the effective refractive index of the third medium and directly to the phase of the field. Figure 4 shows the transmittance calculated using Eq. (12) for a structure containing a photonic crystal with 5 pairs of layers, as well as for a hypothetical structure with the same photonic crystal, but without taking into account the dependence of the phase of the field on the wavelength (the phase of the field is assumed to be zero). It can be seen that the phase of the field has a significant effect on both the magnitude of the transmittance and the width of the bandgap: the bandgap narrows without taking into account the phase of the field.

It can be noted that the obtained Eqs. (8)–(11) completely and unambiguously determine the input wave impedance of a photonic crystal  $Z_{\text{in}} = 1/n_3$ . Similarly, formulas for the output impedance can be obtained: the difference is that it is necessary to consider a Bloch wave incident on the boundary of a photonic crystal instead of a transmitted Bloch wave for selecting the correct value of

the Bloch wavenumber  $K$ . This will lead to the fact that the opposite sign should be taken before the second terms in Eqs. (10) and (11). The resulting Eqs. (8)–(11) can be used for the calculation and analysis of photonic structures, in particular, when using the impedance method, in which the photonic crystal is considered as a single layer [20,21]. It should be noted that a formula similar to Eq. (8) was obtained in Ref. [21], but it is not unambiguous, and there is no algorithm for identifying the correct solution.

Thus, it is shown that the configuration of the near field is strongly rearranged in the band gap in the layer bordering the photonic crystal, which directly affects the transmittance. Analytical formulas for the distribution of the optical field inside the layer bordering the photonic crystal are obtained. It is shown that the use of a photonic crystal is the most preferable in comparison with homogeneous reflective materials (metals), since the phase of the field for a standing wave has a significant dependence on the wavelength ( $3.8 \text{ rad}/\mu\text{m}$ ) in this case, which makes it easy to adjust the field configuration, and the field configuration is determined by the selection of photonic crystal parameters (passive control). The demonstrated approach makes it possible to obtain a required distribution of the optical field inside the dielectric layer, which is important in problems of excitonics, nonlinear optics, photovoltaics, magnonics, etc.

## Funding

This study was supported by the Russian Science Foundation (project No. 23-12-00310).

## Conflict of interest

The authors declare that they have no conflict of interest.

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*Translated by A.Akhtyamov*