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Ultrashort Plasmonic Pulse in Carbon Nanotube

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The dynamics of plasmonic modes excited in a carbon nanotube by an ultrashort electromagnetic pulse is theoretically investigated. The obtained nonlinear differential equations describe the interaction of the exciting electromagnetic pulse and conduction electrons in the carbon nanotube during the propagation of the plasmonic pulse along its axis. It is shown that the exciting electromagnetic pulse with the Gaussian envelope is transformed into a plasmonic cnoidal wave or a plasmonic soliton depending on the ratio of the electromagnetic pulse parameters and the carbon nanotube parameters.

Keywords: carbon nanotube, ultrashort plasmonic pulse, cnoidal wave, kink, soliton.

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Introduction

The rapid development of microelectronics in connection with ever-increasing volumes of processed data has led to an explosive growth in the number of logic elements in integrated circuits [1]. This increase in the number of elements inevitably leads to a reduction in their size, and the quantum properties of both nanostructures and processed signals start to manifest themselves as the sizes approach nanometer scales [2]. The concentration of an electromagnetic field in nanometer volumes of matter induces nonlinear interaction of electrons and photons in nanostructures [3–11].

Nanoplasmonics is one of the rapidly developing recent research trends in nanophotonics [12]. However, energy dissipation, which translates into large losses in the process of transmission and processing of plasmonic signals in metallic nanowires and nanostructures, hinders the transition from semiconductor to plasmonic circuitry. Carbon (graphene) nanostructures, which include carbon nanotubes (CNTs) [3–6,13–16] and logic elements based on them [17,18], may serve as an alternative to plasmonic metallic nanostructures. The dynamics of electrons in the conduction band of curvilinear nanostructures is currently being investigated both theoretically and experimentally [19,20]. The energy losses of signals transmitted and processed in carbon nanostructures are significantly lower than those corresponding to metallic nanostructures. In view of this, the future development of plasmonic circuitry is likely to be focused on carbon nanostructures.

In the present study, the nonlinear properties of CNTs with an armchair configuration of carbon atoms, which translates into metallic conductivity, are investigated. The dispersion relation for conduction electrons in such CNTs depends on the azimuthal and longitudinal components of the electron quasi-momentum, and the electromagnetic field is characterized by a system of two nonlinear equations for the azimuthal and longitudinal components of the vector potential. Cyclic boundary conditions allow one to introduce a relation between the electron quasi-momentum components and obtain a single nonlinear equation for the longitudinal potential component. Depending on the ratio of parameters of ultrashort electromagnetic pulses and CNTs, this equation has solutions in the form of a cnoidal wave or a soliton. An ultrashort plasmonic pulse with a Gaussian envelope may transform in CNTs with metallic conductivity into a soliton propagating along the nanotube axis with a velocity depending on its amplitude. The considered nonlinear effects in CNTs provide an opportunity to design new nanoelements of integrated circuits operating at optical frequencies.

Dispersion relation

In the approximation of tight binding of nearest-neighbor atoms in the graphene crystal lattice, the dispersion relation takes the form [21]

$$E = \tilde{E} \pm E, \tag{1}$$

where the plus and minus signs correspond to the energy of electrons in the conduction band and the energy in the valence band, respectively. The terms of dispersion relation (1) have the form

$$\tilde{E} = E_0 + \gamma_{AA'} g(k_x, k_y),$$

$$\Delta E = \gamma_{AB} \sqrt{g(k_x, k_y)}$$

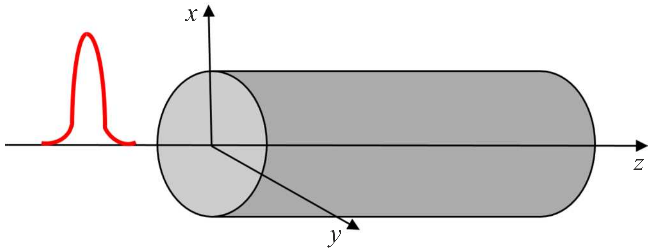

Figure 1. Carbon nanotube with an ultrashort electromagnetic pulse applied to it along axis z.

— half-width of the spectral gap, where

$$g(k_x, k_y) = 1 + 4\cos\frac{k_y a}{2}\left(\cos\frac{\sqrt{3}k_x a}{2} + \cos\frac{k_y a}{2}\right) \quad (2)$$

— geometric function, a — lattice constant of graphene, and E_0 — energy of a carbon atom. The overlap integrals in dispersion relation (1) assume the following values: $\gamma_{AA'} = 0.2\gamma_{AB} = 0.54\,\text{eV}$, $\gamma_{AB} = 2.7\,\text{eV}$ [16].

Periodic boundary conditions $\mathbf{R}\mathbf{k} = (n\mathbf{r}_1 + m\mathbf{r}_2)(\mathbf{1}_x k_x + \mathbf{1}_y k_y) = 2\pi s$, where $r_1 = r_2 = a$ and $s = 1, 2, \ldots, m$, for a CNT (Fig. 1) with chirality indices (n, m) [13] provide an opportunity to determine the relation between the components of electron vector $\mathbf{k}$:

$$\frac{\sqrt{3}k_x}{2} = \frac{1}{n+m}\left[\frac{2\pi}{a}s + (n-m)\frac{k_y}{2}\right],$$

which allows one to rewrite geometric function (2) in the form

$$g_{nms} = 1 + 4\cos\left(\frac{a}{2\hbar}p_y\right) \times \left[\cos\left(\frac{2\pi s}{n+m} + \frac{n-m}{n+m}\frac{a}{2\hbar}p_y\right) + \cos\left(\frac{a}{2\hbar}p_y\right)\right],$$

where $p_y = \hbar k_y$ is the electron quasi-momentum. At $n \neq 3q$ and $m \neq 0$, where $q = 1, 2, 3, \ldots$, gap $\Delta E_{nms} \neq 0$ is present in the electron spectrum; i.e., the CNT is semiconducting. The electron spectrum becomes gapless for zigzag$(n, 0)$ CNTs at $n = 3q$ and armchair(m, m) CNTs at any m at the points of contact between the valence and conduction bands; i.e., these CNTs feature metallic conductivity [3,13,14,16].

Semiclassical approximation

Let us consider the dynamics of a plasmonic pulse excited in a CNT by an ultrashort electromagnetic pulse (Fig. 1). The electromagnetic field in the semiclassical approximation is characterized by the equation for vector potential [22]

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi}{c}\mathbf{j} \quad (3)$$

with gauge

$$\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t},$$

where the current density vector is

$$\mathbf{j} = -e\int \mathbf{v} f \frac{d^3p}{(2\pi\hbar)^3},$$

$\mathbf{v} = \frac{\partial E}{\partial p}$ — electron velocity, and

$$\nabla^2 \to \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}$$

— Laplacian in the cylindrical coordinate system. The electron current in this CNT is excited by an electromagnetic pulse. In this case, electrons undergo no drift motion, but oscillations of the electron density emerge under the influence of the electromagnetic pulse field. Electron oscillations and the electromagnetic pulse field are hybridized, which leads to the generation of a plasmonic pulse propagating along longitudinal CNT axis z.

Electron distribution function f satisfies relation

$$\frac{\partial f}{\partial t} - \frac{e}{c}\frac{\partial\mathbf{A}}{\partial t}\frac{\partial f}{\partial\mathbf{p}} + \frac{\partial E}{\partial\mathbf{p}}\frac{\partial f}{\partial\mathbf{r}} = S_t f.$$

Within the relaxation time approximation [3,4], the collision integral in Eq. (4) may be written as

$$S_t f = \frac{1}{t_r}(f_0 - f),$$

where $t_r \cong 3\cdot 10^{-13}\,\text{s}$ is the relaxation time, $f_0 = [1 + \exp(E/k_\text{B}T)]^{-1}$ is the equilibrium Fermi distribution function, k_B is the Boltzmann constant, and T is temperature, $k_\text{B}T = 2.6\cdot 10^{-2}\,\text{eV}$. Equation (4) for the distribution function takes the following form on the CNT surface:

$$\frac{\partial f}{\partial t} - \frac{e}{c}\frac{\partial A_\varphi}{\partial t}\frac{\partial f}{\partial p_\varphi} - \frac{e}{c}\frac{\partial A_z}{\partial t}\frac{\partial f}{\partial p_z} + \frac{\partial E}{\partial p_\varphi}\frac{1}{r}\frac{\partial f}{\partial\varphi} + \frac{\partial E}{\partial p_z}\frac{\partial f}{\partial z} + \frac{1}{t_r}f = \frac{1}{t_r}\frac{1}{1+\exp(E/k_\text{B}T)}, \quad (5)$$

where $A_\varphi(t, z)$ and $A_z(t, z)$ are the components of the vector potential of the CNT surface mode field.

With the dependence of energy on coordinates $E(\varphi, z)$ taken into account, the current density vector in a CNT in Eq. (3) may be presented as

$$\mathbf{j} = -\frac{e}{(2\pi\hbar)^3}\int dp_r dp_\varphi dp_z\left(\mathbf{1}_\varphi\frac{\partial E}{\partial p_\varphi} + \mathbf{1}_z\frac{\partial E}{\partial p_z}\right)f, \quad (6)$$

where $\mathbf{1}_{\varphi,z}$ are unit vectors of the cylindrical coordinate system. Current density vector (6) has azimuthal component j_φ and longitudinal component j_z, which is directed along longitudinal CNT axis z; i.e., depending on the phase ratio of the current components, it is a right-handed or left-handed helix. It follows from expression (6) for the current density that electron current component j_r is not excited in a CNT, since the equation for radial potential component A_r has no source:

$$\frac{\partial^2 A_r}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 A_r}{\partial t^2} = 0.$$

Let us write the equations for vector potential components A_φ and A_z in a CNT with account for expression (6) for the current density vector:

$$\frac{\partial^2 A_\varphi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 A_\varphi}{\partial t^2} = \frac{e}{c\pi\hbar^2 a}\int_{-\pi\hbar/a}^{\pi\hbar/a} dp_\varphi \int_{-\pi\hbar/a}^{\pi\hbar/a} dp_z \frac{\partial E}{\partial p_\varphi} f, \tag{7}$$

$$\frac{\partial^2 A_z}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 A_z}{\partial t^2} = \frac{e}{c\pi\hbar^2 a}\int_{-\pi\hbar/a}^{\pi\hbar/a} dp_\varphi \int_{-\pi\hbar/a}^{\pi\hbar/a} dp_z \frac{\partial E}{\partial p_z} f. \tag{8}$$

An external electromagnetic field with vector potential components A_φ and A_z interacts with electrons in a CNT, and an electromagnetic field with the same vector potential components is generated by electron current components j_φ and j_z; i.e., the mode with potential component A_r is not generated in CNTs.

In a thin (with a diameter on the order of a few nanometers) CNT, the field of surface modes excited by an electromagnetic pulse with a field distribution independent of angular coordinate φ may be presented in the form of a combination of modes with zero azimuthal indices: E-mode with components (E_r, H_φ, E_z) and H-mode with components (H_r, E_φ, H_z) propagating along nanotube axis z [18]. Thus, the electromagnetic field on the CNT surface is a combination of surface modes with vector potential components A_φ and A_z (Eqs. (7) and (8)).

Electron distribution function $f\big(E(p_\varphi, p_z)\big)$ in a CNT depends on both azimuthal p_φ and longitudinal p_z quasi-momentum components; i.e., azimuthal j_φ and longitudinal j_z components of the electron current are excited. This, in turn, leads to the generation of an electromagnetic field with potential components A_φ and A_z. When excited by a pulse in the H-mode, the electromagnetic field is characterized by potential component $A_\varphi \leftrightarrow (H_r, E_\varphi, H_z)$. The longitudinal potential component characterizes the components of generated magnetic and electric fields: $A_z \leftrightarrow (H_r, E_\varphi, H_z)$; i.e., magnetic field component H_r is also generated. Since only the electromagnetic modes with potential components A_φ and A_z interact with conduction electrons in CNTs, an excitation electromagnetic pulse propagating along the z axis of a CNT should have a field with at least one of these potential components in order to interact with electrons.

Ultrashort pulses in a CNT

In a CNT with an armchair(m, m) carbon atom configuration, substitution of variables [3]

$$p_x \cos 30^\circ = \sqrt{3}p_x/2 \to p_\varphi,$$

$$p_y \sin 30^\circ = p_y/2 \to p_z \equiv p$$

yields geometric function

$$g_{ms} = 1 + 4\cos\left(\frac{s\pi}{m}\right)\cos\left(\frac{a}{\hbar}p\right) + 4\cos^2\left(\frac{a}{\hbar}p\right),$$

where $s = 1, 2, \ldots, m$ is the subband index. Let us rewrite dispersion relation (1) for a CNT with the armchair configuration in the form

$$E_{ms} = E_0 + \gamma_{AA'} g_{ms} \pm \gamma_{AB}\sqrt{g_{ms}}. \tag{9}$$

Let us also assume that a plasmonic pulse in a CNT is characterized by vector potential $A \equiv A_z$. Equation (5) for the distribution function then takes the form

$$\frac{\partial f}{\partial t} - \frac{e}{c}\frac{\partial A}{\partial t}\frac{\partial f}{\partial p} + \frac{\partial E_{ms}}{\partial p}\frac{\partial f}{\partial z} + \frac{1}{t_r} f = \frac{1}{t_r} f_{0ms}, \tag{10}$$

where

$$f_{0ms} = \frac{1}{1 + \exp(E_{ms}/k_B T)}.$$

The solution of Eq. (10) for electron distribution function f may be obtained using the method of characteristics [4], which yields

$$f_{ms} = f_{0ms}\big(p - c^{-1}eA(t)\big).$$

Let us perform substitution $p \to p - c^{-1}eA$ for the quasi-momentum in dispersion relation (9). Equation (8) for the potential then takes the form

$$\frac{\partial^2 A}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = \frac{2e}{\hbar c a^2}\int_{-\pi\hbar/a}^{\pi\hbar/a} dp \frac{\partial E_{ms}}{\partial p} f_{0ms}, \tag{11}$$

where geometric function

$$g_{ms} = 1 + 4\cos\left(\frac{s\pi}{m}\right)\cos[\hbar^{-1}a(p - c^{-1}eA)] + 4\cos^2[\hbar^{-1}a(p - c^{-1}eA)]$$

in the expression for electron energy E_{ms} depends on the potential. The expression for energy $E_{ms}(A)$ includes potential $A(t, z)$; i.e., Eq. (11) is nonlinear.

Substituting integration variable $dp = dE_{ms}\frac{\partial p}{\partial E_{ms}}$ in integral

$$r = \int_{-\pi\hbar/a}^{\pi\hbar/a} dp \frac{\partial E_{ms}}{\partial p} f_{0ms}$$

on the right-hand side of Eq. (11), we obtain

$$r = k_B T \int dE \frac{1}{1 + \exp(E)} = k_B T\{E - \ln[1 + \exp(E)]\},$$

where $E = E_{ms}/k_B T$. Let us insert the values of E_{ms} in the middle $p = 0$ and at the boundary of the Brillouin zone into the right-hand part multiplied by 2:

$$r = 2\big\{E - k_B T \ln[1 + \exp(E/k_B T)]\big\}\Big|_{E_{ms}(0)}^{E_{ms}(\pi\hbar/a)}.$$

At $p = 0$, the geometric function in the expression for electron energy E_{ms} is

$$g_{ms0}(0) = 1 + 4\cos\left(\frac{s\pi}{m}\right)\cos\left(\frac{ea}{c\hbar}A\right) + 4\cos^2\left(\frac{ea}{c\hbar}A\right),$$

while the same function at $p = \pi\hbar/a$ takes the form

$$g_{msa}\left(\frac{\pi\hbar}{a}\right) = 1 - 4\cos\left(\frac{s\pi}{m}\right)\cos\left(\frac{ea}{c\hbar}A\right) + 4\cos^2\left(\frac{ea}{c\hbar}A\right).$$

The right-hand side of Eq. (11) in subband s of the CNT conduction band takes the form

$$r_{ms} = 2\gamma_{AA'}(g_{msa} - g_{ms0}) + 2\gamma_{AB}\left(\sqrt{g_{msa}} - \sqrt{g_{ms0}}\right) + 2k_{\mathrm{B}}T\ln\left[\frac{1+\exp(E_{ms0}/k_{\mathrm{B}}T)}{1+\exp(E_{msa}/k_{\mathrm{B}}T)}\right],$$

where $E_{ms0} = E_{ms}(g_{ms0})$, $E_{msa} = E_{ms}(g_{msa})$.

Assuming $\sqrt{g_{msa}} \approx \sqrt{g_{ms0}}$ for subband s of the conduction band and taking relation $k_{\mathrm{B}}T \ll \gamma_{AA'}$ into account, we neglect the second and the third terms in the expression for r_{ms}. Summing over the subbands, $\sum_{s=1}^{m} |\cos(s\pi/m)| = 1$, we obtain an equation for dimensionless potential $\bar{A} = (ea/\hbar c)A$ in the case of excitation of electrons in the conduction band of a CNT:

$$\frac{\partial^2 \bar{A}}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \bar{A}}{\partial t^2} = -q_{as}^2\cos(\bar{A}), \tag{12}$$

where

$$q_{as}^2 = \frac{16e^2\gamma_{AA'}}{c^2\hbar^2 a}.$$

Figure 2 presents the numerical solution of Eq. (12) for dimensionless potential $\bar{A}(z, t_c)$ with a CNT excited by an electromagnetic pulse with a Gaussian envelope

$$\bar{A}(0, t_c) = \bar{A}_0\exp(-t_c^2 T_0^{-2})[1 - \cos(\omega_0 t_c)] + \pi/2,$$

where $\bar{A}_0 = (ea/\hbar c)A_0$ is the potential amplitude and $t_c = ct$.

If we consider the dynamics of a plasmonic pulse in the telecommunication range, the carrier mode frequency, which assumes a value of $\omega_t = 1.216 \cdot 10^{15}\,\mathrm{s}^{-1}$ ($\lambda_0 = 1.55\,\mu$m), yields scale factor $0.8 \cdot 10^5$ at the value of ω_0 that was used for numerical calculation ($\omega_0 = 0.5 \cdot 3 \cdot 10^{10} = 1.5 \cdot 10^{10}\,\mathrm{s}^{-1}$). Thus, the CNT length in Fig. 2 is $z_{\max} = 1.25\,\mu$m, and electromagnetic excitation pulse duration $T_0 = 33.3 \cdot 10^{-12}$ s. If the CNT length increases by a factor of 10, dimensionless potential $\bar{A}(z, t_c)$ undergoes damped oscillations (Fig. 3). The emergence of potential oscillations after a tenfold increase in the CNT length is illustrated in Fig. 4 in a 2D format.

In order to perform analytical analysis of the dynamics of plasmonic waves in CNTs, we introduce variable $\tau = t - z/v_g$, where $v_g = \mathrm{const}$ is the group velocity of a plasmonic pulse in a CNT. Equation (12) for the dimensionless potential of a plasmonic pulse may then be presented in the form

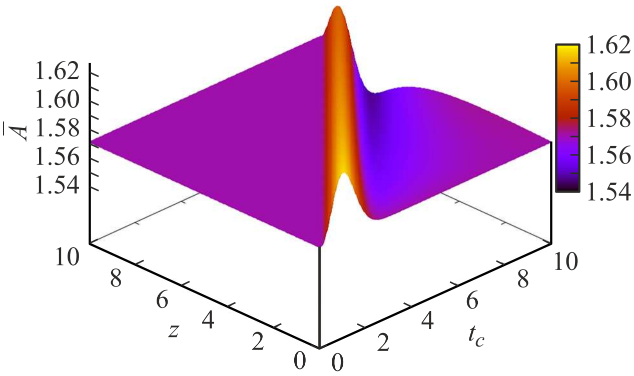

Figure 2. Dimensionless potential $\bar{A}(z, t_c)$ of a plasmonic pulse in a CNT at $z_{\max} = t_{c\,\max} = 10$ (this corresponds to CNT length $z_{\max} = 1.25\,\mu$m), $q_a = 0.5$, $T_0 = 1$, $\omega_0 = 0.5$; arbitrary units.

$$\frac{d^2\bar{A}}{d\tau^2} = -\omega_{as}^2\cos(\bar{A}), \tag{13}$$

where

$$\omega_{as}^2 = \frac{16e^2\gamma_{AA'}v_g^2}{\hbar^2 a(c^2 - v_g^2)}.$$

The solution of Eq. (13) has the form of a cnoidal wave

$$\bar{A} = \mathrm{sn}(\Omega\tau + F_0, \tilde{k}), \tag{14}$$

where $\tilde{k} = \sqrt{2}\omega_{as}/\Omega \le 1$ — modulus of an elliptic integral,

$$F_0 = \int_0^{\bar{A}_0} \frac{d\xi}{\sqrt{1 - \tilde{k}^2\sin\xi}}$$

— elliptic integral of the first kind [23],

$$\Omega = \left[(d\bar{A}/d\tau)_0^2 + 2\omega_{as}^2\sin\bar{A}_0\right]^{1/2},\ \bar{A}_0 = (ea/\hbar c)A_0,$$

$$(d\bar{A}/d\tau)_0 = \Omega\bar{A}_0\cos\bar{A}_0[1 - \tilde{k}^2\sin^2\bar{A}_0]^{1/2} \text{ if } \tau = 0.$$

When modulus $\tilde{k} \to 0$, elliptic sine $\bar{A} = \mathrm{sn}(\Omega\tau + F_0, \tilde{k})$ is transformed into trigonometric sine $\bar{A} = \sin(\Omega\tau + F_0)$; if $\tilde{k} \to 1$, elliptic sine is transformed into kink $\bar{A} = \tanh(\Omega\tau + F_0)$ at $\Omega\tau + F_0 < 1$ or antikink $\bar{A} = \mathrm{cotanh}(\Omega\tau + F_0)$ at $\Omega\tau + F_0 > 1$. Thus, depending on the ratio of parameters of an excitation electromagnetic pulse and a CNT, the dimensionless potential of a plasmonic pulse takes the form of either a nonlinear periodic (cnoidal) wave or a kink/antikink.

The strength of the electric field of a plasmonic pulse in a CNT is determined by taking a derivative with respect to τ of potential in the form of a cnoidal wave (14),

$$E_z = -c^{-1}\partial A/\partial\tau = -E_a\mathrm{cn}(\Omega\tau + F_0, \tilde{k})\mathrm{dn}(\Omega\tau + F_0, \tilde{k}), \tag{15}$$

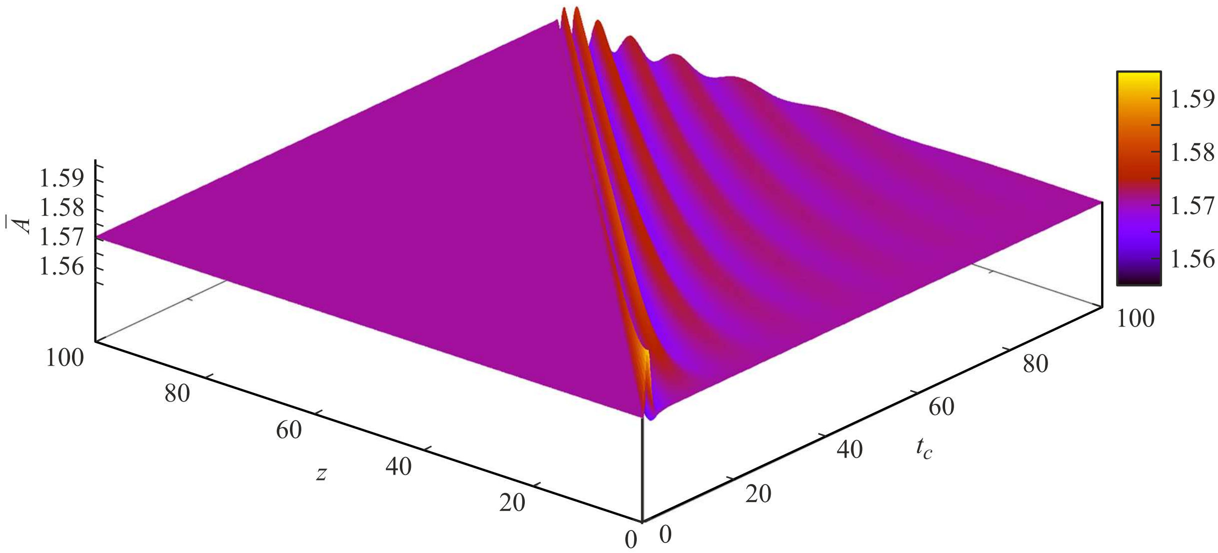

Figure 3. Dimensionless potential $\bar{A}(z, t_c)$ of a plasmonic pulse in a CNT at $z_{\max} = t_{c\,\max} = 100$ (this corresponds to CNT length $z_{\max} = 12.5\,\mu$m), $q_a = 0.5$, $T_0 = 1$, $\omega_0 = 0.5$; arbitrary units.

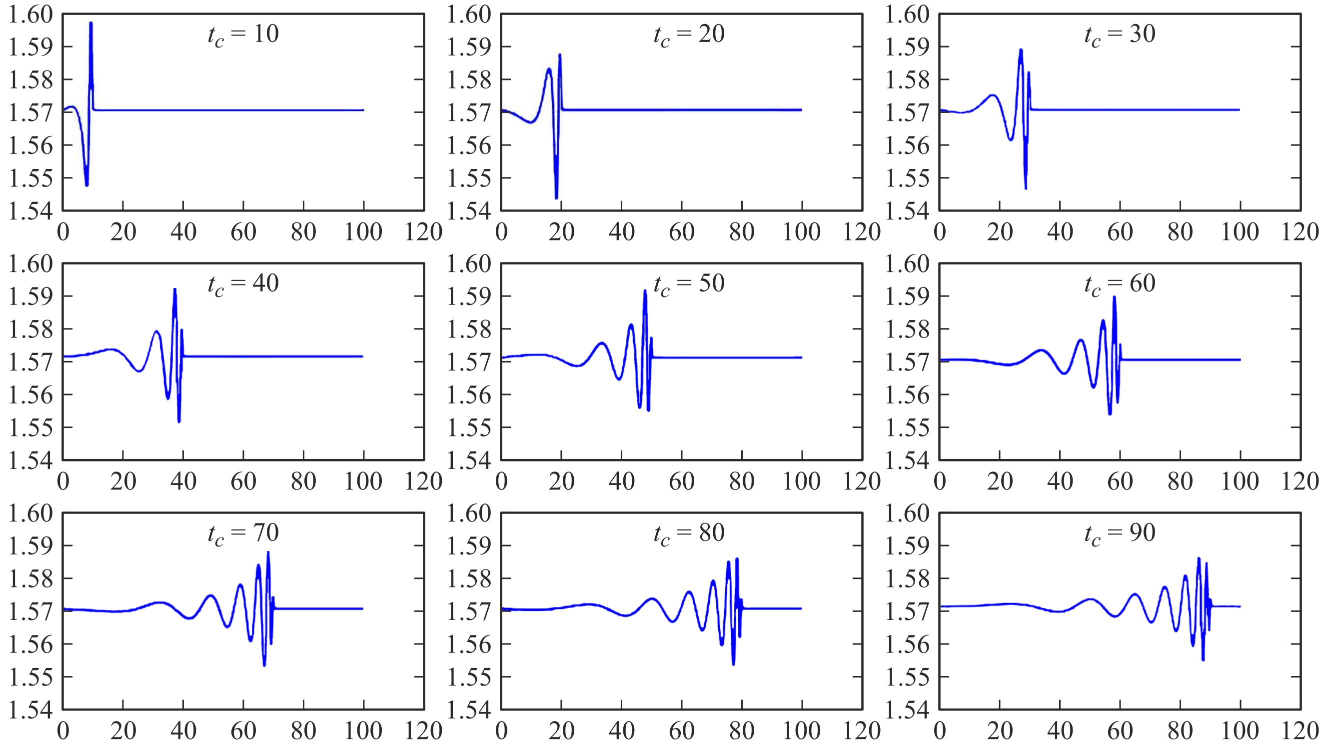

Figure 4. Emergence of oscillations at the trailing edge of a plasmonic pulse with an increase in CNT length from $1.25\,\mu$m to $12.5\,\mu$m. Normalized time t_c and dimensionless potential $\bar{A}(z, t_c)$ of a plasmonic pulse are plotted on the abscissa and ordinate axes; arbitrary units.

where $E_a = \hbar\Omega/ea$; at $\tilde{k} = 1$, we obtain a soliton solution

$$E_z = E_a \operatorname{sech}^2[\Omega(t - z/v_g) + F_0]. \qquad (16)$$

Amplitude E_a of soliton (16) depends on its velocity v_g, which is included in the expression for Ω. Let us find the velocity of a plasmonic soliton for an excitation electromagnetic pulse potential of the form

$$A = A_0 \exp(-t^2 T_0^{-2}) \sin(\omega_0 t), \ \ a\ (\partial A/\partial t)_0 = A_0\omega_0.$$

Expression $\tilde{k} = \sqrt{2}\omega_{as}/\Omega = 1$ yields

$$v_{gs} = c\left(1 + \frac{32e^2\gamma_{AA'}}{a\hbar^2\omega_0^2}\,\frac{1 - \sin\bar{A}_0}{A_0^2}\right)^{-1/2},$$

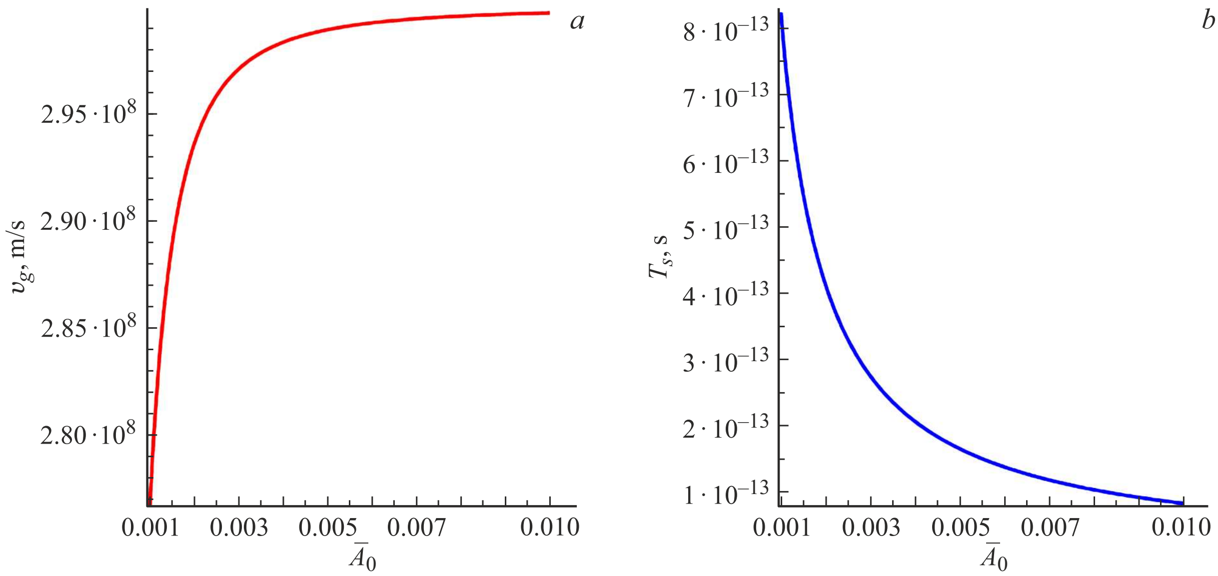

Figure 5. Dependences of (a) velocity v_g and (b) duration T_s of a plasmonic soliton pulse in a CNT on normalized amplitude $\bar{A}_0$ of the excitation electromagnetic pulse potential; here, $\gamma_{AA'} = 0.54\,\text{eV}$, $\omega_0 = 1.216 \cdot 10^{15}\,\text{s}^{-1}$ $(\lambda_0 = 1.55\,\mu\text{m})$, $a = 0.246\,\text{nm}$.

i.e., the velocity of soliton (16) depends on potential amplitude $\bar{A}_0 = (ea/\hbar c)A_0$.

It follows from the analysis of dynamics of pulses in CNTs that, in the case at hand, when an excitation electromagnetic pulse interacts with conduction electrons, oscillations at the harmonics of the electromagnetic field of the excitation pulse vanish upon its transformation into a plasmonic soliton. An electromagnetic pulse with a Gaussian envelope of the longitudinal optical mode is then transformed into a plasmonic soliton propagating along the longitudinal nanotube axis. Duration

$$T_s = \Omega^{-1} = (\omega_0^2\bar{A}_0^2 + 2\omega_{as}^2 \sin\bar{A}_0)^{-1/2}$$

of plasmonic soliton pulse (16) depends on the values of electromagnetic field potential amplitude $\bar{A}_0$ and its derivative $(d\bar{A}/d\tau)_0$ at $\tau = 0$, on the energy density of interaction between nearest-neighbor atoms $\gamma_{AA'}$ in a CNT, and on frequency ω_0 of the carrier mode of an excitation electromagnetic pulse.

The dependences of velocity

$$v_g = c\left(1 + \frac{32e^2\gamma_{AA'}}{a\hbar^2\omega_0^2}\,\frac{1-\sin\bar{A}_0}{\bar{A}_0^2}\right)^{-1/2}$$

and duration

$$T_s = (\omega_0^2\bar{A}_0^2 + 2\omega_{as}^2 \sin\bar{A}_0)^{-1/2}$$

of a plasmonic soliton pulse in a CNT on the amplitude of dimensionless potential $\bar{A}_0$ are shown in Fig. 5. It follows from the analysis of the dependences of soliton velocity v_g (Fig. 5, a) and duration T_s (Fig. 5, b) in a CNT that when the initial amplitude of potential A_0 of an excitation electromagnetic pulse increases 10-fold, the velocity of a soliton grows and tends to the speed of light in vacuum, while its duration decreases by a factor of 8.

Conclusion

The dispersion relation specifies the dependence of energy on the azimuthal and longitudinal components of quasi-momentum of conduction electrons in CNTs. Periodic boundary conditions for CNTs allow one to introduce a relation between these azimuthal and longitudinal components, which simplifies the dispersion relation.

The introduction of a dependence of the electron distribution function on the vector electromagnetic field potential makes it possible to characterize the nonlinear effects of interaction of the electromagnetic field and conduction electrons in CNTs. The semiclassical approach provides an opportunity to obtain nonlinear differential equations characterizing the dynamics of a plasmonic cnoidal wave and a plasmonic pulse in CNTs.

When a CNT with an armchair atomic configuration is excited by an electromagnetic pulse at a telecommunication frequency, either a plasmonic cnoidal wave or a plasmonic soliton is generated in this nanotube as a result of nonlinear interaction of the field of the excitation electromagnetic pulse with conduction electrons in the CNT. An ultrashort electromagnetic pulse with a Gaussian envelope is transformed into a plasmonic soliton pulse with its amplitude and velocity depending on the parameters of the field of the excitation electromagnetic pulse and the CNT parameters.

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Conflict of interest

The authors declare that they have no conflict of interest.

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