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Transportation of unipolar electromagnetic pulses in a coaxial waveguide

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The possibility of efficient passage of a unipolar pulse through a coaxial waveguide has been confirmed. During the demonstration experiment, it was shown that when the potential is excited on the waveguide plates by short pulses, the unipolarity of the pulses passing through the cable is maintained. The problem of the passage of a field of a uniformly moving charge through a waveguide is numerically simulated. It is shown that the main wave passes through the coaxial waveguide almost without loss. The pulse area remains non-zero not only inside the coaxial waveguide, but also near the outer boundary of the ideal waveguide.

Keywords: coaxial waveguide, titanium-sapphire laser, unipolar pulses, charge field diffraction.

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1. Introduction

Extremely short electromagnetic pulses find application in diagnostics and control of ultrafast processes in micro-objects (down to the atomic level), data transmission, and other fields [1,2]. The issues of both generation of such pulses and their transport to the object of interest remain relevant. Since a coaxial waveguide has no cutoff frequency for principal waves, waves of any frequency may propagate in it at the speed of light in vacuum [3]. This makes them promising for transport of unipolar electromagnetic pulses, which act on micro-objects unidirectionally (and thus more efficiently than bipolar ones) [4]. However, as far as we know, systematic studies into the transport of unipolar electromagnetic pulses have not been conducted yet.

The aim of the present study is, first, to verify the passage of unipolar pulses through a coaxial waveguide experimentally. The second goal is related to the fact that the field of an electric charge moving in a straight line at a constant velocity represents a unipolar pulse moving at the velocity of this charge together with it [5]. In this connection, the problem of detachment of a pulse part associated with the charge upon its entry into a coaxial waveguide and its subsequent propagation at the speed of light and exit from the waveguide is solved theoretically.

2. Experimental demonstration of retention of unipolarity in propagation of short unipolar pulses in a coaxial waveguide

A coaxial waveguide consists of two metal cylinders of different diameters. The smaller cylinder is centered inside the larger one. In radio engineering, the inner cylinder is often just a solid wire with a dielectric around it, which insulates it from the outer cylindrical sheath.

The passage of short pulses without loss of unipolarity may be demonstrated in a simple experiment: if a galvanic contact of the source of an electric pulse with the central conductor and the sheath of a cable is provided, it has the capacity to transmit short unipolar pulses. A high-speed photodiode was connected to coaxial cables of various lengths and illuminated by short pulses from a titanium-sapphire laser. Acting as a square-law detector recording the intensity of laser radiation, the photodiode became a source of short unipolar electric pulses. The other end of the cables was connected to the DC input of a high-speed digital oscilloscope with a 0–1 GHz frequency band. The oscilloscope recorded voltage as a function of time. The coaxial cables were imperfect and introduced distortion.

Figure 1, a shows example oscilloscope records obtained with the photodiode connected to the oscilloscope input directly (a) and via a cable 3 m in length (b). The half-amplitude pulse duration in the case of direct connection (panel (a)) is 300 ps. This corresponds to a pulse length in space of 9 cm. Estimating the pulse duration based on the specified photodiode bandwidth of 3 GHz, which is three times greater than the oscilloscope bandwidth, we find a value three times shorter than 100 ps, which corresponds to a pulse length in space of 3 cm. The cable length (3 m) exceeds this value by a factor of 100. This example demonstrates that the shape of pulses changes after passage through the cable. Their amplitude decreases (and duration increases), but pulses remains unipolar. According to the results of oscilloscope record processing, the electric area

$$S_E = \int \mathbf{E} dt,$$

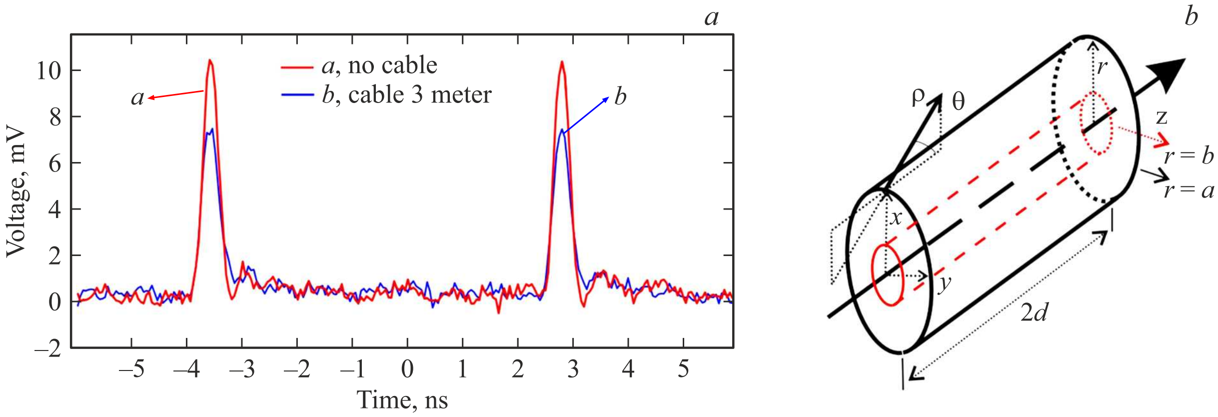

Figure 1. (*a*) Pulses from the photodiode connected to the oscilloscope input directly (curve *a*) and via a 3-meter coaxial cable (curve *b*). (*b*) Diagram of field passage through a coaxial waveguide; the arrangement of cylinders in a coaxial waveguide with cylindrical symmetry.

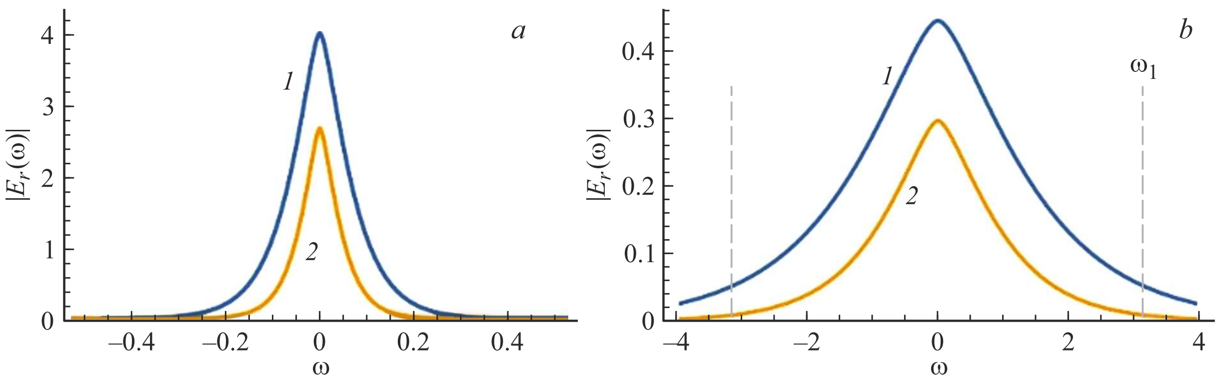

Figure 2. Frequency spectrum of a E field pulse for $z = z_0$ at two points along $r = b = 2$ and $a = 3$ (curves *1* and *2*) and for the velocity (*a*) and (*b*). The cutoff frequency for the chosen parameters is $\omega_1 \approx \pi$.

of a pulse passing through the cable (**E** is the electric field strength) remains virtually unchanged.

3. Charge field diffraction at the open end of a coaxial waveguide

In the second problem, the inner cylinder of a coaxial waveguide of a finite length was hollow, and the charge moved along its axis at a constant velocity. In view of axial symmetry of the problem, the charge field at the waveguide entrance and the scattered field have the same symmetry (Fig. 1, *b*). A uniformly moving charge produces one orbital magnetic field component:

$$H_{\varphi,in} = r^{-1}\Phi_{in}, \quad \tilde{H}_{\varphi,in}(T,r) = q\frac{V}{\eta r^2}(1+T^2)^{-3/2},$$

$$T = \frac{Vt - z}{\eta r}, \qquad \Phi_{in} = qe^{ik_V z}|\Omega|K_1(|\Omega|), \quad \Omega = \eta k_V r, \tag{1}$$

where $k_V = k/V_0$, $V_0 = V\sqrt{\varepsilon\mu}$, $\eta = \sqrt{1-V_0^2}$, $\varepsilon = 1$ is the permittivity of the transparent medium, $\mu = 1$, and $q = 1$ [5]. Here, $\tilde{H}_{\varphi,in}$ is the temporal field amplitude, $H_{\varphi,in}$ is its Fourier transform for spectral parameter $k = \omega\sqrt{\varepsilon\mu}$, and the system of units with a unit speed of light is used. The electric field has two components the amplitudes of which are expressed through the magnetic field amplitude:

$$E_r = -i\sqrt{\mu/\varepsilon}(kr)^{-1}\frac{\partial}{\partial z}(rH_\varphi),$$

$$E_z = i\sqrt{\mu/\varepsilon}(kr)^{-1}\frac{\partial}{\partial r}(rH_\varphi). \tag{2}$$

We write the boundary-value problem of diffraction for amplitude

$$\psi(z,r) = rH_\varphi = \Phi - \Phi_{in},$$

where $\Phi(z,r) = rH_{\varphi,tot}$ is the total magnetic field amplitude for a given k that is regular at $r \to 0$:

$$\left(\frac{\partial^2}{\partial z^2} + k^2\right)\phi + \left(\frac{\partial^2}{\partial r^2} - r^{-1}\frac{\partial}{\partial r}\right)\phi = 0. \tag{3}$$

The boundary condition on metal surfaces of the waveguide for an ideal metal is vanishing of the longitudinal

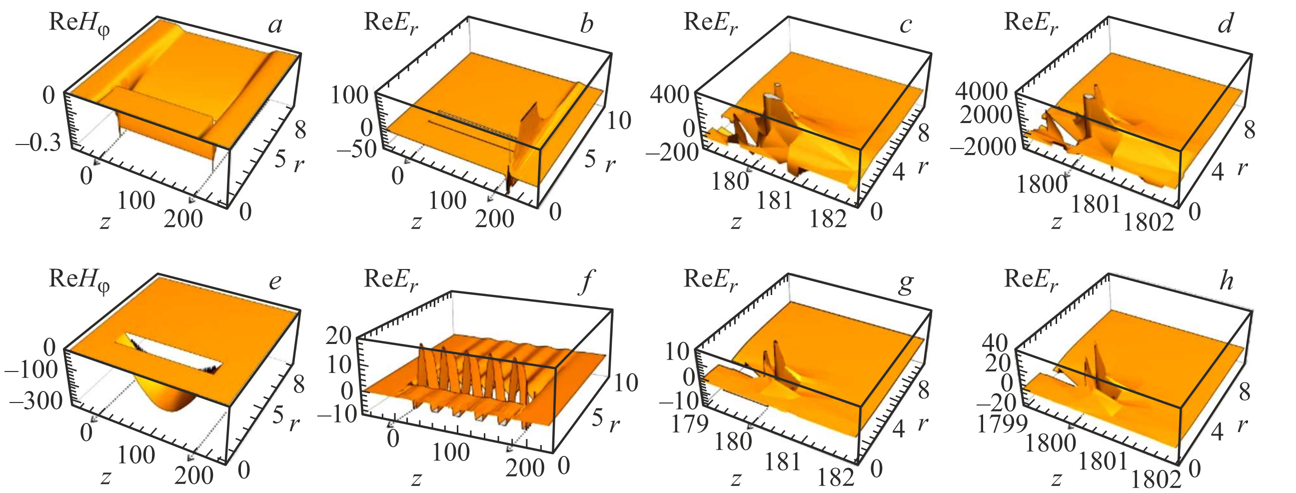

Figure 3. Distribution of scattered magnetic H_φ (a and e) and electric E_r fields for two waveguide lengths: $2d = 180$ and $2d = 1800$, (b–d) and(f–h). Arrows indicate the waveguide boundaries along z. The lines extending along z in (b) indicate the position of cylindrical plates of the coaxial waveguide. The top row (panels a–d) corresponds to $k = 0$ (electric field area distribution), while the bottom row (panels e–h) corresponds to the first resonant value $k = \pi/2d$.

component of the total electric field, $E_{z,tot} = 0$ (or, as follows from (2), $\partial\Phi/\partial r = 0$):

$$\frac{\partial}{\partial r}\psi(z,r) + \frac{\partial}{\partial r}\Phi_{in}(z,r) = 0,$$

$$r = a, b, \qquad z_{m0} < z < z_{m1}. \tag{4}$$

Condition (4) is also justified for a non-ideal metal at $k \to 0$, since the Drude permittivity of metal in this case is

$$\mathrm{Re}\varepsilon_m = 1 - \omega_p^2/\omega^2 \to \omega, \tag{5}$$

and reflection from the metal boundary may be regarded as reflection from free-electron plasma [6] without absorption in the skin layer for a thin layer of metal.

The derivation of the boundary condition at the waveguide edge was detailed in [7]. It is most convenient to use it for the magnetic field amplitude:

$$\psi(z,r) + \Phi_{in}(z,r) = 0, \quad z = z_{m0},\ z_{m1}, \quad r = a, b. \tag{5}$$

The electric field at the waveguide edge is, in the general case, irregular, $E_{r,tot} \sim |r-b|^{-1/2} \to \infty$; however, it is regular for select values of the spectral parameter corresponding to the resonant values of a finite waveguide ($k_n = \pi n/2d$), and so $E_{r,tot} \to \mathrm{const}$.

Incident charge field Φ_{in} satisfies Maxwell's equations, but does not satisfy Sommerfeld's conditions at $r \to \infty$, since a free charge does not radiate and the field is localized near it. However, a waveguide induces scattered field ψ the equation for which (3) should be supplemented with Sommerfeld's boundary conditions at $R(z,r) = \sqrt{z^2 + r^2} \to \infty$ (Fig. 1, b):

$$\frac{\partial}{\partial R}\psi = \mp k\psi, \qquad z \to \mp\infty, \qquad \frac{\partial}{\partial r}\psi = ik\psi,$$

$$z = \mathrm{const}, \qquad r \to \infty. \tag{6}$$

Boundary-value problem of diffraction (3)–(6) specifies completely the scattered field that propagates with the phase velocity of light: $k_z = k$. The phase velocity of the incident field is equal to the charge velocity: $k_z = k/V_0$.

The total field inside the waveguide may be expanded in a spectrum of longitudinal and transverse modes satisfying the boundary conditions:

$$\Phi = \sum_{n,l=0}^{\infty} e^{i\omega t + k_z z}\Phi_n^{(\kappa_1)}(z,r).$$

Here, $k_z = \sqrt{k^2 - \kappa_l^2}$ is the longitudinal wave number, and transverse wave number κ_l corresponds to the spectrum of zeros of the Bessel and Neumann functions, specifying the cylindrical wave profile. Outside the waveguide, the field is expanded in a continuous spectrum of cylindrical waves, and harmonics with $\kappa_l > 0$, $l > 0$ decay inside the waveguide. Harmonics with $\kappa_0 = 0$ correspond to the principal wave with transverse polarization ($E_z = 0$). Waves at spectrum point $k = \kappa_l$, $l > 0$ correspond to waves with longitudinal polarization ($E_z \neq 0$).

The depth of penetration of E-waves into the waveguide is determined by the spectrum of incident radiation. Figure 2 shows the charge field spectrum for two values of charge velocity. It can be seen that the spectrum width is small at $V = 0.1$, and high-frequency harmonics decay over length $a/10$. If the waveguide radius is $a = 3$, this is much smaller than its length, $2d = 180$. In contrast, harmonics with resonant longitudinal modes $k = k_n$ for the principal wave are most sensitive to the part of the spectrum containing them.

A direct numerical solution of boundary-value problem (3)–(6) without the expansion in harmonics of longitudinal

$(k = k_n)$ and transverse $(k = \kappa_l)$ modes (as is commonly done when an analytical solution is obtained [7]) for different values of k from the charge field spectrum demonstrates that the transport of a pulse with a non-zero electric field area $(k = 0)$ is possible, and it is concentrated near the output aperture of the coaxial waveguide not only at $b < r < a$, but also outside the waveguide at (Fig. 3, *b*, where the field magnitude at $k = 0$ is small for $0 < z < 2d$ compared to the field magnitude at $z > 2d$).

The pulse area decreases over distances on the order of the aperture width of the coaxial waveguide, $z - 2d \geq a - b$. This quantity does not depend on the waveguide length, but a 10-fold increase in the waveguide length leads to an order-of-magnitude increase in the pulse area at the same distance from the output (see Figs. 3, *c* and *d*). The magnitude of magnetic field inside the waveguide for the resonant mode (Fig. 3, *e*) is large compared to the peripheral regions and oscillates, which is associated with interference of two oppositely directed principal waves reflected from the waveguide ends. Faster oscillations of the electric field inside the waveguide (Fig. 3, *f*) are attributable to the compensation of oscillations of the charge field moving at velocity $V_0 = 0.1$, which results in variation of the field phase at rate $k_V = k_1/V_0$.

Conclusion

The possibility of efficient long-distance transmission of a pulse with a non-zero electric field area through a coaxial waveguide was demonstrated experimentally and theoretically.

The amplitudes of magnetic and electric fields at the waveguide output were determined via numerical calculation of the boundary-value problem of diffraction of the field of a uniformly moving charge. It was demonstrated that the fraction of waves with frequencies exceeding the cutoff frequencies for non-principal waves is small at low charge velocities. As the velocity increases, the fraction of the field with subcritical frequencies (at which propagation is possible only in the form of principal waves) decreases. As the waveguide length increases, the number of longitudinal resonant modes falling within the charge field spectrum increases, thus enhancing the efficiency of pulse area transport.

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Conflict of interest

The authors declare that they have no conflict of interest.

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