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On the modern understanding of the„EPR paradox“

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The concepts of quantum nonlocality, quantum superposition, the absence of a priori values of observables before measurement, and the ultimate precision of quantum measurements are analyzed in connection with the „EPR paradox“ and subsequent studies stimulated by it. The relationship between the Heisenberg uncertainty principle and the precision of measurements is discussed in connection with the disappearance of interference in „which way?“ schemes and the „EPR paradox“İt is shown that the absence of a priori values of observables before measurement and the existence of quantum nonlocality - in the sense of instantaneous reduction of quantum states of distant objects and the connection between their measured values - are beyond doubt.

Keywords: quantum nonlocality, quantum superposition, quantum entangled states, ultimate precision of quantum measurements, „which way?“ interference schemes.

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1. Introduction

The fundamental work of Einstein, Podolsky, and Rosen [1–3] has set the trend of research attempts at understanding the quantum world for an entire era. However, as often happens, a great breakthrough was rooted in an ordinary incident. Albert Einstein found Werner Heisenberg's uncertainty principle suspicious. The latter interpreted it as the impossibility of simultaneous accurate measurements of canonically conjugate observables. But how are Heisenberg's quantum uncertainties related to the precision of measurements? After all, quantum uncertainty is inherent to the object under examination, while accuracy is a characteristic of the measurer itself. Heisenberg saw the reason for this connection in the force impact of a macroscopic detector on a quantum system. A well-known example is the so-called „microscope“ proposed by Heisenberg himself [4]: single electrons in a double-slit interference circuit are illuminated from the side by photons, the scattering of which allows one to determine the slit through which a scattering electron has passed. According to Heisenberg, interference vanishes due to the fact that the momentum transferred by a photon to an electron knocks it off the „correct“ interference path. This implies that by measuring the coordinate, an observer exerts a force impact on the momentum, thus preventing the determination of its value undistorted by the measurement.

This line of argument immediately raises two questions. First, is measurement, like weighing potatoes at the market, always associated with a force impact? After all, astronomical observations, in line with Einstein's famous quote, are unlikely to threaten the existence of the Moon and the stars. However, this refers to macroscopic objects rather than quantum ones. Second, if an observer knows quantitatively how the object was distorted by measurement, the a priori undistorted value may be reconstructed by solving the inverse problem. This is the subject of the theory of measuring and computing systems [5,6], which also operates within the framework of classical physics. And what about the quantum world? The answer to the first question is obviously negative there. Let us discuss it in more detail.

2. On quantum measurements without a force impact

The suppression of interference of single quanta observed when the slit of a double-slit circuit through which a quantum has passed becomes known (experiments of the „which way?“ type [7]) is one of the most intriguing phenomena of the quantum world. Although this result is well established, the mechanism of suppression remains unclear. New experimental designs and demonstrations of this mysterious effect continue to attract attention [8].

The history of „which way?“ experiments goes back to the first observation of single photon interference in the Young double-slit interferometer [9]. In the words of R. Feynman [10], „I will take just this one experiment, which has been designed to contain all of the mystery of quantum mechanics, to put you up against the paradoxes and mysteries and peculiarities of nature one hundred per cent. Any other situation in quantum mechanics, it turns out, can always be explained by saying, „You remember the case of the experiment with the two holes? It's the same thing „. It does contain the general mystery. “

It is possible to find out through which slit a particle (e.g., an electron) has passed using the already mentioned „Heisenberg microscope“ [4]. Heisenberg himself attributed the vanishing of interference to the impact of a measurer on the object of measurement (interfering or, more precisely, non-interfering electrons). But how important is the force impact of the measurer? Or the very information about the particle's trajectory is key?

The experiment may be updated by recording the passage of photons through the same Young interferometer and installing mutually orthogonal crossed polarizers in the slits [11–13]. The polarization state of the detected photon then allows one to determine the slit through which it has passed. The polarization meter exerts no direct influence on photons, but the lack of interference is easily attributable to mutually orthogonal polarizations of detected particles, which naturally do not interfere. The very installation of polarizers in the slits also alters the state of photons, affecting the measured object.

To minimize the impact of the measurer, the authors of [7] observed interference of individual rubidium atoms. Standing light waves produced alternative trajectories. Microwave pulses did not distort mechanically the trajectories of heavy rubidium atoms, but provided an opportunity to clarify these trajectories. Irradiation marked the state of the resonant transition of a rubidium atom. Different trajectories corresponded to different states of this transition. Interference was suppressed. The force impact of microwave radiation on atoms is negligible, and the authors of [7] concluded that the main reason for the vanishing of interference is not the effect of the measurer, but the information obtained as a result of measurement. This is an important conclusion, but, as in the case of polarizers in slits, one may argue that the reason for interference suppression is in the alteration of the state of atoms by the measurer with the consequence that atoms in different states stop interfering.

An experimental design free from this drawback was proposed in [14]. Interfering or non-interfering photons do not interact with the measurer in any way, since their trajectories are clarified by detecting entangled photons that are not associated with the interferometer. However, if one observer has measured the polarization state of his photon, the trajectory of the second photon becomes known. Therefore, there should be no interference.

The experiment may be even simpler. Let us direct one particle from a pair of spatially correlated entangled particles to two slits. Will there be interference? Naturally, no. Why? Because the second particle of the pair allows one to determine easily through which slit the first particle has passed. Notably, without any force impact. After all, the state vector collapses instantaneously (in the very least, at a superluminal speed [15]) upon detection of the first particle, and, according to special relativity, a superluminal force impact is impossible.

What conclusion can be inferred from these results? The following rule is confirmed once again: either the trajectories are unknown, or there is no interference. It would be interesting to gain an insight into the nature of this law. However, since the answer to this fundamental question is unknown, one can only marvel at the mystery of quantum phenomena.

Thus, quantum measurements do not necessarily have to be accompanied by the types of interactions known in physics. One can safely assume that the reason for interference suppression is not the effect of the measurer on quantum particles, but the potential to obtain information about the path taken by each specific particle.

Notably, one is free to measure, e.g., the coordinate of one particle of an entangled pair and the momentum of the second particle with an accuracy that depends only on the resolution of measuring instruments and is completely unrelated to quantum uncertainties. Another example of such a measurement is provided in [16,17]. The authors of these studies proved that the momentum and coordinate of a photon may be measured with a product of confidence intervals being significantly smaller than the one dictated by the uncertainty principle.

What are the implications of all this? First, Einstein was absolutely right in his suspicions about Heisenberg's interpretation of the uncertainty principle. But does this undermine the foundations of quantum mechanics? Not in the slightest, since the following occurs when one detects entangled particles. The measurement of, e.g., the momentum of the first particle causes a reduction of the quantum state of the second one, and it acquires a specific momentum corresponding to the recorded momentum of the first particle. According to the uncertainty principle, the uncertainty of coordinate of the second particle widens. This coordinate may be measured with arbitrary accuracy, but this measurement will not be informative, since we learn just one of the possible values within a vast confidence interval. Thus, such an experiment does not pose any contradictions to quantum concepts.

At the same time, the question of a predetermined outcome of a quantum measurement and the existence of Einstein's „hidden variables" cannot be resolved this way. The Bell's theorem [18] coupled with subsequent thought and physical experiments may provide an answer to it.

The Bell's theorem and experiments verifying it have clearly proven the inconsistency of the so-called „local realism," which was confirmed by the awarding of the Nobel Prize in Physics in 2022 [19]. What is local realism? It is local in the sense that interaction between distant objects may only be mediated by the types of interaction known in physics. Realism implies the a priori existence of certain values of measured quantities before the moment of measurement. If the existence of quantum uncertainty is admitted, it is attributed, in the vein of classical statistical physics, to a lack of knowledge about the quantum object. In other words, there is something hidden that determines the outcome of the measurement. The non-violation of various types of Bell inequalities (BIs) is the criterion of validity of this hypothesis [20–24]. However, two reasons for violations are possible here: nonlocality and/or quantum superposition in the sense of the lack of a priori values of observables. It follows from the results of a number of other experiments [25,26] that both factors are likely to be in play. John Bell [18] derived his inequalities based on two premises: the reality of existence of values of the measured quantities before the measurement and the independence of a measurement from the adjustments made to a distant detector. To quote him directly, „*the result of the measurements of one system be unaffected by operations on a distant system with which it has interacted in the past*“.[1] When understood as the lack of a measured property before measurement, superposition implies the denial of the first premise, while nonlocality implies the denial of the second one.

3. Bell's inequality in the Clauser–Horne–Shimony–Holt form

Let us briefly recall the derivation of the BI in the Clauser–Horne–Shimony–Holt (CHSH) form [27–29] with emphasis on the aspects of interest to us. In the simplest case [22,30], we take arithmetic formula $s_i = a_i(b_i + b'_i) + a'_i(b_i - b'_i)$, where $a_i^{(\prime)}, b_i^{(\prime)} = \pm 1$ are the values of dichotomic random variables obtained as a result of four measurements, which are combined into a single „4-dimensional“ one with index i. Quantity s_i may assume only two values: $+2$ and -2, since one of the expressions in brackets will always be equal to 0. This yields equality $|s_i| = 2$ and, in view of multiplication by probabilities performed to obtain average values, inequality $|s_i| \leq 2$.

It should be emphasized that $a_i^{(\prime)}$, $b_i^{(\prime)}$ are independent of each other, since inequality $|s_i| \leq 2$ might be violated if there were $s_i = a_i(b_{i(a_i)} + b'_{i(a_i)}) + a'_i(b_{i(a'_i)} - b'_{i(a'_i)})$ with $b_{i(a_i)}^{(\prime)} \neq b_{i(a'_i)}^{(\prime)}$. Thus, the unconditional fulfilment of inequality $|s_i| \leq 2$ implicitly requires the values of a random variable to satisfy a number of conditions simultaneously: these values should exist jointly in each 4-dimensional measurement, and the number of values obtained must be equal to 4, since this number becomes larger at $b_{i(a_i)}^{(\prime)} \neq b_{i(a'_i)}^{(\prime)}$, and the inequality is violated.

The joint existence of values is usually interpreted as „realism,“ while the $b_{i(a_i)}^{(\prime)} = b_{i(a'_i)}^{(\prime)}$ requirement is seen as a requirement of „locality“ of properties. Thus, physical requirements of locality and realism are sufficient for deriving this inequality. According to De Morgan's well-known logical rules, a violation of this inequality implies a violation of either locality, or realism, or both. This is the reason why a violation of local realism is often identified in physics papers as the „physical cause“ of violation of the corresponding inequalities, although a violation of inequalities does not follow logically from this violation of local realism; on the contrary, a violation of either locality, or realism, or both follows from a violation of inequalities.

[1] „the result of a measurement on one system [should] be unaffected by operations on a distant system with which it has interacted in the past.“

If the mentioned four-dimensional measurement is repeated a sufficiently large number of times ($N \gg 1$), the following inequality will be satisfied:

$$S_i = \left|\sum_{i=1}^{N} s_i\right| \leq 2N, \tag{1}$$

which yields the CHSH inequality:

$$|\langle AB\rangle + \langle AB'\rangle + \langle A'B\rangle - \langle A'B'\rangle| \leq 2. \tag{2}$$

It may also be deduced from the existence of four-dimensional classical probability distributions [31]. In certain cases, an equivalent inequality for probabilities is more convenient. Let us denote the possible correlations of values in each component of (1) as $(++)$, $(--)$, $(+-)$, and $(-+)$ and the number of times each of these options is observed in a series of N 4-dimensional measurements for values $a_i^{(\prime)}$ and $b_i^{(\prime)}$ of a pair of random variables $A^{(\prime)}$ and $B^{(\prime)}$ as $n^{++}_{a^{(\prime)}b^{(\prime)}}, n^{--}_{a^{(\prime)}b^{(\prime)}}$, $n^{+-}_{a^{(\prime)}b^{(\prime)}}$, and $n^{-+}_{a^{(\prime)}b^{(\prime)}}$. Then, $n^{++}_{a^{(\prime)}b^{(\prime)}} + n^{--}_{a^{(\prime)}b^{(\prime)}} + n^{+-}_{a^{(\prime)}b^{(\prime)}} + n^{-+}_{a^{(\prime)}b^{(\prime)}} = N$. Let us also denote the number of correlations and anti-correlations for the measured values of a pair of random variables $A^{(\prime)}$ and $B^{(\prime)}$ as $N^{cor}_{a^{(\prime)}b^{(\prime)}} \overset{\text{def}}{=} n^{++}_{a^{(\prime)}b^{(\prime)}} + n^{--}_{a^{(\prime)}b^{(\prime)}}$ and $N^{ant}_{a^{(\prime)}b^{(\prime)}} \overset{\text{def}}{=} n^{+-}_{a^{(\prime)}b^{(\prime)}} + n^{-+}_{a^{(\prime)}b^{(\prime)}}$. The moment of a pair of random variables $A^{(\prime)}$ and $B^{(\prime)}$ is defined as

$$\langle A^{(\prime)}B^{(\prime)}\rangle \overset{\text{def}}{=} \frac{\left(N^{cor}_{a^{(\prime)}b^{(\prime)}} - N^{ant}_{a^{(\prime)}b^{(\prime)}}\right)}{N} = P^{cor}_{A^{(\prime)}B^{(\prime)}} - P^{ant}_{A^{(\prime)}B^{(\prime)}}$$

for a representative sample (a sufficiently large N). Since the product of values $a_i^{(\prime)}b_i^{(\prime)}$ is always positive and equal to 1 for correlated pairs and is always equal to -1 for anticorrelated pairs,

$$\langle A^{(\prime)}B^{(\prime)}\rangle = \frac{\left(\sum_{i=1}^{N} a_i^{(\prime)}b_i^{(\prime)}\right)}{N} = P^{cor}_{A^{(\prime)}B^{(\prime)}} - P^{ant}_{A^{(\prime)}B^{(\prime)}}.$$

Since with a sufficiently large N

$$P^{cor}_{A^{(\prime)}B^{(\prime)}} = \frac{N^{cor}_{a^{(\prime)}b^{(\prime)}}}{N}, \quad P^{ant}_{A^{(\prime)}B^{(\prime)}} = \frac{N^{ant}_{a^{(\prime)}b^{(\prime)}}}{N},$$

we obtain $\langle A^{(\prime)}B^{(\prime)}\rangle = P^{cor}_{A^{(\prime)}B^{(\prime)}} - P^{ant}_{A^{(\prime)}B^{(\prime)}} = 2P^{cor}_{A^{(\prime)}B^{(\prime)}} - 1$, and (2) may be rewritten as

$$0 \leq P^{cor}_{AB} + P^{cor}_{AB'} + P^{cor}_{A'B} - P^{cor}_{A'B'} \leq 2. \tag{3}$$

As was noted above, inequalities are derived from the simultaneous fulfillment of a number of conditions: the joint existence of values in each 4-dimensional measurement and the four-dimensionality of quantities. In physical terms, the first requirement implies that these values existed independently of the acts of measurement and, consequently, before these measurements. After all, measurements are not performed simultaneously in experiments with entangled pairs of particles. This is commonly called the „realism of properties“ (RP). The four-dimensionality of quantities physically implies the independence of measurements and, consequently, their locality (L). Therefore, it can be argued that Bell's inequalities in the CHSH form follow from RP and locality: $(\text{RP} \wedge \text{L}) \Rightarrow$ BIs-CHSH. According to De Morgan's rules, a violation of inequalities then leads to a violation of either the „realism of properties," or locality, or both: $\neg$BIs-CHSH $\Rightarrow (\neg\text{RP} \vee \neg\text{L})$. Importantly, the derivation of these inequalities does not necessitate the violation of locality when they are violated. Thus, if the RP requirement cannot be satisfied in a physical theory for some objective reasons, an explanation for the results of physical experiments with violation of inequalities is already found, and there is no need to look for such an explanation in nonlocality. Nonlocality is fitting only if its mechanisms are clearly explained in the theory. It should not be postulated as some „spooky action," although it manifests itself in an instantaneous reduction of the wave function upon measurement. But is it actually action? Hardly, since it is instantaneous (i.e., according to special relativity, a force impact is infeasible). And what about information action? On the one hand, according to the no-communication theorem, the transmission of information at superluminal speeds is also impossible [32]. On the other hand, the wave function is reduced instantly in an entangled pair of particles. But what is the status of a wave function? Is it a part of objective reality or just a calculational concept?

As for quantum superposition, its manifestations in pure form and a proof of its existence are feasible, apparently, only in a purely local setting. How to accomplish this? The authors of [26] proposed simultaneous observations of two quantum effects: suppression of mutual correlation of photons [33,34] and preparation of quantum squeezed states [35,36]. The effects themselves are localized in a single point in space, which allows one to exclude any influence of nonlocality. Manifestations of simultaneously observed effects are impossible if mixed photons have the same phase difference (sine and cosine of the phase difference). Thus, this phase difference has no definite value and is in a state of quantum superposition. It beats reminding that the lack of certain values of measured quantities before the moment of measurement (i.e., the presence of quantum superposition) is one of the foundational concepts of the Copenhagen interpretation. Other experimental evidence of this fact may be challenged by invoking supposedly unknown types of nonlocal interactions that are not bounded by the light cone and, accordingly, the speed of light. These are the various types of nonlocal theories [37] that explain in purely formal language both the violation of BIs and numerous quantum paradoxes. For example, the interference of individual photons in a Young double-slit experiment may be interpreted as nonlocal „knowledge" of a photon passing through one slit about the existence of the other. The experiment proposed in [26] provides compelling evidence to refute such assertions. No kind of nonlocal „knowledge" of a photon about its future fate may explain the expected rotation of the axes of an uncertainty body or the squeezing ellipse in the phase plane [26]. Thus, the lack of a definite value of the phase difference of individual photons cannot in any way be challenged by any hypothesis of nonlocal „realism" in the sense of a priori existence of specific values of measured quantities. This narrows significantly the range of possible interpretations of quantum theory, but does bot reduce it to just the Copenhagen one. The relational paradigm may also provide an adequate explanation [38].

4. Conclusion

Although the current explanation of the EPR „paradox" is trivial in nature, its significance in the development of quantum physics cannot be overestimated. It was this highly cited work that initiated a whole series of further studies attempting to find an adequate interpretation of quantum mechanics. A generally accepted result has not been presented yet. And if the existence of quantum superposition in the sense of a lack of certain a priori values of observables may be considered proven, the objective existence of quantum nonlocality in the sense of instantaneous reduction of the wave function of spatially distant objects depends precisely on the objectivity of existence of the wave function itself. If it does exist in reality, it is unreasonable to doubt quantum nonlocality. And if the wave function is just a computational technique similar to a slide rule, then there is no such nonlocality? Let us consider a simple example. If a measurement of the energy or momentum (or angular momentum) of one of the particles entangled in these parameters yields a statistical result that is not known in advance, the outcome of measurement for the second particle is predetermined and depends unambiguously on the first one. How did the second distant particle „learn" of it if it did not communicate with the first particle through any known types of interaction? It is clear that this all comes down to the observation of conservation laws, which, in turn, are derived from the homogeneity and isotropy of spacetime, as is demonstrated by Noether's theorem [39]. Thus, it is spacetime itself that is responsible for quantum nonlocality, and the existence of it should not be doubted.

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Conflict of interest

The author declares that he has no conflict of interest.

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