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The influence of the polarity of half-cycle pulses on the dynamics of microresonators in a three-level medium

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Optics of unipolar half-cycle light pulses has been a rapidly developing area of modern physics in recent years. Such pulses have various interesting applications, since they allow ultrafast control of the properties of a medium and ultrafast attosecond switching of the state of a medium. When such pulses collide in a medium, a spatial distribution of the population difference of the energy levels of the medium arises, which is a dynamic microresonator with Bragg-like mirrors in the form of lattices of atomic populations. In this paper, based on the numerical solution of the system of material equations for the density matrix of a three-level medium together with the wave equation, the dynamics of microresonators in collisions of half-cycle attosecond pulses in a medium is studied depending on the polarity of the pulses. It is shown that the shape of the induced microresonators differs significantly from each other when the colliding pulses have the same and opposite polarities. It is demonstrated that this dynamics is qualitatively similar to the dynamics of these structures in a two-level medium, previously studied by the authors.

Keywords: extremely short pulses, attosecond pulses, dynamic microresonators.

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Introduction

Obtaining short pulses of attosecond duration is relevant in today's physics, since such pulses are used to control the movement of electrons in atoms, molecules, solids, and other applications [1–5]. But such pulses consist of several half-waves of electric field strength. Pulses of extremely short duration are required to study the dynamics of faster processes occurring at times less than the period of electron rotation along the electron's orbit in an atom. Pulses of extremely short duration in a given spectral range consist of only one half-wave of electric field strength and are obtained if all but one field's half-wave is removed from the original multi-cycle pulse. The obtained pulse is called a unipolar or half-cycle pulse, since it consists of only one field half-wave [6].

For half-cycle pulses, their important characteristic — the electrical area of the pulse may be different from zero. It is defined as an integral of the electric field strength $\boldsymbol{E}$ over time t at a given point of space $\boldsymbol{r}$ [7,8]:

$$\boldsymbol{S}_E(\boldsymbol{r}) = \int \boldsymbol{E}(\boldsymbol{r}, t)dt. \quad (1)$$

The electric area has the meaning of a mechanical pulse transmitted to a charged particle, it determines the degree of excitation and ionization of a quantum system when the pulse duration is shorter than the Bohr period of an electron in an atom [9–12]. As a result of the rapidly transmitted pulse to the system in one direction, half-cycle pulses can be used in various applications of ultrafast optics. [6,13].

In recent years, nonlinear optics of unipolar pulses has become a separate extensively developing field of modern physics. The unipolar and quasi-unipolar pulses, similar in shape with the unipolar pulses, with a distinct field strength half-wave and trailing edge of the opposite polarity may be obtained under fast deceleration of electrons in various targets [14–18], as a result of non-linear processes occurring in plasma [19], Fourier synthesis of spectral harmonics of a broadband supercontinuum [20], under non-linear interaction of femtosecond pulses with nanocrystals [21] and magnetic hysteresis media [22], as well as solitons [23], due to super-radiance of polarization pulse in a heterogeneous medium and also using other methods [24,25]. Recent results in obtaining and applying unipolar, half-cycle pulses are summarized in papers [26–30], in chapter in the collective monograph [31] and the cited literature.

One of the worth-mentioning applications of half-cycle pulses is the possibility, recently predicted by the authors, of light-inducing and control of dynamic microresonators (DM) that occur when the half-cycle pulses collide in a medium [32–35]. If extremely short pulses collide, the population difference in the overlap region in the medium may have almost constant value. Outside of this region, the population difference can either change its value stepwise and then have a different constant value, or a quasi-periodic grating of atomic populations may be formed — Bragg-like mirrors. In this case, this structure is a DM, the parameters of which can be changed with an increase in the number of collisions between pulses.

The physical mechanism of formation of such structures is the interference of the electric cross-areas of incident pulses [36–38]. In stronger fields, the coherent interaction of incident pulses with the medium plays an essential role in their formation [39]. It occurs when the pulse duration and time intervals between them are shorter than the relaxation time of the medium polarizationT_2 [40]. In this case, the incident pulses induce polarization waves traveling towards each other, existing during the phase memory time T_2. Each subsequent pulse, interacting with these polarization waves, creates a population difference grating in the medium, which plays the role of Bragg-like mirrors in DM. Formation of such structures is described in details in [41]. Therefore, we will not dwell on this issue below. Such structures may attract interest in terms of implementation of an attosecond optical switching mechanism [42], in physics of spatiotemporal photonic crystals [43], integration and differentiation of the femtosecond pulses envelope [44], creation of optical memory systems [45–47] and other applications of super-fast optics [41].

In early studies, the dynamics of DM was studied analytically in the low-field approximation for a multilevel medium. Numerical calculations were carried out mainly for a two-level medium [37–39]. Numerical modeling performed for a three-level medium revealed a qualitative matching of the system dynamics for a two-level medium [49]. In [39], a detailed analysis of the dynamics of DM in low-field approximation using perturbation theory was carried out. The results of numerical calculations also demonstrated consistency with the results of analysis within the framework of this approach.

Numerical analysis for the two-level medium was carried out in high fields [39]. Parameters of colliding pulses were selected so as to act like 2π- or 4π-pulses of self-induced transparency (SIT) on the two-level medium. At that, dynamics of the system significantly depended on the pulses polarity. If the colliding pulses had the same polarity, then the medium was practically not excited after the pulse had passed through it, which is natural for SIT pulses.

If the colliding pulses had the opposite polarity, then the DM emerged only in the pulse overlapping region [39]. In this case, the field strength at the center of the medium is always zero upon collision, therefore, at this point the medium was completely in the ground state. And at the edges there appeared a grating of population differences, which had only a few spatial periods. With the growth of collisions between the pulses the spatial frequency of these gratings increased. Apparently, this feature — DM localization only near the pulses overlapping region — is quite typical for collision of opposite polarity SIT pulses. Previously, a similar feature was observed in the collision of single-cycle pulses consisting of two half-waves of opposite polarity, each of which acted as 2π- pulse SIT in a two-level medium [35].

However, when powerful single-cycle and subcycle pulses are acting on real multilevel media, it is possible to populate other levels of the medium, i.e. microresonators should occur at each resonance transition of the medium, as was also predicted by a simple analytical approach for the low-fields [37–39]. At the same time, the effect of polarity of colliding pulses on DM behavior in a multilevel medium has not been investigated.

In this paper, based on the numerical solution of the system of equations for the density matrix of a three-level medium, together with the wave equation for the electric field strength, the behavior of DM versus colliding pulses polarity and their amplitude was studied. 4 series of numerical computations in low-fields and high-fields were carried out. At that, the initially colliding pulses had both, the same polarity and opposite polarity. In both cases it was demonstrated that DM had different behavior with pulses of the same and opposite polarity. The occurrence of DM localized near the pulse overlapping region was demonstrated when pulses of opposite polarity collided at each resonant transition of a three-level medium. These results are qualitatively consistent with the results of computations for a two-level medium, thereby expanding the scope of their applicability.

Theoretical model

To study the dynamics of DM, a series of numerical computations was carried out, where a system of material equations for a three-level medium density matrix was used. It was solved numerically together with the wave equation for the electric field strength. This system of equations is as follows [50]

$$\frac{\partial}{\partial t}\rho_{21} = -\rho_{21}/T_{21} - i\omega_{12}\rho_{21} - i\frac{d_{12}}{\hbar}E(\rho_{22}-\rho_{11}) - i\frac{d_{13}}{\hbar}E\rho_{23} + i\frac{d_{23}}{\hbar}E\rho_{31}, \quad (2)$$

$$\frac{\partial}{\partial t}\rho_{32} = -\rho_{32}/T_{32} - i\omega_{32}\rho_{32} - i\frac{d_{23}}{\hbar}E(\rho_{33}-\rho_{22}) - i\frac{d_{12}}{\hbar}E\rho_{31} + i\frac{d_{13}}{\hbar}E\rho_{21}, \quad (3)$$

$$\frac{\partial}{\partial t}\rho_{31} = -\rho_{31}/T_{31} - i\omega_{31}\rho_{31} - i\frac{d_{13}}{\hbar}E(\rho_{33}-\rho_{11}) - i\frac{d_{12}}{\hbar}E\rho_{32} + i\frac{d_{23}}{\hbar}E\rho_{21}, \quad (4)$$

$$\frac{\partial}{\partial t}\rho_{11} = \frac{\rho_{22}}{T_{22}} + \frac{\rho_{33}}{T_{33}} + i\frac{d_{12}}{\hbar}E(\rho_{21}-\rho_{21}^*) - i\frac{d_{13}}{\hbar}E(\rho_{13}-\rho_{13}^*), \quad (5)$$

$$\frac{\partial}{\partial t}\rho_{22} = -\rho_{22}/T_{22} - i\frac{d_{12}}{\hbar}E(\rho_{21}-\rho_{21}^*) - i\frac{d_{23}}{\hbar}E(\rho_{23}-\rho_{23}^*), \quad (6)$$

$$\frac{\partial}{\partial t}\rho_{33} = -\frac{\rho_{33}}{T_{33}} + i\frac{d_{13}}{\hbar}E(\rho_{13}-\rho_{13}^*) + i\frac{d_{23}}{\hbar}E(\rho_{23}-\rho_{23}^*), \quad (7)$$

$$P(z,t) = 2N_0 d_{12}\mathrm{Re}\rho_{12}(z,t) + 2N_0 d_{13}\mathrm{Re}\rho_{13}(z,t), \quad (8)$$

$$\frac{\partial^2 E(z,t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E(z,t)}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P(z,t)}{\partial t^2}. \quad (9)$$

In this system of equations ω_{12}, ω_{32}, ω_{31} — frequencies of resonance transitions, $\hbar$ — reduced Planck constant, d_{12}, d_{13}, d_{23} — dipole moments of transitions. Variables ρ_{11}, ρ_{22}, ρ_{33} — populations for the 1-st, 2-d and 3-d condition

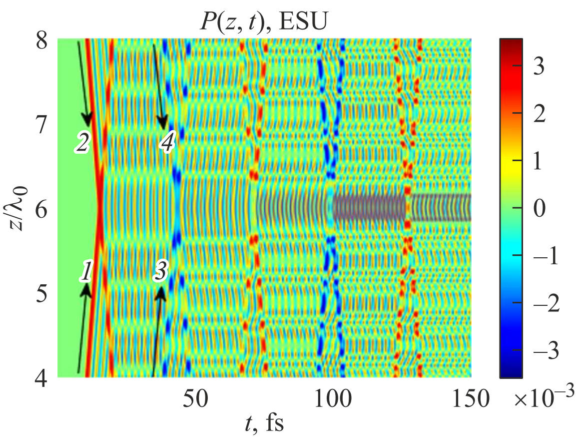

Figure 1. Spatiotemporal dynamics of polarization of a three-level medium $P(z,t)$.

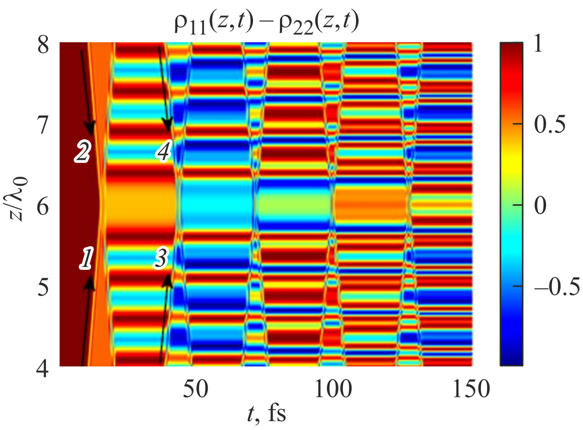

Figure 2. Spatiotemporal dynamics of population difference $\rho_{11}-\rho_{22}$ for a three-level medium.

of the medium, respectively, ρ_{21}, ρ_{32}, ρ_{31} — off-diagonal variables of the density matrix elements influencing the media polarization dynamics, T_{ik} — relaxation time. They are much longer than the duration of the processes under consideration and may be neglected here.

The medium was excited by a pair of counter half-cycle unipolar pulses of Gaussian shape:

$$E_1(z=0,t)=E_{01}e^{-\frac{(t-\Delta_1)^2}{\tau^2}}, \quad (10)$$

$$E_2(z=L,t)=E_{02}e^{-\frac{(t-\Delta_2)^2}{\tau^2}}. \quad (11)$$

Here $\Delta_1=\Delta_2=2.5\tau$ — delays between pulses. The delay values are selected in such a way that the pulses collide in the center of the medium. Zero boundary conditions were selected at the edge of the integration region. This made it possible to implement a sequence of colliding pulses — after the first collision the pulses left the medium, reflected from the boundaries of the integration region and returned to the medium again, colliding in it. 3D propagation of unipolar half-cycle pulses as demonstrated in [51], is expressed through a one-dimensional wave equation in coaxial waveguides.

For numerical computations, the following parameters were used, which remained unchanged. Only amplitudes of the incident pulses and their sign changed. Duration of two pulses was $\tau=580$ as. Three-level medium's parameters had the following values: transition frequency 1−2 $\omega_{12}=2.69\cdot10^{15}$ rad/s (respective transition wavelength $\lambda_{12}=\lambda_0=700$ nm), dipole moment of transition 1−2 $d_{12}=20$ D, transition frequency 1−3 $\omega_{13}=2.5\omega_{12}$, dipole moment of transition $d_{13}=1.5d_{12}$, $\omega_{23}=\omega_{13}-\omega_{12}$, dipole moment of transition 2−3 $d_{23}=0$, concentration of three-level particles $N_0=10^{14}\,\text{cm}^{-3}$, relaxation time $T_{1k}=1$ ns. Pulses (10) and (11) collided in the center of the medium, at a point with coordinate $z=z_c=6\lambda_0$.

Numerical simulation results

In the first series of numerical computations the pulses had amplitudes $E_{01}=E_{02}=1.8\cdot10^5$ ESU and the same polarity. The results of numerical computations of the spatiotemporal dynamics of the medium polarization and population differences for each resonance transition for these parameters are illustrated in Fig. 1−4. The first 4

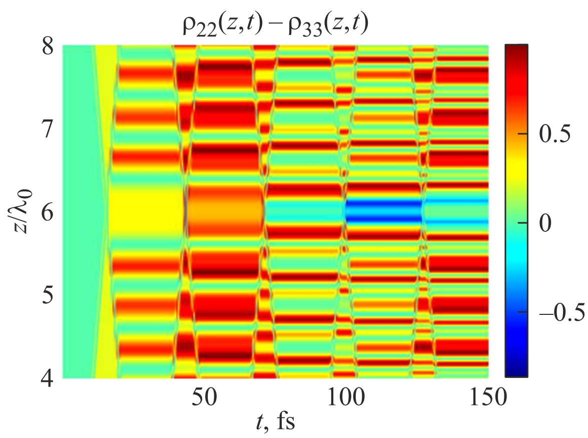

Figure 3. Spatiotemporal dynamics of population difference $\rho_{22}-\rho_{33}$ for a three-level medium.

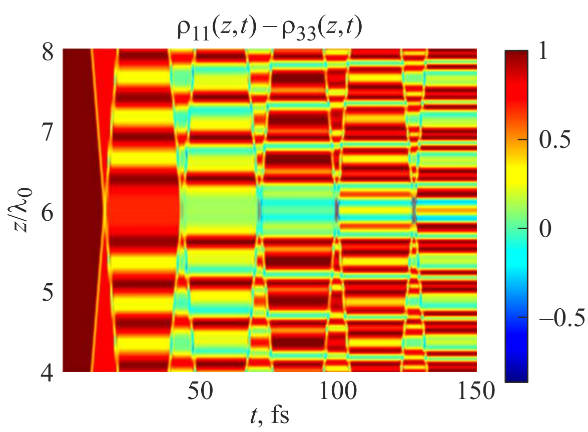

Figure 4. Spatiotemporal dynamics of population difference $\rho_{11}-\rho_{33}$ for a three-level medium.

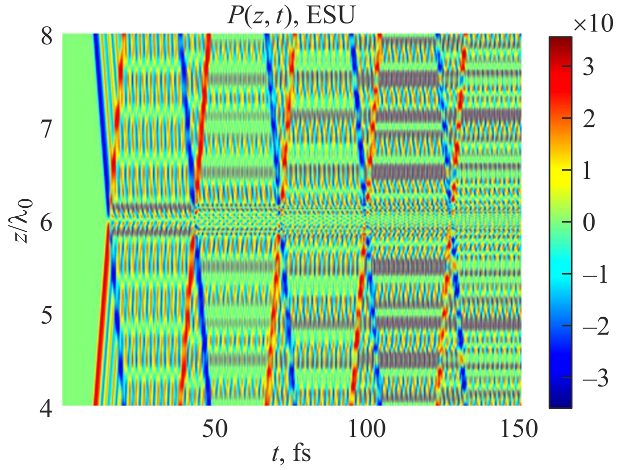

Figure 5. Spatiotemporal dynamics of polarization of a three-level medium $P(z, t)$.

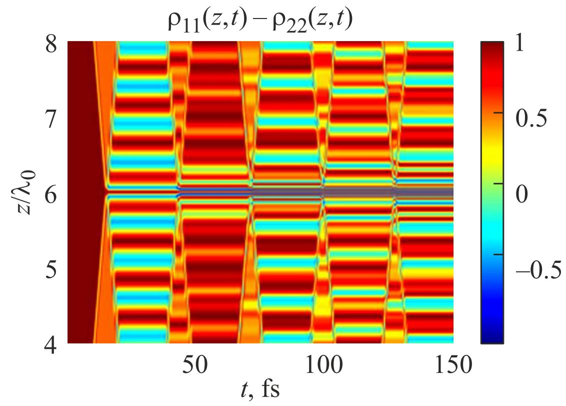

Figure 6. Spatiotemporal dynamics of population difference $\rho_{11}-\rho_{22}$ for a three-level medium.

pulses are shown in numbers, the directions of their propagation at the corresponding moments of time are shown by arrows in Fig. 1, 2. In total, these Figures and below show the dynamics of the system as a result of 5 collisions between pulses. The first collision occurs at a moment of time $t_1 = 15.2$ fs, second — at a moment of time $t_2 = 42.3$ fs, third — at a moment of time $t_3 = 71.7$ fs and etc.

As can be seen from Fig.1, complex polarization vibrations occur in the vicinity of the pulse overlap region of point $z_c = 6\lambda_0$. Meanwhile after first collision, as seen from Fig. 2–4, DM occurs at each transition — in the pulse overlap region the population difference has almost constant value, and a microresonator with Bragg mirrors occurs from the left and from the right. After each subsequent collision, the shape of the microresonators changes as a result of the interaction of incident pulses with fluctuations in the polarization of the medium induced by previous pulses. This dynamics is equivalent to behavior of DM studied earlier in the two-level [34,39] and three-level media when the pulses had similar polarity [45].

Now let's consider the case when the first two colliding pulses have opposite polarity, $E_{01} = -E_{02} = 1.8 \cdot 10^5$ ESU. The rest parameters remained the same. The results of analysis of polarization dynamics and population difference for this case are given in Fig. 5–8.

It can be seen that Bragg-like structures in the form of population difference gratings occur outside the overlap region in this case, as well. However, in the pulse overlap region in point $z_c = 6\lambda_0$ the medium is not excited, since the colliding pulses have opposite polarity, and the field strength in this point is always equal to zero.

However, near this point, a structure consisting of ultra-low-period gratings appears in the pulse overlap region and is localized near the pulse overlap region. With the growth of collisions between the pulses the spatial frequency of this structure is increased. The physical mechanism of its formation is associated with the formation of standing polarization waves localized in the pulse overlap region and their interaction with each subsequent pulse [35,39]. A similar structure, localized in the half-cycle pulses overlap region, also occurred when the half-cycle SIT pulses collided in a two-level medium when the pulses had different

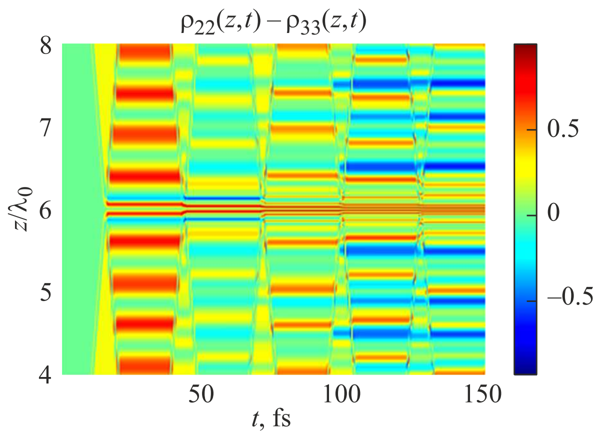

Figure 7. Spatiotemporal dynamics of population difference $\rho_{22}-\rho_{33}$ for a three-level medium.

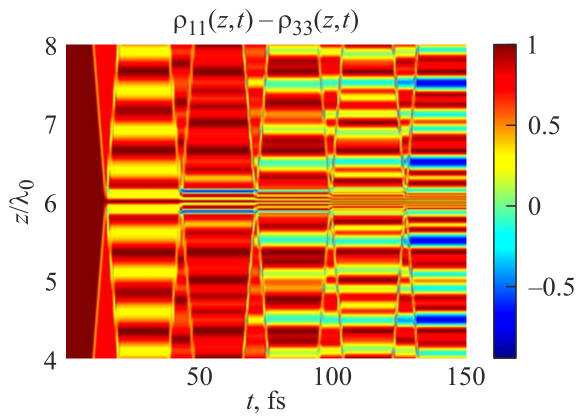

Figure 8. Spatiotemporal dynamics of population difference $\rho_{11}-\rho_{33}$ for a three-level medium.

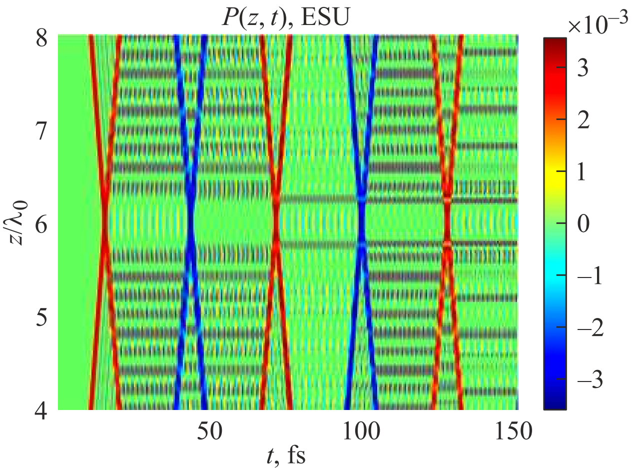

Figure 9. Spatiotemporal dynamics of polarization of a three-level medium $P(z,t)$.

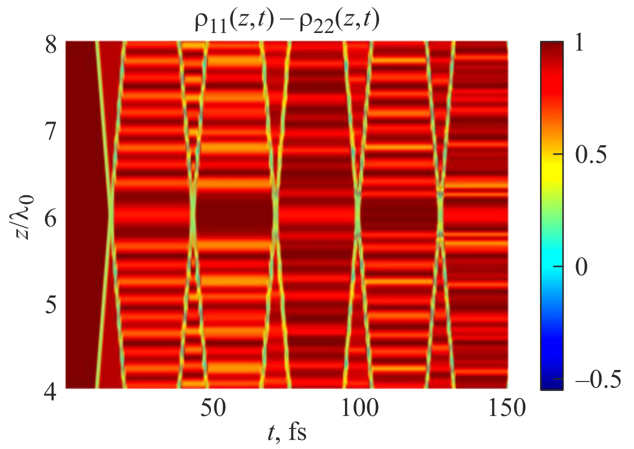

Figure 10. Spatiotemporal dynamics of population difference $\rho_{11}-\rho_{22}$ for a three-level medium.

polarities [39]. However, the pulses in question in our case had an amplitude smaller than SIT pulses.

In the third series of numerical computations the field amplitude was increased almost 2 times and had a value of $E_{01} = E_{02} = 3 \cdot 10^5$ ESU, while the pulses had similar polarity. The rest of parameters remained unchanged. The results of numerical computations of the spatiotemporal dynamics of polarization and population differences for each resonance transition of the medium for these parameters are illustrated in Fig. 9–12.

With selected parameters, after the passage of the first pair of pulses, the medium practically returned to the ground state, the population value of the ground state was about $\rho_{11} \simeq 0.9$. I.e. the incident pulses behaved practically like 2π-SIT pulses. In this case, as seen from Fig. 9–12, the medium practically didn't have any excitation, no any distinct DMs could be observed. This situation is completely equivalent to collision of half-cycle SIT pulses of the same polarity in a two-level medium. [39].

In the final series of numerical computations the field amplitude$E_{01} = -E_{02} = 3 \cdot 10^5$ ESU, and pulses had opposite polarity. The rest of parameters remained unchanged. The results of numerical computations of the spatiotemporal dynamics of polarization and population differences for each resonance transition of the medium for these parameters are illustrated in Fig. 13–16.

Fig. 14 illustrates formation of a localized DM near the pulses overlap region after the first collision. The rest of the medium, outside this region, was practically unexcited, similar structures occurred at the remaining transitions (Fig. 15, 16). After subsequent collisions, the spatial frequency of the gratings in the pulse overlap region goes up. This is equivalent to a case of the two-level medium [39].

The difference in dynamics occurs after the second and subsequent collisions — outside of the pulse overlap region the medium is no longer in the ground state, and Bragg-like population difference gratings of complex shape appear in this overlap region. They arise due to fluctuations in the residual polarization of the medium outside the pulse overlap region. These fluctuations exist at different

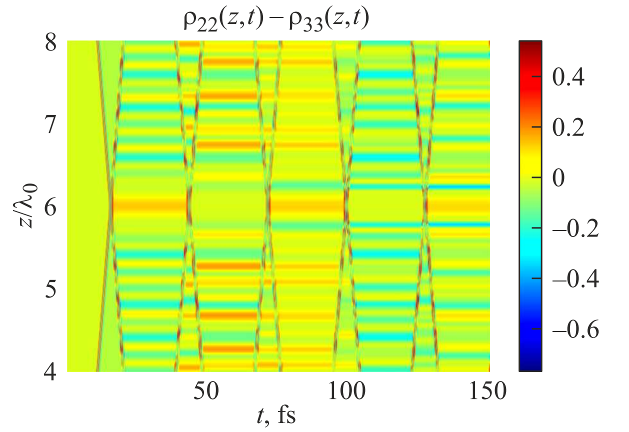

Figure 11. Spatiotemporal dynamics of population difference $\rho_{22}-\rho_{33}$ for a three-level medium.

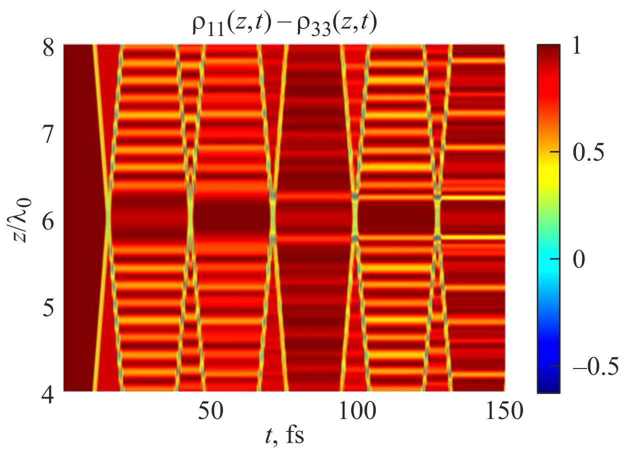

Figure 12. Spatiotemporal dynamics of population difference $\rho_{11}-\rho_{33}$ for a three-level medium.

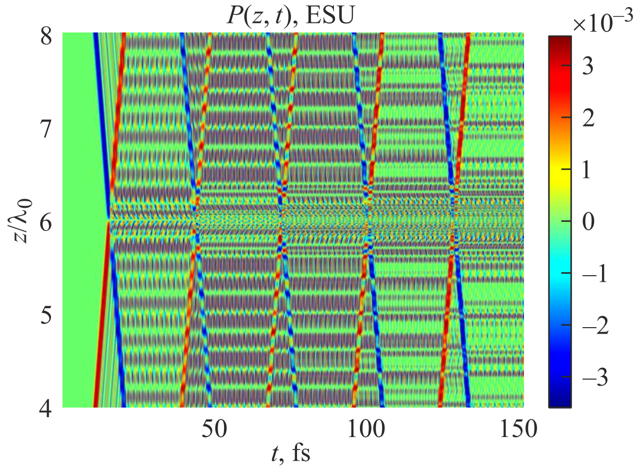

Figure 13. Spatiotemporal dynamics of polarization of a three-level medium $P(z, t)$.

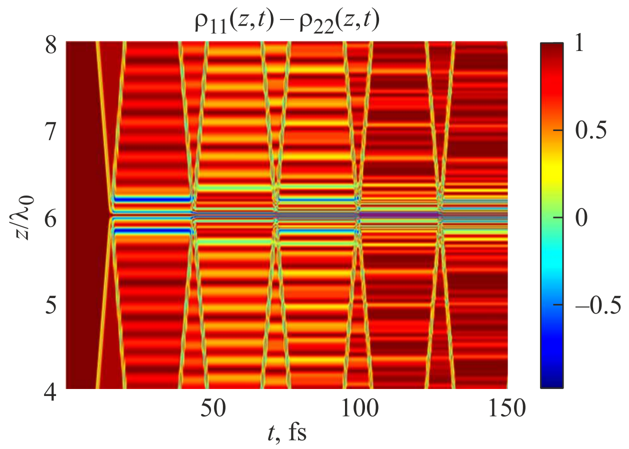

Figure 14. Spatiotemporal dynamics of population difference $\rho_{11}-\rho_{22}$ for a three-level medium.

resonance transitions and persist after the first collision, since the incident pulses behave somewhat different from SIT pulses.

It should be noted, that ionization of the medium is ignored in the suggested model. However, taking ionization into account, as demonstrated in paper [52], doesn't result in disappearance of the gratings. It is even possible to suppress the ionization of the medium with a certain delay between pulses. In computations, we also neglected the effect of the opposite polarity of the excitatory impulses' trailing edge on the system. If this trailing edge is too long and weak, its effect on the system may be neglected [53].

Conclusion

In this paper, the dynamics of microresonators as a function of the polarity of colliding pulses in a three-level medium is studied based on numerical computations. With a small amplitude of the colliding pulses and their identical polarity, a DM with Bragg-like mirrors in the form of population difference gratings appears at each transition of the medium. The population is practically constant in the pulse overlapping region.

If the colliding pulses had the opposite polarity, then the Bragg-like structure remained in the pulse overlapping region. However, in the pulse overlapping region a localized „quasi-resonator" or a grating-like structure emerged with the spatial frequency increasing with the rising number of pulses collisions. Such a localized structure has been observed previously in a two-level medium when pulses of opposite polarity collide [39].

With an almost twofold increase of the pulses amplitude, so that the effect of the incident pulses was close to the action of 2π- SIT pulses for the main transition of $1-2$ medium, the medium remained practically unexcited (with identical polarity of the colliding pulses) in both, the pulse overlapping region, and beyond this region. If the colliding pulses had the opposite polarity, then a localized DM occurred in the area of pulse overlapping after the first collision. Such behavior is equivalent to the earlier observed behavior in the two-level media [39]. After subsequent

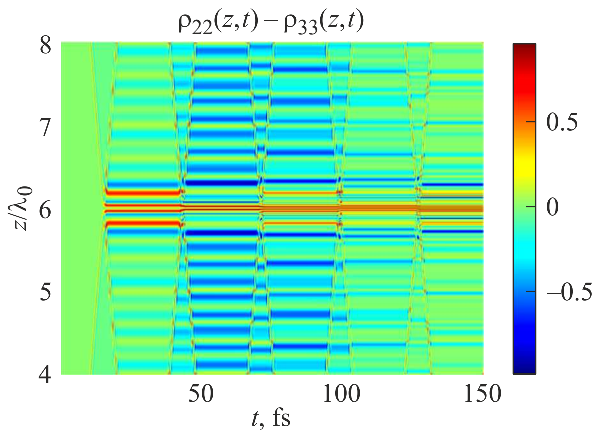

Figure 15. Spatiotemporal dynamics of population difference $\rho_{22}-\rho_{33}$ for a three-level medium.

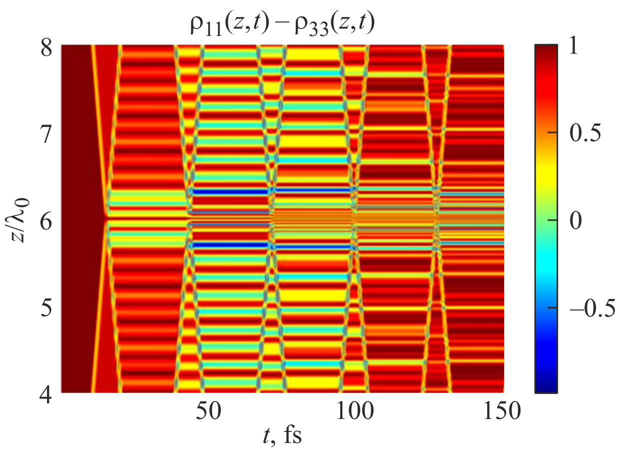

Figure 16. Spatiotemporal dynamics of population difference $\rho_{11}-\rho_{33}$ for a three-level medium.

collisions, the number of periods of the Bragg grating increased due to an increase in the spatial frequency of the structure.

Thus, the dynamics of induced DMs is qualitatively similar to that previously studied for a two-level medium [39]. It is essentially dependent on the polarity of initial half-cycle pulses. In case of pulses of opposite polarity, an ultra-small DM may emerge at each resonance transition of the medium, localized in the pulse overlapping region, the size of which is much smaller than the wavelength of the resonance transition.

The structures studied provide a new direction for research in attosecond physics, optics of unipolar pulses, and physics of space-time photon crystals. The studied phenomena can find their applications for ultrafast optical switching, ultrafast petahertz electronic systems [42] and the creation of optical memory systems based on atomic coherence [45–48].

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Conflict of interest

The authors declare that they have no conflict of interest.

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